Terrain-Based Propagation Model for Rural Area—An Integral Equation Approach

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Abstract—A terrain-based propagation model for vertically polarized radio waves is described, based on the field integral equation for a smooth surface. The model is simplified from a 3D integral equation model to a one-dimensional integral equation by assuming that the surface is magnetically perfectly conducting (a soft surface) with no transverse variations. By assuming no back scattering, the integral equation is turned into a simple integral. The method is tested numerically with known solutions. The integral equation model is also applied to actual terrain profiles, assuming that the surface is magnetically perfectly conducting (a perfect magnetic conductor). It is assumed to be smooth, which is not a strong assumption, since it means that we do not have rapid variations within the integration step. Since this is of the order of a quarter of a wavelength, quite rapid fluctuations may be treated, in fact, the method has been applied to rough surfaces as well. Another assumption simplifying the numerical work, but not essential, is the use of a 2-D surface, i.e., there are no variations of the surface transverse to the direction of propagation. This is, of course, justified by the narrow Fresnel zones defined for flat surfaces, but it is a source of error to neglect contributions from the side. Only comparisons with measurements will indicate the magnitude of the error. Finally, we shall assume only forward type of propagation, neglecting back scattering. The reason for this is not only numerical efficiency, but the fact that we are interested only in the slow fading. Reflections or back scattering from nearby objects give rise to fast Rayleigh distributed fading which is not possible or desirable to predict in detail. Only the average field describing the slow fading is of interest here.

The assumption of perfect magnetic conductivity may seem strange, but for vertical polarization, it corresponds to the case of a reflection coefficient of minus 1. Over real ground, the reflection coefficient approaches minus 1 for grazing incidence, and in practice, the angles are so small that it is a good approximation. It is an experimental fact that, in the microwave region where the Norton surface wave can be neglected, there is only a small difference between results for the two polarizations.

We first briefly derive the integral equation and simplify it using the assumptions mentioned above. It is then compared with some UTD calculations (Luebbers [7]) for a wedge, and finally for an extended frequency range compared with narrowband measurements. By applying some statistics on the measurements, we find the standard deviation (STD) of the errors and find it to be a smoothly growing function of frequency. Finally, the results are discussed together with some comments on the numerical efficiency of the method.

An alternative numerical approach is to solve the parabolic equation ([8], [9]) using finite difference schemes. This has the added advantage that atmospheric inhomogeneities may be taken into account. The results presented in this paper have been compared with the method of Marcus [9] with excellent results.

II. THE INTEGRAL EQUATION PROPAGATION MODEL (IE MODEL)

The electric field over a perfect magnetic conductor is the dual of the magnetic field over a perfect electric conductor, so we are essentially using the dual of the often used Magnetic-Field-Integral-Equation (MFIE) over a smooth surface (see e.g., [10]). The boundary conditions for a perfect magnetic conductor are

\[ \hat{n} \times \vec{H} = 0, \quad \hat{n} \cdot \vec{E} = 0. \]  

1. INTRODUCTION

Propagation over irregular terrain has been the subject of various propagation models for more than 40 years. There has recently been an increased interest in the possibility of automatic prediction of radio coverage based on geographical data bases for cellular radio in the microwave region up to a few GHz. Usually quite simple parametric models, like the Hata model [1] are used, but such a model does not reflect the dynamic variations in the average signal due to shadowing and diffraction. More advanced methods tend to become numerically very cumbersome for many successive obstacles, like Vogeler’s knife-edge diffraction [2]. Recently Janaswamy [3] has applied a Fredholm integral equation to small irregularities. One of the first attempts was performed in 1952 by Hufford [4] who derived an integral equation model based on Green’s theorem. Variations of this method were later used by Ott and Berry [5], Ott et al. [6]. In [5] it was mentioned that the method would experience numerical instabilities for higher frequencies (more than 10 MHz). In this paper we shall show that a Hufford-like integral equation for a perfect magnetic conductor has no numerical problems and we shall test the accuracy with extensive experiments. The surface is assumed to be smooth, which is not a strong assumption, since it means that we do not have rapid variations within the integration step. Since this is of the order of a quarter of a wavelength, quite rapid fluctuations may be treated, in fact, the method has been applied to rough surfaces as well. Another assumption simplifying the numerical work, but not essential, is the use of a 2-D surface, i.e., there are no variations of the surface transverse to the direction of propagation. This is, of course, justified by the narrow Fresnel zones defined for flat surfaces, but it is a source of error to neglect contributions from the side. Only comparisons with measurements will indicate the magnitude of the error. Finally, we shall assume only forward type of propagation, neglecting back scattering. The reason for this is not only numerical efficiency, but the fact that we are interested only in the slow fading. Reflections or back scattering from nearby objects give rise to fast Rayleigh distributed fading which is not possible or desirable to predict in detail. Only the average field describing the slow fading is of interest here.

