1.0 Introduction

The aim of this Chapter is conveyed by its title. It is intended to be an introduction to, but be independent of, the subsequent Chapters which are written with the aim of providing sufficient knowledge to design a small wind turbine to the appropriate international standard, IEC 61400-2. This Chapter begins by considering the amount of energy in the wind that is available for conversion by a wind turbine. Section 1.2 introduces the basic technology of horizontal-axis wind turbines. The main turbine operating parameters are derived in Section 1.3. Typical power curves of both large, grid-connected turbines and smaller machines for remote area power systems are given and analysed in Section 1.4. The variation of wind speed with height and its effect on turbine performance is considered in Section 1.5. Section 1.6 discusses some effects of the inevitable turbulence in the wind. The mechanical and electrical layout of typical turbines is described in the following Section 1.8 describes some size issues for small turbines. The chapter concludes with suggestions for further reading and revision of fundamental fluid mechanics for those attempting to master the subsequent Chapters.

1.1 How Much Energy is in the Wind?

Since the primary purpose of a wind turbine is to convert the kinetic energy (KE) of the wind into (usually) electrical energy, it is useful to begin by considering the amount of energy available. The simple analysis that is used for this purpose also serves as a gentle introduction to the control volume (CV) analyses that will be used extensively in later Chapters, beginning with Chapter 2.

![Energy flow past a circular disk representing the blades.](image)

Suppose the wind is blowing from left to right with a wind speed of $U_0$. For simplicity we assume that the wind is steady (i.e. not varying in time) and uniform (i.e. not varying in position). Some effects of unsteadiness (turbulence) and non-uniformity will be considered later. The air has constant density, $\rho$, meaning that the flow, as are all flows considered in these Notes, is incompressible. At 20°C the density of air at sea level is nearly 1.2 kg/m³ and we can use this value in most situations. Most modern turbines are “horizontal-axis” wind turbines, designated as HAWTs, for which the axis of rotation of the blades is parallel or nearly parallel to the wind. HAWTs operate in a manner very similar to propellers. With the blade radius denoted by $R_1$ we can represent the turbine by a circular disk – the blade disk - whose area $A = \pi R_1^2$. We now determine the amount of kinetic energy in the air that passes the blade disk per unit time. (This is done in the absence of the blades, for reasons that will be explained shortly.) The unit of energy is the Joule, J, so the amount of energy that passes will
be in J/sec, which gives Watts, the unit of power. It is usually power output that concerns the designer and user of wind turbines.

The right side of Figure 1.1 shows an elemental volume of the airflow. Its exact shape is not critical. The volume is about to cross the imaginary line (when viewed side-on) in the wind that represents the blade disk. The volume of the element is the product of its area, \( \Delta A \), and length normal to the disk, \( \delta x \), so its mass is \( \rho \Delta A \delta x \) and its KE is \( \frac{1}{2} \rho \Delta A \delta x U_0^2 \). The time taken for this element to cross the blade disk, \( \delta t \), is given simply by \( \delta x = U_0 \delta t \). The contribution of the element to the total amount of KE that passes in \( \delta t \) is symbolised as \( \delta KE \), and is given by

\[
\delta(\Delta KE) = \frac{1}{2} \rho \Delta A U_0^3 \delta t
\]

so that summing over all elements of area that make up the disk gives the KE passing the disk as

\[
\delta(KE) = \frac{1}{2} \rho A U_0^3 \delta t
\]

For the mathematically inclined, this equation can now be taken formally to the limit as \( \delta t \to 0 \), to give

\[
\frac{d(KE)}{dt} = \dot{W} = \frac{1}{2} \rho A U_0^3
\]

or

\[
\frac{dKE}{dt} = \dot{W} = \frac{1}{2} \rho A U_0^3
\]

where we have used the common thermodynamic symbol for power, \( \dot{W} \), as the time rate change (derivative) of the energy. Equation (1.1) is extremely interesting because it suggests, as indeed is approximately the case, that the output power of any turbine depends on the cube of the wind speed. This simple and fundamental fact must never be forgotten. If this cubic dependence seems strange, it may be useful to remember that the windspeed determines both the amount of energy, proportional to \( U_0^2 \), and the mass carrying that energy through the blade disk per unit time, which is proportional to \( U_0 \). In practice the power output is never as great as that indicated by (1.1) because extraction of all the available KE would require the flow to be decelerated to rest. Furthermore we will see that a turbine cannot capture all the flow that would otherwise pass through the disk, even if it could decelerate this flow to rest, so that the KE calculation done in the absence of the blades will overestimate the actual energy capture. If we then include the effects of finite efficiency of the generator, and losses through the action of viscosity, is reasonable to assume that the power converted into electricity is about 40% of that given by (1.1).

It is important that the derivation of (1.1) be understood because the CV analyses that will be undertaken in later chapters extend the ideas and manipulations used to derive (1.1).

**Example 1.1**

Estimate the power extracted by a 5 m diameter wind turbine at a wind speed of 10 m/sec.
Answer

From Equation (1.1) and the discussion following the equation, assume

\[
\dot{W} = 0.4 \left( \frac{1}{2} \rho A U_0^3 \right)
\]

\[
= 0.4 \times 0.5 \times 1.2 \times \pi \times 2.5^2 \times 10^3
\]

\[
= 4.71 \times 10^3
\]

Now let us check the units: density is in kg/m³; area in m²; and (velocity)³ in m³/sec³. Their product gives kg m²/sec³, which are the units of Watts. Thus our estimate for the power output is 4,710 W or 4.71 kW.

Example 1.2

Sometimes resource surveys for turbine applications give the wind speed in terms of a “power density”, \( PD \), in W/m² which is equal to \( \dot{W}/A \) from Equation (1.1). If the power density is 100 W/m² what is the wind speed?

