3.0 Introduction

The conservation equations of fluid mechanics were used in the last Chapter to derive equations for the power output and thrust for a flow which has no radial dependence. These equations lead to the well-known Lanchester-Betz limit on wind turbine performance, but they do not consider the forces acting on the blades which give rise to the thrust and torque, and hence power. These are obviously the main quantities required in any prediction and analysis of wind turbine performance and they are required for the structural design as well. It is necessary to consider the possibility that these forces, and the flow that causes them, have some radial dependence.

The traditional way to extend the analysis of Chapter 2 is to divide the flow through the blades into a number of concentric annular streamtubes. The conservation equations for these streamtubes are easily-recognised generalisations of those derived in Chapter 2. We then consider the forces acting on the blade elements, the portion of the blades intersected by each streamtube. The velocity and pressure within each streamtube are constant but may vary from one streamtube to the next. Experience shows that typical performance analyses can be done accurately with between 10 to 20 blade elements. The next Section makes explicit a number of the fundamental assumptions of blade element theory, of which the most important is the assumption that blade elements behave as aerofoils. We will review the important aspects of aerofoil theory in the next Chapter. Section 3.2 develops the important conservation equations in annular streamtube form, starting from the analysis of Chapter 2. Section 3.3 considers the forces acting on the blade elements, which are assumed to be aerofoils. The implementation of these equations in a computer program to predict wind turbine performance is covered in Section 3.4. Section 3.5 describes this program and gives a listing in Fortran. The source files of all listed programs are available as explained in the Introduction to these Notes; these files are available in either Fortran 90 or C.

3.1 Some Assumptions of Blade Element Theory

There are two fundamental assumptions necessary to extend the analysis of Chapter 2:

- The flow in each streamtube is independent of that in other streamtubes, and
- The forces acting on each blade element are the same as those on an aerofoil of the same section, angle of attack, and effective velocity.

It is easy to demonstrate that both these assumptions can be in error. Take the first assumption first: if there is a radial dependence of the velocity, then there must be a radial pressure gradient in the flow and each streamtube will therefore exert a force on its two neighbours. Fortunately this force is redistributive, i.e. it sums to zero over the whole flow, so ignoring it should not cause too severe an error.

The second assumption is much more intriguing. Some of its limitations are shown by the following argument. Imagine the two-dimensional analogue of a wind turbine, recalling that an aerofoil is also two-dimensional. If we consider a blade element at radius \( r \), then the two-dimensional analogue will be an infinite “cascade” of aerofoils spaced \( 2\pi r/N \) apart, where \( N \) is the number of blades. The ratio of the element’s chord length \( c \) to this quantity is the local solidity, \( \sigma = Nc/2\pi r \). As \( \sigma \to 0 \), then we would expect the blade to behave as an aerofoil, but at high \( \sigma \), the flow over each blade will be affected by the proximity of the adjacent blades. Among other effects, this proximity is known experimentally to delay separation from an aerofoil – an effect that seems intuitively obvious. Thus
the second assumption requires $\sigma \to 0$, and it is intriguing that rotors designed for optimum performance usually have small $\sigma$, typically in the range $0.1 - 0.2$.

3.2 The Conservation Equations for Annular Streamtubes

These equations can be obtained by applying the vector equations in the last Chapter to the annular streamtube at radius $r$, and radial extent $dr$, as shown in Figure 3.1.

The blade tip radius is $R_1$. The streamtube thickness is $dr_0$ upstream, $dr$ at the blades, and $dr_\infty$ in the far-wake. Note that the streamtube is annular and it is assumed that $dr << r$ typically. Any velocity in the radial direction is ignored, but the circumferential or swirl velocity will be included in the analysis. Firstly we consider the conservation equations.

3.2.1 Conservation of Mass

Dividing the conservation of mass equation (2.1) by the density and applying it to the streamtube whose flow area is approximately $2\pi r dr$, gives

$$U_0^22\pi r_0 dr_0 = U_1^22\pi r dr = U_\infty^22\pi r_\infty dr_\infty$$

or, in a form analogous to (2.3),

$$U_0r_0 dr_0 = U_1r dr = U_\infty r_\infty dr_\infty$$

(3.1)

Figure 3.1. Annular streamtube intersecting a blade element.

