Tree-structured generation of orthogonal spreading codes with different lengths for forward link of DS-CDMA mobile radio

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Tree-structured generation of orthogonal spreading codes with different lengths is presented for orthogonal multiplexing of forward-link code-channels of different data rates in direct sequence code division multiple access DS-CDMA mobile radio. The bit error rate performance under a multi-user environment suffering multipath Rayleigh fading is evaluated by computer simulation.

Introduction: Improving the capabilities of multimedia communications is the target for third generation mobile communications systems. A variety of data services from low to very high bit rates is required; most services offered by current mobile radio systems are limited to voice and voice-band data services. Recently, direct sequence code division multiple access (DS-CDMA) [1] has been drawing much attention for third generation systems [2, 3]. The forward radio link (cell site-to-mobile) can use orthogonal spreading because all code-channels are synchronous, wherein the code-channels are distinguished by the combination of different short spreading codes (with the same code length) and a single long spreading code [1, 3]. Conversely, for the reverse radio link (mobile-to-cell site), all users are asynchronous and therefore, random long spreading codes unique to each user are used. One way to flexibly provide data services of different rates is to assign the multiple code-channels in the forward link to each user according to the data rate requested [3]. This orthogonal multicode assignment requires multiple RAKE combiners at the mobile receivers, each combiner belonging to a different code channel. This increases the complexity of the mobile receivers. Of course, for the reverse link, orthogonal multicode transmission [3] or variable rate transmission using a single long spreading code [2] can be applied. In this Letter, we propose a tree-structured generation method of orthogonal spreading codes with different code lengths for forward link applications and we evaluate its bit error rate (BER) performance by computer simulation under a multi-user environment wherein multiple users requesting different data rates are active under multipath Rayleigh fading.

Code generation method: Let \( C_N \) denote the set of \( N \) binary spreading codes of \( N/2 \)-chip length, \( \{ C_n(n) \}_{n=1}^{2N} \), where \( C_n(n) \) is the row vector of \( N \) elements and \( N = 2^K \) (\( K \) is a positive integer); it is generated from \( C_{2N} \) at

\[
C_N = \begin{bmatrix}
C_N(1) \\
C_N(2) \\
C_N(3) \\
\vdots \\
C_N(N-1) \\
C_N(N)
\end{bmatrix} = \begin{bmatrix}
C_{2N}(1)C_{2N}(2) \\
C_{2N}(1)C_{2N}(3) \\
C_{2N}(1)C_{2N}(4) \\
\vdots \\
C_{2N}(1)C_{2N}(2N) \\
C_{2N}(1)C_{2N}(2N+1) \\
C_{2N}(1)C_{2N}(2N+2) \\
\vdots \\
C_{2N}(1)C_{2N}(2N-1) \\
C_{2N}(1)C_{2N}(2N)
\end{bmatrix}
\]

(1)

where \( C_{2N}(n) \) is the binary complement of \( C_{2N}(n) \) and is the row vector of \( N/2 \) elements. As a result, tree-structured spreading codes are generated recursively as shown in Fig. 1 (\( K = 6 \) is assumed). Starting from \( C(1) = 1, 0 \) is a set of \( 2^K \) spreading codes are generated at the \( K \)th layer (\( k = 1, 2, \ldots, K \)) from the top. The code length of the \( k \)th layer is \( 2^k \) chips and can be used for the code-channels transmitting data at \( 2^{k+1} \) times the lowest rate. It can be understood from eqn. 1 that generated codes of the same layer constitute a set of Walsh functions and they are orthogonal. Furthermore, any two codes of different layers are also orthogonal except for the case that one of the two codes is a mother code of the other; for example, all of \( C(2), C(1), C(1), C(1) \), and \( C(2) \) are mother codes of \( C(3) \), and so are not orthogonal against \( C(3) \). From this observation, we can easily find that if \( C(1) \) is assigned to a user requesting an 8 times higher data rate service, all 14 codes \( \{ C(1), C(2), C(3), \ldots, C(4), C(5), \ldots, C(8) \} \) generated from this code cannot be assigned to other users requesting lower rates; in addition, mother codes \( \{ C(1), C(1) \} \) of \( C(1) \) cannot be assigned to users requesting higher rates (of course, the use of codes of excessively short code lengths may also be impractical). This is a restriction imposed on the proposed tree-structured code assignment in order to maintain orthogonality; however, the use of the \( C(1) \) code is equivalent to the simultaneous use of eight consecutive codes \( \{ C(1), \ldots, C(8) \} \) in the case of orthogonal multicode assignment.