Answer

From (1.1)

\[
PD = \frac{1}{2} \rho U_0^3
\]

so that

\[
U_0 = \left( \frac{2 \times 100 / 1.2}{1} \right)^{1/3} = 5.5 \text{ m/sec.}
\]

1.2 Examples of Wind Turbines

Commercial turbines range in power output from a few watts to several megawatts. Nevertheless, the basic operating principles are the same for turbines of all sizes. For example, the restriction on output power given by the Lanchester-Betz limit, which will be derived in Section 2.5, is independent of size. On the other hand, there are operational issues that do depend on size; for example, starting performance and cut-in speed – the lowest wind speed at which power is extracted – are more important for small machines. There are several reasons for this, of which the most important are:
• Small wind turbines are usually located where the power is required rather than where the wind regime is most favourable, whereas wind farms containing large turbines are normally sited in windy areas,

• The generators of small turbines often cause a significant resistive torque that must be overcome aerodynamically before the blades will start turning. Furthermore, pitch control is rarely used on small wind turbines because of cost. (The precise definition of blade pitch will be given in Chapter 3.) Thus it is not possible to adjust the blade’s angle of attack to the prevailing wind conditions. This problem is particularly acute during starting. Starting and low wind speed performance is discussed in Chapter 6.

• many small turbines rely on furling for overspeed protection - see Chapter 7 - whereas large turbines usually have a brake on the high speed shaft (after the gearbox and before the generator).

A further major difference is that small turbines usually operate with varying shaft speed in an attempt to maintain maximum performance as the wind speed varies. Many large turbines run at constant speed as this allows the generator to maintain synchronicity with the grid.

Figure 1.2 shows a commercially available 5 kW wind turbine designed for use in a remote area power system (RAPS). We will look at its performance and its mechanical layout later in this Chapter. Each blade is 2.5 m long with a rotor diameter of 5.1 m. The turbine’s most noticeable feature, in comparison to the much larger turbine in Figure 1.3, is the tail fin which keeps the blades pointing into the wind. The tail fin or tail vane minimises the yaw angle, of the turbine, $\gamma$, defined as the angle between the turbine’s axis and the wind direction. Yaw reduces the power by a factor of approximately $\cos^3 \gamma$, and so is significant for even moderate values of $\gamma$. It is important, therefore, to minimise yaw. Yaw behaviour will be analysed in Chapter 7 where the stabilising effects of coning will be discussed. Coning occurs when the blades are deflected downwind by the aerodynamic forces on the blade. Alternatively, coning may be deliberately introduced in the way the blades are attached to the hub. At maximum power, the 5kW blades cone by around 120 mm, and it is estimated that this is sufficient to give as much yaw stability as the tail fin. Therefore, the tail fin is strictly necessary only to minimise yaw during starting. The stabilising effects of coning are similar to the roll stability given to most commercial aircraft by the dihedral of the wing; this is the inclination of the wing, so that the tip is higher than the root when the fuselage is level and there is no roll.

The blades are composed of aerofoil sections whose purpose is to produce a lift, which is the primary component of the torque about the turbine axis. The product of this torque and the blades’ angular velocity gives the power extracted from the wind. Blade design is covered in Chapter 3 and the aerodynamics of lift and drag in Chapter 4. It is not very clear from the photographs in Figures 1.2 and 1.3 that the blades are twisted, that is they are more “square on” near the tips, but it is more obvious that the blade width – more correctly the chord – decreases towards the tip. The complete definition of the twist and chord, along with the reasons why they both decrease with radius, will covered in the next Chapters, especially the last section of Chapter 4.

Figure 1.3 shows a large 5 MW wind turbine designed for use in a remote area power system (RAPS). The blades are 20.5 m long and the rotor diameter is 44 m. Most large turbines have three blades, partly because this number is held to be visually more appealing than the alternative of two blades. The blades are normally attached to the hub in a manner that allows them to pitch as the wind speed varies to maintain an appropriate angle of attack. Behind the blades is the nacelle, on top of which is an anemometer and wind vane. The former measures the wind speed for use in determining the appropriate pitch angle for the blades, which is usually adjusted every second or so, and the latter measures the yaw angle to determine whether the yaw motor must be activated to drive the blades into the wind. We will see in Section 1.7 that the mechanical layout of large wind turbines is much more complex than that of small machines.
In developing a wind farm, the first (and far from trivial) task is to determine the wind resource, which may vary significantly around the site because of the surface roughness – see Section 1.5 – and the topography. There remain at least two further important issues: noise and visual pollution. The latter is often addressed using sophisticated software that optimises the layout of the turbines to maximise power extraction and minimise the visual impact of the turbines.

Well designed wind turbines are extremely quiet: one simple data correlation for the sound power level, \( L_P \), gives

\[
L_P = 10^{-7} \hat{W}
\]  

(1.2)

Lawson (1992), that is one-ten millionth of the turbine’s power is output as noise. Another correlation that is more accurate in some cases, is

\[
L_P = 50 \log_{10} V_{tip} + 10 \log_{10} R_1 - 1
\]  

(1.3)

where now \( L_P \) is measured in the more common unit of decibels (dB), Wagner et al. (1996). \( V_{tip} \) is the circumferential velocity of the blade tip in m/sec and \( R_1 \) is measured in m. \( L_P \) is the strength of the source of the sound as a multiple of the standard base level of \( 10^{12} \) Watts. It is used, in combination with an equation for the propagation of the sound, to determine the noise level at any point around the turbine or turbines.