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With these boundary conditions the integral equation becomes
\[ \hat{n} \times \vec{E}(p) = T \hat{n} \times \vec{E}_i(p) + \hat{n} \times \frac{T}{4\pi} \int_S \left( \hat{n}' \times \vec{E}_i \right) \times \nabla' \psi \, ds' \]
(2)
where \( p \) is the point of observation and \( p' \) is a source point on the surface. \( \hat{n}, \hat{n}' \) are the surface normal vectors at \( p, p' \). \( \vec{E}_i \) is the incident field from the sources and \( \psi \) is the spherical Green's function given by
\[ \psi = \frac{e^{-j\beta r}}{r}, \quad r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \]
and \( T \) is a function defined as 2 if \( p \) is on \( S \) and 1 elsewhere. Due to the singularity of \( \psi \) at \( p = p' \) this point should be omitted from the integration, instead the principal value of the integral should be taken.

The integral part of (2) constitutes the field scattered from the nonplanar part of the surface \( S \) to the position \( p \) on the surface. Thus the field \( \vec{E}(p) \) can be expressed by an incident field \( \vec{E}_i \) and a scattered field \( \vec{E}_{\text{scat}} \).

Introducing the equivalent source current notation \( \vec{M}^s \)
\[ \vec{M}^s = -\hat{n} \times \vec{E} \]
(5)
the integral equation for the total space \( V \) can be written as
\[ \vec{M}^s(p) = T \vec{M}^i_i(p) + T \vec{M}_{\text{scat}}(p) \]
\[ = T \vec{M}^i_i(p) + \hat{n} \times \frac{T}{4\pi} \int_S \vec{M}^s \times \nabla' \psi \, ds' \]
\[ = T \vec{M}^i_i(p) + \frac{1}{4\pi} \int_S \left( (\hat{n} \cdot \hat{r}) \vec{M}^s - (\hat{n} \cdot \vec{M}^s) \hat{r} \right) \]
\[ \times \frac{1 + j\beta r}{r^2} e^{-j\beta r} \, ds' \]
(6)
where \( \hat{r} = \frac{\vec{r}}{r} \) is a unit vector from the scatterer point \( p' \) to the point of observation \( p \).

III. SIMPLIFICATIONS OF THE INTEGRAL EQUATION

The solution of the scattered field on the surface will be determined for an irregular surface which is assumed to be constant transverse to the direction of propagation. The propagation over the terrain can then be approximated by the propagation over a 2-D rough surface because the active scatter region is rather narrow transverse to the direction of propagation.

The 2-D surface \( S \) can then be described by the function \( z = \zeta(x, y) = \zeta(x) \). The normal vector \( \hat{n} \) of the surface will lie in the \( x, z \) plane.

The solution at any point \( p \) requires a total solution at the entire surface \( S \). To simplify the equation, the integral in the direction transverse to the \( Tx - Rx \) plane will be deduced analytically. The resulting integral equation is thereby simplified to a solution along the \( Tx - Rx \) axis as illustrated in Fig. 1.