It should be noted that, of course, code generation can start from any \( k \)th layer using a set of \( 2^k \) orthogonal codes other than Walsh functions, e.g. orthogonal Gold codes.

To make the other-cell interference a noise-like interference, each cell's code-channels formed by the orthogonal spreading codes generated above are multiplied by a long random spreading code unique to that cell. If the radio channel is frequency nonselective or if the delay spread is negligible compared to the chip interval, all code-channels remain orthogonal. However, if the radio channel is frequency selective, the orthogonality is destroyed. Computer simulations were used to evaluate the BER performance of the forward link using tree-structured orthogonal spreading code assignment under frequency selective Rayleigh fading.

Computer simulation: The DS-CDMA forward link consists of a number of orthogonal code-channels. The data to be transmitted on each code-channel are first encoded by a rate-1/3 convolutional code, block-interleaved by a \( 2^K \times 16 \)-bit interleaver, and then, quaternary phase shift keying (QPSK) modulated, resulting in 1 frame of 16 slots, each slot containing \( N_s \) symbols. The convolutional code used here has a constraint length of 7 bits and generator polynomials: 554, 624, 764 (octal notation). A number \( N_{\text{slot}} \) of rows in the interleaver, varies according to the data rate while that of the columns remains the same. Each column corresponds to a slot of \( N_{\text{slot}} \) bits and consists of a time-multiplexed pilot of \( N_p \) zeros placed at the beginning and end of the column and coded data of \( 2(N_{\text{slot}}-N_p) \) bits. The time-multiplexed pilots are used at the receiver for multipath channel estimation to perform coherent RAKE combining [3]. Finally, binary PSK spreading is applied to each code channel. Orthogonal spreading codes of different code lengths are generated according to eqn. 1 using \( C(1) = 1 \) and \( K = 6 \). In the simulation, \( N_{\text{slot}} = 2560 \times 2^4 \) and \( N_s = 2560 \times 2^4 \) are used for the code-channels transmitting data at \( 2^4 \times \) the lowest rate denoted by \( R_c \), where \( k = 1, 2, \ldots, 6 \). We assume a chip rate of 4.096Mchip/s, resulting in a data frame length of 10ms. The lowest data rate (\( k = 6 \)) becomes \( R_c = 38.4 \text{kbit/s} \) including overhead data (e.g. cyclic redundancy check code and convolutional code tail) and the corresponding QPSK modulation rate is 64k symbol/s. The long random code used in the simulation was generated by a random binary number generator and had a repetition period of 40960 chips which is equal to 10ms.

A multipath Rayleigh fading channel model with two independent resolution paths having equal average powers, but a propagation delay time difference of eight chips, was assumed (thus, the multipath channel delay spread is 0.98\( \mu \)s). The fading maximum Doppler frequency is 80Hz. A single radio cell site was assumed. At the receiver, a de-spreader resolves the received multipath spread signal into two components and then channel estimation is performed using the time-multiplexed pilot for subsequent coherent RAKE combining [3]. The simulation assumed ideal chip timing. The RAKE combiner output sequence is a soft decision sequence and is de-interleaved for succeeding soft decision Viterbi decoding to recover the transmitted data.
The simulations considered the case of eight users of rate $R$, and one user of rate $R = 8R$ (this condition is represented by $R = 8 + R_x \times 1$ in Fig. 2). For tree-structured orthogonal code assignment, eight codes selected from code set $C_8$ were assigned to eight users requesting a data rate of $R$, and one code from code set $C_1$ to the one user requesting data rate $R$. For multicast assignment, 16 codes selected from code set $C_{16}$ are assigned to nine users (one user uses eight codes simultaneously). We repeated the above simulation using different codes and the results of measurements from Fig. 2 indicate that the forward link using tree-structured comparison, the BER performance for the case where only a single user of data rate $R$, is indicated by curve TCAL (MCA1) and that of the single-user case due to other user interference.