1.3 Turbine Operating Parameters

As with any fluid machine, it is often useful to discuss wind turbine operation in terms of non-dimensional groups that can be obtained from dimensional analysis. Here we introduce the important parameters by taking advantage of Equation (1.1), which strongly suggests that the most important parameter, the power coefficient, \( C_P \), should be defined as

\[
C_P = \frac{\hat{W}}{\frac{1}{2} \rho U_0^3 \pi R_1^2}
\]  

(1.4)

\( C_P \) is to be interpreted as the ratio of the actual power produced to the power in the wind that would otherwise pass the blade disk. Note that:

- \( C_P \) is dimensionless, even though it includes the factor of 1/2.
- For later use, \( C_P \) is not strictly an efficiency, even though it is sometimes treated as one. As will be explained in the next Chapter, it is possible to increase \( C_P \) by increasing the velocity of the wind through the blades. However, \( C_P \) can be interpreted as an efficiency when comparing turbines of the same type, such as all those considered in this Chapter.

Let us use the form of (1.4) in setting up the dimensional analysis. If we make the very general statement that the turbine power should depend on wind speed, air density, turbine radius, blade speed or angular velocity, \( \Omega \) (whose units are radians per second, abbreviated to rad/sec), and the kinematic viscosity of the air, \( \nu \), then we have

\[
f(\hat{W}, U_0, \rho, R_1, \Omega, \nu) = 0
\]  

(1.5)

where \( f \) denotes (the unknown) functional dependence. \( \nu \) is the actual viscosity divided by the density, and has units of m\(^2\)/sec. For sea level conditions, \( \nu = 1.5 \times 10^{-5} \) m\(^2\)/sec at 20\(^\circ\)C. There are many ways
of proceeding with the dimensional analysis, so the following way may not be familiar to all readers. Inexperienced readers are referred to Fox & McDonald (2000) or Smits (2000).

Expression (1.5) contains six parameters or variables and three dimensions, so we expect three non-dimensional groups to result from the following analysis. To ensure that $C_P$ as defined by (1.4), is one of these groups, the "repeating variables" should be $U_0$, $\rho$, and $R_1$. These repeating variables can, in principle, appear in all the non-dimensional groups. Forming these groups then allows (1.5) to be rewritten as

$$f(C_P, \Omega R_1/U_0, U_0R_1/\nu) = 0$$

(1.6)

The second of these groups is the tip speed ratio, which is sufficiently important to have its own symbol $\lambda$. For future reference

$$\lambda = \Omega R_1/U_0 = V_{tip}/U_0$$

(1.7)

is the ratio of $V_{tip}$ to the windspeed. In studying the aerodynamic forces generated on blades (the lift and drag) we will see that $\lambda$ is crucially important in determining the angle of attack of the blade sections or elements. Usually, $\lambda$ lies between 7 and 10 when a turbine is performing optimally. Thus the tips are travelling at a velocity that is many times the wind speed and this can cause them to reach the limit of incompressible flow, about 30% of the speed of sound, which is about 340m/sec at typical sea level conditions. There is no evidence that compressibility is important on operating wind turbines.

The third group should be recognised as a Reynolds number, $Re$, which generally measures the effects of viscosity. The form of the Reynolds number in (1.6) is not used in wind turbine studies because our later consideration of blade aerodynamics will show that the most useful form comes from considering the lift and drag behaviour of the aerofoil sections that comprise the blades. This $Re$ contains the blade chord, $c$ - the width of the blade - and the “total” velocity at the blade, $U_T$. The determination of $U_T$ will be considered in Chapter 3. Thus

$$Re = U_Tc/\nu$$

(1.8)

We will see in Chapter 5 that most of a turbine’s power is produced near the tip, so the value of $Re$ most often quoted is the tip Reynolds number. If $\lambda$ is sufficiently high, $U_T = \lambda U_0$ at the tip, but $U_T = U_0$ everywhere along a stationary or slowly rotating blade. For most large turbines, producing say 100 kW or more, $C_P$ is not strongly dependent on $Re$, but, as we shall soon see, $C_P$ varies significantly with $\lambda$. Reynolds number effects can be significant for small turbines, especially during starting.

The final quantity we need to consider for the present, is the thrust on the blades, $F$. This force is not as important for wind turbines as it is for propellers, which are designed to produce thrust, but it is necessary to determine the horizontal force acting on the top of the turbine tower, which is nearly equal to $F$. Dimensional analysis starts from the appropriately modified form of (1.5)

$$f(F, U_0, \rho, R_1, \Omega, \nu) = 0$$

from which the following form of the thrust coefficient, $C_F$, should be almost immediately recognisable

$$C_F = \frac{F}{\frac{1}{2} \rho U_0^2 \pi R_1^2}$$

(1.9)
From the discussion of power output, we would expect that \( C_F \) is strongly dependent on \( \lambda \) but not usually on \( \textit{Re} \).

![Power Curve for 5 kW Turbine at Sea Level.](image)

**1.4 The Power Curve and the Performance Curve**

However much the aerodynamicist is happy to use non-dimensional groups to describe and analyse performance, the owner of the turbine is more interested in the actual power output as function of windspeed. This is given by the power curve. Figure 1.4 shows the power curve for the 5 kW turbine and Figure 1.5 shows that for Vestas V47 660 kW turbine. Note that each power curve shows the “cut-in” wind speed below which no power is produced and the “rated” wind speed where the advertised power is obtained. This speed is 13 m/sec for the smaller machine and 15 m/sec for the larger. It is common for the rated speed to increase with turbine size because the tower height also increases, and this increases the windspeed at the hub. Both turbines have a region, between 5 and 10 m/sec, where the power increases rapidly, approximating the cubic dependence of Equation (1.1). Because of the sensitivity of power to wind speed, it is important to take note of the rated speed; the easiest way for a manufacturer to “improve” the performance of a turbine is often to increase the rated speed! (At this introductory stage, we are ignoring several important issues regarding the determination of the power curve. Some of these will be raised in Section 1.6.)