When a source radiating a vertically polarized electric field is placed in the \( x, z \) plane the surface source current \( \vec{M}^s \) has the following symmetries
\[ \vec{M}^s_x(y') = -\vec{M}^s_z(-y') \]
\[ \vec{M}^s_z(y') = -\vec{M}^s_x(-y') \]
\[ \vec{M}^s_y(y') = -\vec{M}^s_y(-y'). \]
(7)
As a consequence of these properties and the fact that the \( y \) component of \( \vec{n} \) is zero, the scatter contributions in the \( z \) and \( x \) directions, from each side of the \( x \)-axis, will be of equal amplitude and have opposite signs. Thus when determining the scattered field at the surface along the \( x \)-axis, the contribution of \( \vec{M}_{\text{scat}} \) in the \( x \) and \( z \)-directions will be canceled when the integration in the \( y \) direction is performed symmetrically on both sides of the \( x \)-axis. The only remaining contribution to the scatter integral is the contribution in the \( y \)-direction. This is given by
\[ \vec{M}_{\text{scat}}(x) = \frac{1}{4\pi} \int_S \left( \vec{M}^s_x(\hat{n} \cdot \hat{r}_2) + \frac{y'}{r_2} (\hat{n} \cdot \vec{M}^s) \right) \]
\[ \times \left( 1 + j\beta r_2 \right) e^{-j\beta r_2} ds' \]
\[ \approx \frac{1}{4\pi} \int_S \vec{M}^s_x(\hat{n} \cdot \hat{r}_2) \left( 1 + j\beta r_2 \right) e^{-j\beta r_2} ds' \]
(8)
The approximation in (8) is exact for a cylindrical wave originating from a line source directed along the \( y \)-axis where \( \hat{n} \cdot \vec{M}^s = 0 \). The approximation is valid when the most significant contributions come from the vicinity of the axis of propagation, which will be the case when the surface is constant transverse to the axis of propagation.

Due to the fact that \( \vec{M}^s \) originates from a point source it must be reasonable to assume a local spherical phase distribution of \( \vec{M}^s \) with center at the source position \( p_0 \). The phase at the arc \( r_1 = R_1 \) in Fig. 2 is thus constant and equal to the phase \( \phi_1(x', 0) \) of \( \vec{M}^s(x', 0) \). The phase \( \phi_2(x', y') \) along the line in the \( y \) direction is then approximated by the sum of the on-axis phase \( \phi_1(x', 0) \) and the spatial phase distance \( \beta \Delta r_1 \)
\[ \Delta \phi_1(x', y') = \phi_1(x', y') - \phi_1(x', 0) \]
\[ \approx \beta \Delta r_1(x', y') \]
\[ \approx \beta \frac{y'^2}{2R_1}, \quad y' < R_1 \]
(9)
where \( R_1 = r_1(x', 0) \) and \( r_1 = R_1 + \Delta r_1 \). The requirement \( y' < R_1 \) is due to the validity of the series expansion of \( \Delta r_1 \).

Similarly \( r_2 \) can be divided into a constant and a varying part \( r_2 = R_2 + r_2 \) where the varying part can be approximated
The total path length difference $\Delta r_{12}$ can be expressed as

$$\Delta r_{12} = \Delta r_1 + \Delta r_2 \approx \frac{R_1 + R_2}{2R_1R_2} y'^2$$

(11)

Since the phase variation of the integral contributions in the $y'$-direction is more significant than the amplitude variation, the amplitudes of the contributions to scattering integral will be considered constant in the $y'$-direction. The integral is then reduced as follows

$$M_{\text{scat}}(x) = \frac{1}{4\pi} \int_{-\infty}^{\infty} M^s(x') (\hat{n} \cdot \hat{r}_2)$$

$$\times \frac{1}{\lambda_0} \int_{-\infty}^{\infty} \frac{j\beta R_2}{R_2} e^{-j\beta R_2} d\phi' d\psi'$$

$$\approx \frac{1}{4\pi} \int_{-\infty}^{\infty} M^s(x') (\hat{n} \cdot \hat{r}_2)$$

$$\times \frac{j\beta R_2}{R_2} e^{-j\beta R_2} d\phi' d\psi'$$

$$= \frac{1}{4\pi} \int_{-\infty}^{\infty} M^s(x') (\hat{n} \cdot \hat{r}_2)$$

$$\times \sqrt{\frac{R_1 R_2}{R_1 + R_2}} e^{-j\frac{\pi}{4}}$$

(12)