3.2.2 Conservation of Momentum

Because we are only interested in the force in one direction, it is easier to revert to scalars. The contribution to the axial thrust, $F$, from the streamtube is:

$$dF = \rho U_0 U_0^22\pi r_0 dr_0 - \rho U_\infty U_\infty^22\pi r_\infty dr_\infty$$

$$= 2\pi \rho U_0^2 r dr [U_0 - U_\infty]$$

using Equation (3.1). Note that this is the total force acting on the $N$ blade elements that intersect this streamtube. This can be rewritten as

$$= 4\pi r \rho U_0^2 a(1 - a) dr$$

(3.2)
Figure 3.2. Velocities for blade element at radius r.

Where $a$ is the axial interference factor defined in Section 2.5

$$a = 1 - \frac{U_1}{U_0} \quad (3.3)$$

so that the larger the value of $a$ the more deceleration occurs as the air goes through the blades.

### 3.2.3 Conservation of Angular Momentum

In vector form, the contribution to $dT$, the torque on the blade elements from the streamtube can be obtained from (2.7). The equation can be turned into a scalar equation for the contribution to $T$, the torque acting about the axis of rotation:

$$dT = \rho r_w W_w 2 \pi r_w dr_w \quad (3.4)$$

assuming that there is no swirl upstream of the blades. (This is an important assumption that we will look at in more detail in Chapter 4.) Downstream of the blades, the angular momentum of the streamtube is conserved so $r W_1 = r_w W_w$. Using this relationship and conservation of mass

$$dT = 2 \pi \rho U_0 (1 - a) W_1 r^2 dr$$

$$= 4 \pi \rho U_0 (1 - a) a' \Omega r^3 dr \quad (3.5)$$

where $W_1 = 2a' \Omega r$ defines (twice) the rotational interference factor. The geometric significance of $a$ and $a'$ are discussed in the next Section. Note that the average $W$ seen by the blades is

$$W = \frac{(W_0 + W_1)}{2} = \frac{W_1}{2} = a' \Omega r \quad (3.6)$$
3.3 Forces Acting on a Blade Element

Combining the analysis of the previous two sections, we know the velocity components at each blade element. For a blade element at radius $r$, the situation is summarised in Figure 3.2. We ignore any radial velocity, that is, velocities into or out of the page. The velocity in the wind direction is $U_1$ and that in the circumferential velocity is the sum of $(1 + a') \Omega r$ and $W$ as defined in Equation (3.6). Adding these velocities vectorially gives $U_T$, which is usually called the “total” or “effective” velocity as seen by the blade element. $\alpha$ is the angle of attack, which is sometimes called the angle of incidence. The figure also defines three important angles: $\theta_p$ is called the twist, and is the angle between the plane of rotation of the blade and the element’s chord line. Sometimes $\theta_p$ is termed the pitch angle, but we will use “pitch” to signify a constant, global change in $\theta_p$ caused by alteration of the blade’s attachment at the hub. The final angle $\phi$, has no general name; it is the angle between $U_T$ and the plane of rotation. From the geometry

$$\theta_p + \alpha = \phi$$  \hspace{1cm} (3.7)

Note very carefully that Figure 3.2 is NOT meant to indicate the location of the effective velocity relative to the blade element or the line of action of the forces. In aeronautical applications this line of action is very important; for example in determining the longitudinal stability of an aircraft. For wind turbines, however, its location is of much less significance.

Figure 3.3 shows the resulting lift and drag. By definition, the lift acts at right angles to $U_T$ while the drag acts in the direction of $U_T$. We will see in Chapter 4 that the typical magnitude of the lift is many times that of the drag. Since the primary purpose of the forces on the blade element is to produce a torque about the axis of rotation, or equivalently, a circumferential force in the direction of rotation, the figure indicates the necessity of maximising the lift and minimising the drag. Very simply, drag acts to reduce the torque produced by the lift. We will see that the key to wind turbine performance is the ratio of the lift to drag, rather than their individual values.