For zero input, the behaviour is described by the following discrete-time state equations [3, 4]:

$$x_{k+1} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -r^2 & 2r \cos \theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k$$

$$f = f(x_k) = Ax_k + [0 \ g]^T s_k$$

$f$ denotes the net effect of the nonlinearity of the quantiser, $g$ is a scale-factor, and the resonator has poles at $\pm \phi$ in the $z$-plane. The nonlinearity, $f$, is given by $u = uQ(u)$ where $Q(u) = \text{sgn}(u)$ for a two-level quantiser. Sequence $s_k$ is the binary $\{+1, -1\}$ output sequence generated by the quantisation of the non-linear oscillation.

**Nonlinear oscillations:** For zero input, commonly occurring oscillations have simple output sequences such as $s_k = \ldots + + + + + \ldots$, $s_k = \ldots + + + - - - - - \ldots$, although very complex oscillations can occur for some $\theta, \phi$ values [3–6], particularly for lossless resonators ($r = 1$) and resonators with negative damping ($\phi > 1$).

In many cases, it is observed that after any initial transients, the state-variable amplitudes are bounded by $x_k$. That is, the states remain within a square: $E = -g \leq x_1 \leq +g$, $E = -g \leq x_2 \leq +g$. Some conditions on the parameter values $(r, \phi)$ for which these zero-input oscillations are bounded by this square have been given previously [4, 6]. In this Letter these conditions are extended to cover the full range of $(r, \phi)$ values. The zero-input trajectories are (i) strictly-bounded if, for $x_k \in \mathbb{I}_1$, $x_k \in \mathbb{I}_2$, $\forall k > 0$ (ii) ultimately-bounded if, for $x_k \in \mathbb{I}_2$, $\exists k_n < 0$ such that $x_k \in \mathbb{I}_2$, $\forall k > k_n$. The strictly-bounded condition means that any oscillation having a state within $\mathbb{I}_2$ has all its states within $\mathbb{I}_2$. The ultimately-bounded condition allows for an initial transient extending outside $\mathbb{I}_2$. Sufficient conditions are derived below for the trajectories from any state in $\mathbb{I}_2$ to be strictly-bounded for all $r, \phi$.

Despite extensive simulations, no zero-input oscillations have been observed having all states outside $\mathbb{I}_2$, so it is conjectured that they do not occur. If this conjecture is true, the strictly-bounded and the ultimately-bounded conditions are each sufficient for all zero-input oscillations to be bounded by the square for any given pair $(r, \phi)$ of parameter values meeting the conditions.

**Nonlinear map:** For zero input, at each time step, the state space is transformed by a linear transformation defined by the matrix $A$, which may be considered as a rotation through $\pi/2$, together with a shear. This is illustrated for $\gamma = 0.9$ by Fig. 1a and $c$ for $\theta = 75^\circ$ and $30^\circ$, respectively, ($ABC \Rightarrow AB'CD'$). This is followed by a

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**References**

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**Zero-input oscillation bounds in a bandpass $\Sigma\Delta$ modulator**

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Sufficient conditions for the state-variables of zero-input oscillations of a bandpass $\Sigma\Delta$ modulator structure to remain within a prescribed square region in the state-space are derived for the full range of parameter values of the digital resonator within the modulator.

**Introduction:** Nonlinear oscillations in bandpass $\Sigma\Delta$ modulators cause spurious tones or inferior signal-to-noise ratios, and their suppression or amplitude limitation is essential [1, 2]. Determining upper-bounds on their amplitudes is an important, though generally difficult requirement. Theoretical bounds are often not available, so estimation by simulation is usual.

Some bounds on the zero-input oscillations of a particular bandpass $\Sigma\Delta$ modulator structure are given here. The structure [2, 4] comprises a second-order digital resonator, and for zero input, the behaviour is described by the following discrete-time state equations [3, 4]:

$$x_{k+1} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + f \begin{bmatrix} 0 & 1 \\ -r^2 & 2r \cos \theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k$$

$$f = f(x_k) = Ax_k + [0 \ g]^T s_k$$

This is followed by a

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