After 10 m/sec, the power increases less rapidly for both turbines for the same reason; safety. It is important to be able to control a turbine at high wind, so that it does not attempt to extract more power than can be absorbed by the generator. High power levels may also cause unacceptable structural loads on the blades and other components. There are a number of control strategies, such as controlling the angle of attack by pitching the blades. We will investigate some of them in Chapters 3 - 5. For large turbines, there is also a “cut-out” wind speed at which the turbine is shut down for safety reasons. This is 25 m/sec for the V47. At this speed, a brake is activated, and not released until the wind has died down. At high wind speeds, smaller turbines are often “furled”, that is turned out of the wind direction by the collapse of the tail fin. However, the turbine shown in Figure 1.2 relies on control of the generator’s field current to reduce output in high winds. In the event of an electrical failure, a centrifugally activated band brake stop the blades.
On larger turbines as the windspeed increases from below the cut-in, the brake is released, and, the blades are pitched into the wind (we will understand this phrase more fully after Chapter 5) ready to begin turning when the wind picks up. In contrast, most small tribunes do not have pitch adjustment and so must rely on the aerodynamic forces generated at high angles of attack, to overcome the resistive torque of the drive train and generator. Some of these aspects will be discussed further in Chapter 6, where we will see the importance of good low wind speed performance of small turbines.

![Power Curve for the Vestas V47 Turbine.](image)

**Figure 1.5.** Power Curve for the Vestas V47 Turbine.

<table>
<thead>
<tr>
<th>$U_0$ (m/sec)</th>
<th>$\dot{W}$ (kW) for $\rho = 1.21$ kg/m$^3$</th>
<th>$\dot{W}$ (kW) for $\rho = 1.06$ kg/m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>2.7</td>
<td>0.6</td>
</tr>
<tr>
<td>5.0</td>
<td>43.1</td>
<td>36.2</td>
</tr>
<tr>
<td>6.0</td>
<td>95.4</td>
<td>81.8</td>
</tr>
<tr>
<td>7.0</td>
<td>164</td>
<td>141</td>
</tr>
<tr>
<td>8.0</td>
<td>248</td>
<td>215</td>
</tr>
<tr>
<td>9.0</td>
<td>346</td>
<td>300</td>
</tr>
<tr>
<td>10.0</td>
<td>446</td>
<td>392</td>
</tr>
<tr>
<td>11.0</td>
<td>534</td>
<td>480</td>
</tr>
<tr>
<td>12.0</td>
<td>598</td>
<td>554</td>
</tr>
<tr>
<td>13.0</td>
<td>634</td>
<td>607</td>
</tr>
<tr>
<td>14.0</td>
<td>651</td>
<td>637</td>
</tr>
<tr>
<td>15.0</td>
<td>657</td>
<td>652</td>
</tr>
<tr>
<td>16.0</td>
<td>659</td>
<td>657</td>
</tr>
<tr>
<td>17.0</td>
<td>660</td>
<td>659</td>
</tr>
<tr>
<td>18.0</td>
<td>660</td>
<td>660</td>
</tr>
</tbody>
</table>

**Table 1.1** Power curve for Vestas V47 660 kW wind turbine at two different densities.
Introduction to Wind Turbine Technology

The V47 rotates at constant 28.5 r.p.m. to maintain synchronicity with the electricity grid. The rotor diameter is 47 m, so the data in the Table 1.1 can be used to determine the operating characteristics: a plot of $C_P$ against $\lambda$. This is shown in Figure 1.6 using the data for both densities. It is immediately obvious that the non-dimensional plotting has significantly collapsed the data for the two densities. The curve is typical of all well designed wind turbines; $C_P$ is small at low $\lambda$, reaches a maximum usually when $\lambda$ is in the range of 7 - 10, and then decreases. The upper end of the operating range, at about $\lambda = 18$ in Figure 1.6, is called the runaway point. Any turbine which loses its electrical load will seek to operate at runaway. If this occurs in high wind, then the centrifugal stresses in the blade can be, literally, shattering. It is, therefore, important to have overspeed protection on any turbine. We will see in later chapters that the operating range between the optimum power point (maximum $C_P$) and runaway becomes increasingly difficult to analysis as wake expansion and thrust increase.

We will also see later that the strong effect that $\lambda$ has on turbine performance arises through its control of the angle of attack of the blade elements that make up the blades. Since the value of $C_P$ determines how much power is extracted at a given wind speed, Figure 1.6 indicates that a constant speed turbine cannot operate at maximum efficiency over a large range of wind speed; obviously the turbine’s designers have opted to forgo some of the extractable power for the simplicity of constant speed operation.

Figure 1.6 shows a maximum $C_P$ of about 0.46, but we have to realise that the power listed in Table 1.1 is the output electrical power which is less then the input aerodynamic power by the product of the efficiencies of the (mechanical) drive train and the (electrical) generator. Estimating the combined efficiencies as 0.9, indicates that the V47’s maximum efficiency is within 15% of the Lanchester-Betz limit, the supposed maximum efficiency of this type of turbine (Chapter 2). It is unlikely that we will be able to get significantly closer to the Lanchester-Betz limit.

As distinct from the majority of large turbines, small turbines usually operate at variable speed. In principle, it is then possible to adjust the rotor speed as the wind speed varies to maintain the maximum $C_P$. The rotor speed of the turbine shown in Figure 1.2 is not given by the manufacturers, but the $C_P$ tabulation in Table 1.2 shows $C_P$ to decrease with increasing $U_0$ suggesting that the
optimum $\lambda$ is not maintained. However, the turbine’s blades have a maximum $C_p$ of 0.45 in terms of power extracted from the wind; the variation shown in Table 1.2 reflects a deliberate decision aimed at improving the controllability of the turbine in high winds; the cost in lost power is not large because of the relatively rare occurrence of high winds as will be seen in Section 1.6.