![Figure 3.3. Lift and drag for blade element](image-url)

Because wind turbines operate at high values of $\lambda$, typically in the range of 7 to 10, the magnitude of $\Omega r$ at the tip is about ten times that of $U_1$. At the hub, $\Omega r$ is nearly zero, so that $\theta_p$ must vary significantly with radius to maintain the angle of attack, $\alpha$, at reasonable values to avoid flow...
separation. In the next Chapter on aerofoil behaviour, we will see that efficient operation of wind turbines occurs over a limited range of $\alpha$.

The basic assumption is that the lift and drag acting on the blade element are the same as those on an aerofoil of the same section, angle of attack, and effective velocity. From the definitions of the lift and drag coefficients, $c_l$ and $c_d$ respectively, in Equation (4.1) of the next Chapter,

$$LIFT = \frac{1}{2} \rho U_1^2 c \cos \phi \quad \text{and} \quad DRAG = \frac{1}{2} \rho U_1^2 c \sin \phi$$  \hspace{1cm} (3.8)

where $c$ is the chord; its precise definition will be given in Chapter 4. We now resolve the lift and drag into the circumferential and axial components of interest to the wind turbine designer.

The total thrust on $N$ blade elements is

$$dF = \rho U_1^2 c N (c_l \cos \phi + c_d \sin \phi) \, dr/2$$  \hspace{1cm} (3.9)

and the torque due to the circumferential force is

$$dT = \rho U_1^2 c N (c_l \sin \phi - c_d \cos \phi) \, r \, dr/2$$  \hspace{1cm} (3.10)

Eqns (3.9) and (3.10) are the basic blade element equations. Over the years, a large number of modifications to these equations have been proposed, of which we will note only one, meant to account for the finite number of blades ($N < \infty$) on any real turbine. The necessity of some correction of this form comes from realising that the streamtube analysis has assumed that the velocities and pressures are uniform in the circumferential direction, whereas this may not be the case if the number of blades is finite. In other words, the axial velocity at the blade element may be different from $U_1$ which is the streamtube’s average velocity. A simple and commonly used correction for this effect is through “Prandtl’s tip loss factor”, $P$; $dr$ in (3.9) and (3.10) is replaced by $Pdr$ where

$$P = 2 \cos^{-1} \left( e^{-f} \right) / \pi$$  \hspace{1cm} (3.11a)

and

$$f = N(R_1 - r)/(2R_1 \sin \phi)$$  \hspace{1cm} (3.11b)

In practice, $P$, which is always less than unity, makes only a small difference to the predicted turbine performance and is often neglected.

**Example 3.1**

Suppose a turbine is operating at its maximum efficiency where $a = 1/3$ and $a'$ is negligible. If $\lambda = 7$, and ignoring tip loss effects, estimate the value of necessary $\theta_c$ to achieve $\alpha = 6^\circ$ at $r/R = 0.25$ and $r/R = 1$.

**Answer for $r/R = 0.25$**

In the blade element diagram of Figure 3.2, $U_1/U_0 = 2/3$, $r\Omega / U_0 = 1.75$, so that $\phi = 20.85^\circ$. From Equation (3.7), $\theta_c = 14.85^\circ$. 

Answer for \( r/R = 1.0 \)

In the blade element diagram of Figure 3.2, \( U_1/U_0 = 2/3 \), \( r\Omega /U_0 = 7.0 \), so that \( \phi = 5.44^\circ \). From Equation (3.7), \( \theta_p = -0.56^\circ \).

These results show the necessity for a significant variation in twist along a well-designed wind turbine blade.

### 3.4 Combining the Equations for the Streamtube and the Blade Element

We now combine Equations (3.2) and (3.5) for the wake with (3.9) and (3.10) for the blade element. One way of doing this is through the following relationship for \( \phi \)

\[
\tan\phi = \frac{1}{X} \left( \frac{1 - a}{1 + a'} \right)
\]

which follows from the velocity triangle Figure 3.2. \( X \) is the local speed ratio

\[
X = \frac{r\Omega /U_0}{\lambda R_1}
\]

If values are assumed for \( a \) and \( a' \), \( \phi \) can be found from (3.12) and then \( c_l \) and \( c_d \) determined and so on. This leads to an iterative process because (3.2) and (3.9) can be combined to give \( a_n \), the new estimate for \( a \) for the \( n \)th iteration, as

\[
a_n(1 - a_n) = f_u = U_1^2 (c_l \cos\phi + c_d \sin\phi)\sigma/(4U_0^2 P)
\]

where the quantities on the right hand side of (3.13) are those from the \( (n-1) \) iteration. In addition

\[
\sigma = \frac{Nc}{(2\pi r)}
\]

is the local solidity mentioned in Section 3.1. \( f_u \) in (3.13) plays the same role for the blade element as \( C_F \) for the whole turbine in the analysis of Chapter 2. Thus, for example, it is easy to modify (3.13) for the high thrust region in the same way that Equation (2.19) modifies (2.18).

Combining (3.5) and (3.9) gives

\[
a' = U_1^2 (c_l \sin\phi - c_d \cos\phi)\sigma/(4U_0^2 PX)
\]

as the new estimate for \( a' \). This allows \( a \) and \( a' \) to be adjusted iteratively for each blade element until the momentum and angular momentum fluxes balance the thrust and torque, respectively, on a blade element.

### 3.5 Fortran 90 Program for Wind Turbine Analysis

Some Conse...
or their meanings are obvious from the equations of the last two Chapters. To be consistent with the equations we have derived, all lengths are normalised by the tip radius and all velocities by the wind speed.

The main program, main.bet.f90 reads the relevant blade parameters from the file blade.in. These include the number of blades (Numb), the number of blade elements per blade (nbes) and the hub and tip radius. All blade elements have the same radial extent. The overall blade pitch is also set in blade.in. The main program then calls the subroutine in tcdist.f90 to determine the chord and twist of the blade elements, whose radial extent, dr, is constant for all elements. The particular chord and twist are for a blade that was extensively field- and wind tunnel tested by Anderson et al. (1982). Polynomial fits to their tabulated distributions are used in tcdist.f90.

Having fixed the blade geometry, the main program begins an outer loop to allow the user to vary the windspeed (U0) and then the tip speed ratio (lambda) in the inner loop. All the blade element calculations are done in the subroutine in the program betn.f90 and should be self-explanatory. The blade element calculations determine the lift and drag coefficients from the subroutine in N0012.f90 which contains data fits for the linear region of the NACA 0012 profile. These fits are restricted both in angle of attack (alpha) and Reynolds number (Re) AND SHOULD NEVER BE USED OUTSIDE THEIR RANGE OF VALIDITY.

The iteration is extremely simple: the new and old values of a (a) and a’ (adash) are simply averaged to get the appropriate values for the next iteration.

---

```
module blade
implicit none
contains

subroutine chortwist(nbes, rt, rh, rc, delr, twist, pitch, c)
imPLICIT NONE

! Program to give chord and twist distribution of the blade used by M. B. Anderson et al.
! Performance and wake measurements on a 3 m diameter HAWT, 4th Intl Symp. Wind
! Energy Systems (1982). The chord is normalised by the tip radius and the twist is in degrees

! Declaration of dummy arguments. Variables are:
! nbes number of blade elements!
! rt radius of tip
! rh radius of hub in the same units
! rc(i) radius of centre of blade element i
! delr increment in radius across blade element
! pitch blade pitch or setting angle in degrees
! c(i) chord of blade element i

integer, intent(IN) :: nbes
real, intent(IN) :: rt, pitch
real, intent(INOUT) :: rh
real, intent(OUT) :: rc(:), delr, twist(:), c(:)

integer :: i
real :: chd, x, pi, c_rh, twist_rh

integer :: :: i
real :: :: chd, x, pi, c_rh, twist_rh

! Chord distribution - 4th order polynomial fit to data in Anderson et al.(1982).
chd(x) = 0.2694289 - x*(0.9746212 - x*(1.7212092 - x*(1.4594518 - 0.4740908*x)))
```
\[
\begin{align*}
\text{rh} &= \text{rh}/\text{rt} \\
\text{delr} &= (1.0 - \text{rh})/\text{nbes} \\
\text{c}_{\text{rh}} &= \text{chd}(\text{rh}) \\
\text{c}(\text{nbes}+1) &= \text{chd}(1.0) \\
\text{twist}_\text{rh} &= t1(\text{rh}) + \text{pitch} \\
\text{twist}(\text{nbes}+1) &= t1(1.0) + \text{pitch}
\end{align*}
\]