**Example 1.3**

It is proposed to install a RAPS in a village in Tibet at an altitude of just over 5,000 m. As the RAPS contains a wind turbine, it is necessary to estimate the reduction in power caused by the increase in altitude.

**Answer**

From the material presented in this Chapter, we would expect that $C_p$ should remain constant as the altitude varies. Thus the change in power occurs through the change in density, much like that shown in Table 1.1 and Figure 1.6. One way to estimate the density change is from the “International Standard Atmosphere” (ISA) which is often used in aerodynamics to account for altitude effects on aircraft performance. Table 1.3 gives the ISA variation of temperature, density, and kinematic viscosity with altitude.

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$\rho$ (kg/m³)</th>
<th>$\nu$ (m²/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
<td>1.225</td>
</tr>
<tr>
<td>1,000</td>
<td>8.5</td>
<td>1.112</td>
</tr>
<tr>
<td>2,000</td>
<td>2.0</td>
<td>1.007</td>
</tr>
<tr>
<td>3,000</td>
<td>-4.5</td>
<td>0.909</td>
</tr>
<tr>
<td>4,000</td>
<td>-11.0</td>
<td>0.819</td>
</tr>
<tr>
<td>5,000</td>
<td>-17.5</td>
<td>0.738</td>
</tr>
<tr>
<td>6,000</td>
<td>-24.0</td>
<td>0.606</td>
</tr>
</tbody>
</table>

Table 1.3 Properties of the International Standard Atmosphere (from Smits (2000, Table C5))
The ISA gives the sea-level density of air as 1.225 kg/m$^3$ (and the temperature as 15°) and the density at 5,000 m as 0.738 kg/m$^3$. Thus the power reduction should be by a factor of about $0.738/1.225 = 0.602$ - a substantial 40%. The corresponding change in the kinematic viscosity is by a factor of nearly 40%. This will cause a similar decrease in the Reynolds numbers, which probably will not have a large effect on optimal power but may influence starting performance.

1.5 The Variation of Power Output with Height

In Example 1.3, we saw the significant effect of altitude when it causes a large variation in density. We now look at the effect of tower (or nacelle) height, $h$. Typical values of $h$ are in the range 20 – 50 m, and are, therefore, small compared to the altitudes (say 5,000 m!) over which the air properties change significantly. We can, therefore, ignore property changes in discussing height.

There are two main expressions used to describe the height dependence of the mean wind speed, which we now call $U$ rather than $U_0$ as used previously. (The main reason to distinguish between $U_0$ and $U$ arises when we consider the effect of the blades on the wind speed in Chapter 2, for which Section 1.1 provides the background. Furthermore, $U = U_0$ only when $h$ is the hub height.) The simplest is the power law

$$U(h) = U(h_r) \left( \frac{h}{h_r} \right)^m$$

(1.10)

where $h_r$ denotes a “reference height” usually 10 m, and $m$ is an exponent that depends on the roughness of the surface. It is usually asserted that the logarithmic “law”

$$U(h) = U(h_r) \left( \frac{\ln(h/z_0)}{\ln(h_r/z_0)} \right)$$

(1.11)

is more accurate. Here $z_0$ is the “roughness length”. Typical values of $m$ and $z_0$ are given in Table 1.4.

<table>
<thead>
<tr>
<th>Type of terrain</th>
<th>$z_0$ (m)</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>open country</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>farmland with few trees etc</td>
<td>0.05</td>
<td>0.16</td>
</tr>
<tr>
<td>farmland with many trees, forests etc, villages</td>
<td>0.3</td>
<td>0.928</td>
</tr>
</tbody>
</table>

Table 1.4 Variation of $z_0$ and $m$ with terrain (from Walker & Jenkins (1997, Table 1.1)).

An average value of $m$ is about 1/6, so that, if $\tilde{W} \sim U^3$, then $\tilde{W}$ increases roughly as $h^{1/2}$. This leads to the rule-of-thumb that doubling the tower height will increase the power output by 40%. In practice, the wind speed dependence on height is more complex that suggested by the relations (1.10) and (1.11). The logarithmic law, for example, is strictly applicable only if $z_0$ does not vary significantly around the site for a distance of probably 100$h$, the site is flat, and there is no heating or cooling of the air; in other words, the flow is “neutral”. Thus $U(h)$ may well depend on wind direction and time of day, and there is evidence that $m$ also depends on $U(h_r)$, Spera (1994, Chapter 3).
1.6 Turbulence and Wind Statistics

Rarely is the wind steady. It usually fluctuates with time in an apparently random way that can be measured by the turbulence intensity defined in terms of the root mean square (rms) of the velocity fluctuations. To quantify this relation, assume that the wind speed at any time $t$, is the sum of the mean $U$, and the fluctuating velocity $u(t)$. Note that the average value of $u(t)$ is zero. The turbulence intensity is defined as

$$I_u = \frac{\sqrt{\overline{u^2}}}{U} = \frac{1}{U} \left[ \int_0^T u^2 dt \right]^{1/2}$$

(1.12)

where $T$ is the sampling time. The overbar on $u^2$ (in the square root) denotes a time average - a convention that will be used again in these Notes, so that, for example, $\overline{u} = 0$. In practice, the output of an anemometer (which measures $U + u$) is usually sampled at a fixed frequency and the integral in (1.12) is approximated by a summation.