\text{WRITE} (*, *)
\text{WRITE} (*, *)’ Radius(cm) chord(cm) twist (deg)’
\text{WRITE} (6, 50) 100*\text{rh}, 100*\text{rt}*\text{c}_{\text{rh}}, \text{twist}_\text{rh}
\text{WRITE} (20, 50) 100*\text{rh}, 100*\text{rt}*\text{c}_{\text{rh}}, \text{twist}_\text{rh}

\text{DO} i = 1, \text{nbes} \quad \text{! Calculate chord and twist at the centre of each element.}
\quad \text{IF} (i == 1) \text{THEN}
\quad \quad r_{c}(1) = \text{rh} + \text{delr}/2
\quad \text{ELSE}
\quad \quad r_{c}(i) = r_{c}(i-1) + \text{delr}
\quad \quad \text{END IF}
\quad \quad \text{c}(i) = \text{chd}(r_{c}(i))
\quad \quad \text{twist}(i) = t1(r_{c}(i)) + \text{pitch}
\quad \text{WRITE} (6, 50) 100*\text{rt}*r_{c}(i), 100*\text{rt}*\text{c}(i), \text{twist}(i)
\quad \text{WRITE} (20, 50) 100*\text{rt}*r_{c}(i), 100*\text{rt}*\text{c}(i), \text{twist}(i)
\quad \text{END DO}
\quad \text{WRITE} (6, 50) 100*\text{rt}, 100*\text{rt}*\text{c}(\text{nbes}+1), \text{twist}(\text{nbes}+1)
\quad \text{WRITE} (20, 50) 100*\text{rt}, 100*\text{rt}*\text{c}(\text{nbes}+1), \text{twist}(\text{nbes}+1)
\quad \text{WRITE} (*, *)
\quad 50 \quad \text{FORMAT}(3x, e12.5))
\quad 200 \quad \text{FORMAT} (3x, f7.4, 3x, f9.5, 3x, f6.2)
\quad \text{RETURN}
\text{END SUBROUTINE} \text{chordtwist}

\text{FUNCTION} t1(x)
\quad \text{! Twist distribution - fit to data in Anderson et al. (1982) by 4th order polynomial to } r/R = 0.7, \quad \text{! then linear.}
\quad \text{IMPLICIT NONE}
\quad \text{REAL} :: t1
\quad \text{REAL} :: x
\quad \text{IF} (x <= 0.7) \text{THEN}
\quad \quad t1 = 54.16632 - x*(307.42939 -x*(719.549614 - x*(785.971096 – x*326.673372)))
\quad \text{ELSE}
\quad \quad t1 = 5.318999 - 7.059999*x
\quad \text{END IF}
\quad \text{RETURN}
\text{END FUNCTION} t1
\text{END MODULE} blade

\text{FUNCTION} t1(x)
\quad \text{! Twist distribution - fit to data in Anderson et al. (1982) by 4th order polynomial to } r/R = 0.7, \quad \text{! then linear.}
\quad \text{IMPLICIT NONE}
\quad \text{REAL} :: t1
\quad \text{REAL} :: x
\quad \text{IF} (x <= 0.7) \text{THEN}
\quad \quad t1 = 54.16632 - x*(307.42939 -x*(719.549614 - x*(785.971096 – x*326.673372)))
\quad \text{ELSE}
\quad \quad t1 = 5.318999 - 7.059999*x
\quad \text{END IF}
\quad \text{RETURN}
\text{END FUNCTION} t1
\text{END MODULE} blade

N0012.f90

\text{MODULE} aerofoil
\quad \text{IMPLICIT NONE}
\quad \text{CONTAINS}
\quad \! Subroutine to calculate the lift, cl, and drag, cd, coefficient of a NACA0012 aerofoil using the
\quad \! correlations of McCroskey (1987).
SUBROUTINE LandD(alpha_in, Re, cl, cd)
    IMPLICIT NONE
    REAL, INTENT(IN) :: alpha_in, Re ! Declare dummy arguments
    REAL, INTENT(OUT) :: cl, cd
    REAL :: alpha, cd0, delcd ! Declare local variables
    alpha = alpha_in

    ! The linear range is |alpha| < 12 degrees. An value of alpha =12.0 in the output file LandD.out
    ! (in betn.f90) indicates an invalid calculation.
    IF (alpha > 12.0) alpha = 12.0
    IF (alpha < -12.0) alpha = -12.0
    IF (Re < 500,000) WRITE (*, *) " Reynolds number is too low!!!"
    cl = alpha*(0.1025 + 0.00485*LOG10(Re/10**6)) ! Equn (4.3) for Cl.
    cd0 = 0.0044 + 0.018*Re**(-0.15) !! Equn (4.4) for minimum Cd.
    delcd = (cl/1.2)**2*0.009 ! A data fit to obtain Cd at other angles.
    cd = cd0 + delcd
    RETURN
END SUBROUTINE LandD

END MODULE aerofoil

MODULE calc_mod
    IMPLICIT NONE
    CONTAINS
    SUBROUTINE calc(Numb, nbes, delr, rhub, rtip, rc, twist, c, alpha, U0, Lambda)
        USE aerofoil
        IMPLICIT NONE
        ! Subprogram to implement blade element/one-dimensional wake analysis for a horizontal-
        ! axis wind turbine with any number of blades of any length.
        INTEGER, INTENT(IN) :: Numb, nbes ! Declare dummy arguments.
        REAL, INTENT(IN) :: delr, U0, Lambda, rtip, rhub
        REAL, INTENT(IN) :: rc(:), twist(:), c(:)
        REAL, INTENT(INOUT) :: alpha(:)
        INTEGER, PARAMETER :: max = 50 ! Declare local variables.
        REAL, PARAMETER :: TOL = 0.0005
        INTEGER :: i, j
        REAL :: a, adash, cosphi, sinphi, phi, Ut, X
        REAL :: diffa, sigma, suma, sumadash
        REAL :: smallf, P, Re, faca, cl, cd
        REAL :: newa, newadash, thrust, torque, power, U1
        REAL :: delthr, deltord, delpower, gam, PI
        REAL :: olda, oldadash
        REAL, PARAMETER :: visc = 1.5e-05
        PI = 4.0*atan(1.0)
thrust = 0.0           ! Initialise variables.
torque = 0.0
power = 0.0
a = 0.3
adash = 0.0

DO i = 1, nbes             ! Begin main loop.
  X = Lambda*rc(i)                         ! Calcute local speed ratio and local solidity.
sigma = Numb*c(i)/PI/rc(i)/2.0

  DO j = 1, 200     ! For each blade element, allow up to 200 iterations.
    phi = atan((1 - a)/(1 + adash)/X)     ! Find phi from Equn (3.12)
    cosphi = cos(phi)
    sinphi = sin(phi)
    smallf = Numb*(1 - rc(i))/sinphi/2.0
    P = 2*acos(exp(-smallf))/PI

    ! Evaluate Prandtl's tip loss factor, Equn (3.11).