The definition (1.12) immediately gives rise to an operational difficulty: what is the appropriate $T$ to use in determining $I_u$? In principle, we want $T$ to be sufficiently large that any increase would not alter the value of $I_u$. In practice, this usually cannot be achieved, and as a compromise, the most common $T$ used in wind turbine analysis is 10 minutes. For example the IEC Standard 61400 for determination of the power curve – see Figs. 1.5 and 1.6 – requires this value of $T$.

$I_u$ depends on $m$ and $z_0$, increasing from around 0.1 (10%) for smooth terrain up to 0.2 (20%) or more for rough terrain at high $z_0$. Turbulence also depends on height, usually decreasing with increasing $h$. It can also affect the determination of turbine power – see Exercise 1.21 at the end of this Chapter - and have a large effect on the turbine loads – see Chapter 7. It will be seen that, in order to undertake load analysis, it is necessary to know the probability of the wind speed. This probability can be viewed in two ways, the probability density function, $p(U)$, and the cumulative probability, $P(U)$. The former measures the occurrence of a particular wind speed, whereas the latter gives the probability that the wind speed is less than $U$. Mathematically, the two are related by $dP/dU = p$.

The most common assumption, and the one made in IEC 61400, for the form of $p(U)$ and $P(U)$ is the so-called Rayleigh distribution

$$P(U) = 1 - e^{-\pi(U/2C)^2}$$

and

$$p(U) = \frac{\pi U}{2C^2} e^{-\pi(U/2C)^2}$$

(1.13)

Figure 1.7. Probability of wind speed.
Note the use of the overbar in (1.13) to denote the average value of $U$ as we now have to distinguish between the instantaneous $U$ and its average, $U; U = \bar{U} + u(t)$. In most of these Notes, we do not have to make that distinction, so we will follow (and have followed!) convention and omit (have omitted) the overbar unless that would cause confusion.

The Raleigh distribution is a special case of the Weibull distribution which is commonly used for approximating the wind speed probability distribution. Figure 1.7 shows that the Rayleigh distribution is in reasonable agreement with wind speed statistics obtained at the entrance to Newcastle Harbour. Knowing $p(U)$, allows calculation of the average power output for a particular site, according to

$$\bar{W} = \int_0^{\infty} \dot{W}(U) \rho(U) dU$$

(1.14)

The ratio of the average power to the rated power is called the capacity factor, which is a crucial parameter in determining the economics of wind farms in particular. In practice the capacity factor varies widely, reaching nearly 50% at a few very favourable locations around the world.

Small wind turbines, on the other hand, are usually sited next to the load they supply, rather than in the windiest possible location. This means that good low wind speed performance is crucial to maximising the utilisation of the available energy. Fig. 1.8 combines the power curve from Fig. 1.4 with $p(U)$ from Fig. 1.7 to give the integrand of (1.14) as a function of $U$. By comparing Fig. 1.8 and 1.4, the importance of the wind speeds around the mean is obvious. For example, $U = 4$ m/sec gives a contribution to the average power larger that from 14 m/sec! In Exercise 1.23, you will determine the capacity factor for the turbine and wind speed data shown in Figure 1.8.

Finally, it is important to realise or remember that probability distributions relate to the amplitude of the wind speed not to its frequency. In other words, if, for example, it is necessary to analyse wind turbine response to a gust, which involves the wind speed variation with time, then more information is needed than is contained in $p(U)$ and $P(U)$. We return to these issues in Chapter 7.

1.7 The Electrical and Mechanical Layout of Wind Turbines

For both large and small wind turbines, most of the electrical and mechanical components are housed in the nacelle on top of the tower. The main component is, of course, the generator, but as shown in Figs. 1.9 and 1.10, there may be many others. Both figures show a gearbox, which is not common on small machines and is slowly losing favour on large machines as well. The large turbine is significantly more complex, with an anemometer and wind vane (not shown) used in conjunction with the hydraulic yaw gear to drive the turbine into the wind. This particular turbine also has blade pitch adjustment.
Pitch adjustment is generally held to be too expensive for small turbines, but some mechanism of overspeed protection is required. Many small turbines (unlike the one shown in Figure 1.10) are designed to furl at high speeds. Furling is the process by which the blade disk (the swept area of the blades) is moved from being normal to the wind (for maximum power production – see Exercise 2) to a much smaller angle to the wind. This can be done by yaw – motion in the horizontal plane – or by pitch – a vertical motion of the turbine. One method for furling is to displace the turbine axis horizontally from the yaw axis (the pivot to the tower) and the tail fin is spring loaded to allow collapse at sufficiently high wind speed. There are a number of possible problems with furling, such as:

- The difficulty achieving furling consistently at the same windspeed for a variety of load cases,
- Furling may cause a non-zero yaw at lower windspeeds with the consequent reduction in power output, and
- The transient forces associated with furling may be large.

Nevertheless, its common use necessitates the treatment in Chapter 7 where we discuss turbine performance and safety at high wind speed.

The turbine in Figure 1.10 has a field-excited generator and control of the field current is the primary method of overspeed protection. The use of a field-excited generator also has the advantage that the field is not activated until the blades reach a sufficiently high speed during starting. Thus the resistive torque of the generator is low during the critical starting sequence, whereas permanent magnet alternators often have a significant “cogging torque” which must be overcome by the blades as they begin to rotate logging torque is required to force the rotor through the “permanent” magnetic field.

Figure 1.9. Nacelle layout of Vestas V47 660 kW wind turbine. The main components other than those indicated are: 2. main shaft; 3. blade hub; 5. blade bearing; 8. disk brake; 12 yaw drive motor.
1.8 Some Size Issues with Small Turbines

The IEC 61400-2 defines a small turbine as having a swept area of less than 200 m², which corresponds to a power output of about 120 kW. In addition, there is a further subdivision in that turbines of less than 2 m² (about 1.2 kW) do not need to have their tower included in the certification process. Clausen & Wood (2000) have made a further subdivision as shown in Table 1.5

<table>
<thead>
<tr>
<th>Category</th>
<th>Power (kW)</th>
<th>R₁ (m)</th>
<th>max. Ω (rpm)</th>
<th>typical uses</th>
<th>generator type(s)</th>
<th>example websites</th>
</tr>
</thead>
<tbody>
<tr>
<td>micro</td>
<td>≤ 1.2</td>
<td>1.5</td>
<td>700</td>
<td>electric fences, yachts</td>
<td>Permanent magnet (PM)</td>
<td>(1)</td>
</tr>
<tr>
<td>mid-range</td>
<td>1 - 5</td>
<td>2.5</td>
<td>400</td>
<td>remote houses</td>
<td>PM or induction</td>
<td>(2)</td>
</tr>
<tr>
<td>mini</td>
<td>20 - 100</td>
<td>5</td>
<td>200</td>
<td>mini grids, remote communities</td>
<td>PM or induction</td>
<td>(3)</td>
</tr>
</tbody>
</table>

Table 1.5. Operating Parameters of Small Wind Turbines (adapted from Clausen & Wood (2000)).

Example web-sites:
(1) http://www.windenergy.com/
(3) http://www.enercon.de/
http://www.venwest.iinet.net.au/
The significance of the division is more apparent from Table 1.6 which shows the size-dependence of the main turbine parameters for a constant tip speed ratio and blade density at the same wind speed.

It is clear from Table 1.6 that micro-turbines have the poorest starting performance which is often exacerbated by the use of permanent magnet alternators, which can have a significant cogging torque. If the resistive torque is significant, then starting performance can be improved, and the cut-in wind speed lowered, by increasing the number of blades. This is, presumably, why many very small turbines are multi-bladed. On the other hand, if there is no significant resistive torque, then starting is independent of the number of blades, \( N \), simply because the starting torque is proportional to \( N \), as is the rotational inertia. In fact, ensuring good starting performance is not easy, as the following example shows. The 5 kW turbine shown in Fig. 1.10 has a net resistive torque from the field-excited generator and drive-train of about \( \text{Nm} \), which is sufficient to prevent the blades from starting at less than about 2.5 m/sec. This torque compares with the rated torque of over 100 \( \text{Nm} \). We will investigate starting and low wind speed performance in Chapter 6.

### Table 1.6. Dependence of Important Parameters on Blade Radius (adapted from Clausen & Wood (2000)).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Power Law Dependence on ( R_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds Number (Re)</td>
<td>1</td>
</tr>
<tr>
<td>Power Output</td>
<td>2</td>
</tr>
<tr>
<td>Noise Output</td>
<td>2</td>
</tr>
<tr>
<td>Centrifugal Loads</td>
<td>2</td>
</tr>
<tr>
<td>Starting Torque</td>
<td>3</td>
</tr>
<tr>
<td>Inertia of Blades</td>
<td>5</td>
</tr>
</tbody>
</table>

**1.9 Further Reading**

There are a number of good introductory books on wind turbines, such as Walker & Jenkins (1997) (but watch out for the numerous typographical errors), Kentfield (1996), with a chapter on furling, and Spera (1994), still in print, which contains an excellent introduction to aerodynamics by R.E. Wilson. An old favourite, but no longer in print, is Eagleston & Stoddard (1987) with its detailed development of blade dynamics.

Wind energy is fortunate in having a number of excellent web sites, such as the Danish Wind Turbine Manufacturers site http://www.windpower.dk. Click on the button “Guided Tour” in the home page for a large amount of information relevant to this Chapter and these Notes. Related information is available from the American Wind Energy Association site http://awea.org along with a list of, and contact details for, American manufacturers of small turbines.

A number of manufacturers of large turbines produce CDs of their products and Lior International NV http://www.lior-int.com sell a CD which has much visually-based useful information on large wind turbines.

Readers intending to proceed to the subsequent Chapters are warned that their knowledge of basic fluid mechanics will be tested. If you wish to revise, or to learn, the necessary background material, then you should consult an introductory text, such as Fox & McDonald (2000) or Smits (2000).
1.10 References


Exercises

1. Check the units of Example 1.2.

2. If a turbine is operated at a yaw angle of \( \gamma \) (between the wind direction and the turbine axis) the power output is known to decrease roughly as \( \cos^3 \gamma \). Using the definition of \( \dot{W} \) from Equation (1.4), explain how this dependence might come about. Answer: the blades in yaw respond primarily to the velocity normal to them, which is \( U \cos \gamma \).

3. The maximum solar insolation (radiant energy from the sun) that reaches the earth’s surface is about 1 kW/m\(^2\). In terms of the power density used in Example 1.2, what wind speed corresponds to this level? Answer: 11.9 m/sec.

4. If the shaft torque on a turbine is denoted as \( T \) Nm, what is the appropriate formulation of the torque coefficient, \( C_T \)? Answer:

\[
C_T = \frac{T}{\frac{1}{2} \rho U_0^2 R}
\]

What is the relationship between \( C_T \) and \( C_P \)? Answer: \( C_P = \lambda C_T \).

5. Determine \( V_{\text{tip}} \) for the Vestas V47 660 kW turbine (65.6 m/sec) and estimate the sound power level using Equation (1.3). Answer: 103 dB.

6. The turbine depicted in Figure 1.2 has a shaft speed of 280 r.p.m. at a wind speed of 10 m/sec. Determine \( V_{\text{tip}} \) (74.8 m/sec) and estimate the sound power level from Equation (1.3). Answer: 97 dB. Note that the sound power level of the human voice is about 90 dB.