    alpha(i) = phi*180.0/PI - twist(i)
    Ut = sqrt((1-a)*(1-a) + (X*(1 + adash))**2)
    Re = Ut*U0*c(i)*rtip/visc

    ! Find angle of attack, effective velocity, and Reynolds number.

    call LandD (alpha(i), Re, cl, cd)

    ! Balance axial momentum in wake to thrust on blade, Equn (3.13) to find new value of a.
    faca = Ut*Ut*sigma*(cl*cosphi + cd*sinphi)

    ! Determine circulation using Equn (4.8).
    gam = Ut*c(i)*cl*(1 - cd*cosphi/(cl*sinphi))/2.0
    IF (faca > 1.0) THEN

    ! Use Equn (3.13) when a < 0.5.
    newa = (1 + sqrt( faca - 1))/2

    ELSE

    ! Use Glauert's empirical correction when a > 0.5, Equn (2.19).
    newa = (1 - sqrt(1 - faca))/2
    END IF

    ! Use Equn (3.14) to get adash for next iteration.
    newadash = Ut*Ut*sigma*(cl*sinphi - cd*cosphi)/4/X/(1 - a)/P
    IF (newadash < 0.001) newadash = 0.001
    IF (newadash > 0.1) newadash = 0.1

    ! Convergence is based on checking successive values of a.
diffa = ABS(a - newa)
IF (diffa < TOL ) EXIT ! The calculations have converged.
olda = a
oldadash = adash
a = (olda + newa)/2.0
adash = (oldadash + newadash)/2.0
END DO

! Determine contribution of blade element to blade's torque and thrust.

delthr = Numb*Ut*Ut*c(i)*delr/PI
deltor = delthr*rc(i)*(cl*sinphi - cd*cosphi)
delthr = delthr*(cl*cosphi + cd*sinphi)

thrust = thrust + delthr
deltorque = deltor*Lambda
defp = deltor

U1 = 1.0 - a
WRITE (*, 50) rc(i), U1, gam, delthr, defp
WRITE (27, 50) rc(i), alpha(i), cl, cd

END DO
WRITE (*, 100) power, thrust
WRITE (30, 100) power, thrust
50 FORMAT(5(3x, f7.4))
100 FORMAT(/," Cp = ", f7.4, " Ct = ", f7.4, /)
RETURN
END SUBROUTINE calc
END MODULE calc_mod

---

**main_bet.f90**

PROGRAM bladewake
USE blade
USE calc_mod
IMPLICIT NONE

! Program to implement blade element/one-dimensional wake analysis for a horizontal-axis wind turbine with any number of blades of any length. Program and subprograms written by D.H. Wood

! Main variables are:
- a - axial interference factor
- adash - rotational interference factor
- alpha - angle of attack of blade element
- c - chord of blade element
- cd - drag coefficient
- cl - lift coefficient
- P - Prandtl's tip loss factor
- gam - circulation of blade element
- Numb - number of blades
- nbes - number of blade elements
- phi - angle between Ut and plane of rotation
- rhub - radius of hub
- rtip - radius of tip
- rc - radius of midpoint of blade element
- sigma - local solidity
! \text{twist} - \text{angle between chord line and plane of rotation}
! \text{Ut} - \text{effective velocity at blade element}
! \text{U0} - \text{wind speed}

\text{INTEGER :: i, j, nbes, Numb}
\text{REAL, PARAMETER :: TOL = 0.0005}

\text{REAL :: Pl, delr, X}
\text{REAL :: U0, Lambda, pitch, rtip, rhub}
\text{REAL :: newa, newadash, error, massflux}

! Note use of ALLOCATABLE arrays.