7. The wing of a Boeing 747 has a tip chord of 4.03 m. What is the tip \( Re \) when:

(i) taking-off at sea-level with a speed of 84 m/sec (22.6 million).
(ii) cruising at 240 m/sec at an altitude where \( v = 3.2 \times 10^6 \) m\(^2\)/sec (320 million).
8. The 660 kW turbine whose characteristics are shown in Table 1.1 and Figures 1.5 and 1.6 has a tip chord of 0.4 m. Estimate the tip $Re$ at maximum performance. Answer: 1.9 million.

9. The blades of a 600 W turbine have a tip chord of 42 mm and is designed to start at $U_0 = 3$ m/sec. Estimate the tip $Re$ for the stationary blades at this windspeed. Answer: 9,000.

10. Estimate the tip radius and tip $Re$ for this turbine if its rated speed is 10 m/sec at $\lambda = 7.5$ and $C_p = 0.4$. Answers: 0.9 m, 171,000.

11. If the turbine for the previous exercise does start at 3 m/sec, what power would it extract? Answer: 16 W.

12. The blades of a 2.5 m radius blade have a tip chord of 75 mm. What is the $Re$ at rated conditions, $\lambda = 10$, and $U_0 = 10$ m/sec. Answer: 510,000. If the speed of sound is 342 m/sec at sea level conditions, what is the tip Mach number. Answer: 0.292.

13. What additional non-dimensional group would arise if blade chord had been added to the general equation for power or thrust?

14. The V47 turbine whose characteristics are given in Table 1.1 has a rotor diameter of 47 m, and rotates at 28.5 r.p.m. From the data in Table 1.1, determine the $C_p$ versus $\lambda$ distribution to verify Figure 1.6.

15. The following table gives the rotor diameter of some Vestas turbines as a function of output power. Is there a discernible change in efficiency as rotor size increase? Answer: no.

<table>
<thead>
<tr>
<th>$W$ (kW)</th>
<th>Rotor diameter (m)</th>
<th>Rated speed (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>225</td>
<td>29</td>
<td>14</td>
</tr>
<tr>
<td>660</td>
<td>47</td>
<td>15</td>
</tr>
<tr>
<td>1500</td>
<td>64</td>
<td>15</td>
</tr>
<tr>
<td>1650</td>
<td>66</td>
<td>17</td>
</tr>
<tr>
<td>2000</td>
<td>80</td>
<td>15</td>
</tr>
</tbody>
</table>

16. Consider a two-bladed 5 m diameter turbine with a hub height of 20 m. Using (1.11), estimate the difference in wind speed at the tips of the blades when they are vertical for the four values of $z_0$ in Table 1.4. What is the significance of these results?

17. Assume that $W \sim U^3$ and the cost of a turbine is $a$ (in an arbitrary currency), and the cost of a tower in the same currency is $bh^n$, where $a$, $b$, and $n$ are “constants” and $h$ is the hub height. Further assume that the variation in $U$ across the blade disk (see the previous exercise) can be ignored. Derive an expression for the power per unit cost, $p = \frac{W}{(a + bh^n)}$, as a function of $h$, using Equation (1.10) for the variation of $U$ with $h$. Defining $r$ as the ratio of turbine cost to the cost of the turbine and tower, $r = a / (a + bh^n)$, show that the optimum tower height $h_{opt}$, that maximises $p$, occurs when $r(h_{opt}) = 3m/n$, provided $3m/n < 1$.

18. Check the websites of some small turbine manufacturers to obtain data for $a$, $b$, and $n$ in the previous exercise. One very good site is the Bergey WindPower site: http://www.bergey.com. For their 10 kW turbine, $a \approx US\$18,000, and for the 10 kW self-supporting tower, $b = 170.0$, and $n = 1.217$ for $h$ in m. (These data were obtained in November, 2000.) Show that the optimum height increases with increasing roughness, and that there is no optimum height for the roughest terrain in Table 1.4.
19. By considering the power curves in Figs. 1.4 and 1.5, comment on the use of $\dot{W} \sim U^3$ in the previous two exercises. Using manufacturer’s data such as in Tables 1.1 and 1.2 (http://www.bergey.com is also an excellent source) determine an alternative exponent for $U$.

20. Using the logarithmic law, (1.11), show that $r$ defined in Exercise 17 is now given by the implicit equation, $r = \frac{3}{n [\ln(h_{opt}/z_0)]}$ with no restriction on $n$ and $z_0$ so that there is always an optimum height.

21. Writing $U(t) = U + u(t)$, and assuming that $C_P$ in (1.4) is independent of $t$ and $U(t)$, and

$$\frac{u^3}{U^3} \ll 3I_u^2$$

show that

$$\dot{W} = \frac{1}{2} \rho U^3 \pi R_t^2 C_P (1 + 3I_u^2)$$

In words: determining $C_P$ from the power curve by ignoring the turbulence, as was done in producing Fig. 1.6 and Table 1.2, will produce an over-estimate. If $I_u = 0.2$, the error is 12%. Comment on this result in view of Exercise 2. Comment further on the empirical fact that the power actually increases as the sampling period is reduced from the standard value of 10 minutes.

22. From the Rayleigh distribution, (1.13) with $U = 5.7$ m/sec, calculate the capacity factor for the Vestas V47 turbine for $\rho = 1.21$ kg/m$^3$ using the data in Table 1.1 and (1.14). Answer: 0.205.

23. From the Rayleigh distribution, (1.13) with $U = 5.7$ m/sec, calculate the capacity factor for the 5 kW turbine for $\rho = 1.21$ kg/m$^3$ using the data in Table 1.2 and (1.14). Answer: 0.219.