\text{REAL, ALLOCATABLE :: rc(:), twist(:), c(:), alpha(:)}

\text{PI = 4.0*atan(1.0)}

\text{OPEN (unit = 10, file = 'blade.in', status = 'old') ! Data file for blade parameters.}
\text{WRITE (*, *)}

\text{OPEN (unit = 20, file = 'chandtw.out') ! File to save chord and twist distribution.}
\text{OPEN (unit = 27, file = 'LandD.out') ! File to save angle of attack, lift and drag.}
\text{OPEN (unit = 30, file = 'PandTSR.out') ! File to save power and tip speed ratio.}

\text{READ (10, *) Numb ! Determine blade geometry.}
\text{READ (10, *) rhub, rtip}
\text{READ (10, *) nbes, pitch}

! Allocate the array length depending on the number of blade elements.
\text{ALLOCATE (rc(nbes), twist(nbes+1), c(nbes+1), alpha(nbes))}

! Determine the chord and twist distribution.
\text{CALL chortwist (nbes, rtip, rhub, rc, delr, twist, pitch, c)}

! Main loop to allow any number of wind speeds.
\text{DO}
\text{WRITE(6, '('' Enter U0: end with -ve: '', $')')}
\text{READ (*, *) U0}
\text{IF (U0 < 0.0) EXIT ! Escape main loop.}

\text{DO ! Inner loop for any number of tip speed ratios.}
\text{WRITE(6, '('' Enter Tip speed ratio: end with -ve: '', $')')}
\text{READ (*, *) Lambda}
\text{IF (Lambda < 0.0) EXIT ! Escape inner loop.}

\text{END DO}
\text{WRITE (*, *)}
\text{WRITE (*, *) ' rc(i) U1(i) gam delF delT'}
\text{WRITE (27, 50) U0, lambda}
\text{WRITE(27, *) ' rc(i) Alpha Cl Cd'}

! CALL subroutine for blade element calculations.
\text{CALL calc(Numb, nbes, delr, rhub, rtip, rc, twist, c, alpha, U0, Lambda)}

\text{END DO}
\text{FORMAT(1, '' For U0 = '', f7.3, '' m/s and Lambda = '', f7.3)}
\text{STOP}
\text{END PROGRAM bladewake}
3.6 Some Consequences of the Blade Element Equations

In preparation for the discussion in the next Chapter on Reynolds number effects, it is instructive to look at Equations (3.14) and (3.15) for optimum performance, in which case $a$ and $a'$ are fixed. Ignoring the drag, these equations lead to the requirement that $Re_c$ remains constant in order to maintain maximum power extraction as wind speed etc vary. The product $Re_c$ is sometimes called the reduced Reynolds number, and indicates that an aerofoil section need not have the highest possible $c_l$; in fact, a smaller $c_l$ can lead to larger $Re$ and hence and to a smaller performance penalty as the wind speed decreases.

We will delay until Chapter 5 the discussion of the implementation of the programs developed in this Chapter, because the last paragraph indicates the need to consider aerofoil behaviour before we proceed. However, this is a good opportunity to consider briefly the chord and twist distributions contained in tcdist.f90. These are plotted in Figure 3.4. We saw earlier in this Chapter the reason for the decrease in twist as radius increases: the need to keep each blade element at an appropriate angle of attack while $U_T$ increases with radius. In the next Chapter we will see again that the appropriate $\alpha$ is that which maximises the ratio of lift to drag. The justification for the approximately inverse relation between chord and radius is not available to us until we consider the blade’s circulation in the next Chapter.

References


Exercises

1. Make a list of the assumptions of blade element theory as presented in this Chapter.
2. What are the units of $dF$ in Equation (3.2) and $dT$ in (3.5)?

3. A turbine is operating at $\lambda = 9$ in a wind of 10 m/sec. Estimate the minimum $U_r$ (near the hub) and maximum $U_r$ at the tip.

4. How would Figure 3.2 change if a propeller, rather than a wind turbine, were being analysed?

5. Redo the calculations of Example 3.1 for $\lambda = 3$ and 10. What do the results suggest for a strategy to control the power output?

6. Interpret the $C_P$ versus $\lambda$ curve (Figure 1.6) for the Vestas V47 in terms of the variation in typical angle of attack.

7. The power curve for the V47 in Figure 1.5 is for constant $\Omega$ operation. Determine the qualitative behaviour of $\alpha$ as $U_0$ increases.