(13)

(14)

In (12) it is observed that $\hat{n} \cdot \hat{r}_2 = 0$ when the scatter point and the observation point is on a planar surface, i.e., no multiple scattering along the surface. Thus, for a locally smooth surface, there is no scattering from the region near the observation point. The near field term can therefore be neglected, as done in (13) in which the assumption of constant amplitude in the $y'$-direction has been used. The integral with respect to $y'$ is now a Fresnel integral and may be integrated (14).

The scattering integral has now been reduced to an on-axis integral over the surface currents. The incident on-axis surface currents can now be determined as

$$\hat{M}_i(x) = \hat{n} \times \hat{E}_i(x)$$

$$= -\hat{n} \cdot \hat{r} E_i(x) \hat{y}$$

(15)

where $E_i$ is dependent on the source and distance.

Equations for the total surface currents $M^s$, the incident surface currents $M^s_i$, and the scattered surface currents $M_{\text{scat}}$ along the direction of propagation have now been derived. The total on-axis equation for the unknown quantity is

$$M^s(x) = T M^s(x) + \frac{T}{4\pi} \int_{-\infty}^{\infty} M^s(x') (\hat{n} \cdot \hat{r}_2)$$

$$\times \sqrt{\frac{\lambda_0 R_1 R_2}{R_1 + R_2}} e^{-j\pi/4} d\phi' d\psi'$$

(16)

The solution is valid for a 2-D surface with gentle slopes and local smoothness.

An investigation of the active scatter region from a surface with gentle slopes and a point source with dipole characteristics has been performed, and it turned out that the contributions from $-\infty$ to the transmitter point and from $p$ to $\infty$ are negligible. This means that we can turn the integral equation into a simple integral, described by the following algorithm

$$M_n^s = T M_{\text{on}}^s + \frac{T}{4\pi} \sum_{m=0}^{n-1} M_m^s f(n, m) \Delta x_m$$

(17)

where

$$f(n, m) = (\hat{n} \cdot \hat{r}_2) \frac{j\beta}{R_2} \sqrt{\frac{\lambda_0 R_1 R_2}{R_1 + R_2}} e^{-j(\beta R_2 + \frac{\pi}{4})}$$

$$\frac{\Delta l_m}{\Delta x_m}$$

$$R_1 = \sqrt{x_n^2 + (z_m - h_1)^2}$$

$$R_2 = \sqrt{(x_n - x_m)^2 + (h_2 - z_m)^2}$$

(18)

and the spatial sampling resolution is less than $\lambda_0/2$. Equation (16) can of course be applied to determine the vertically polarized $E$ field at any location above the surface. The principle of the algorithm in (17) is illustrated in Fig. 3.

Equation (17) is implemented in a C-program and has been applied to various surfaces. The program is implemented with a transmitter pattern as a dipole antenna and the estimated path loss predictions are based on the vertical $E$-field. Some of the results are presented in the following sections.

IV. PROPAGATION OVER A SMOOTH WEDGE

The derived IE model for path loss predictions has been applied to a smooth perfectly conducting wedge (Luebbers, [7]), which is assumed to be infinite transverse to the direction of propagation. The wedge has the center at 2.5 km and a height of 50 m as illustrated in Fig. 4.

The path loss predictions are carried out at 100 MHz and 1 GHz with a transmitter height of 10 m and a receiver height
varying from approximately 0 m to 200 m. In Fig. 5, the IE path loss predictions are shown and compared with UTD path loss predictions performed under the same conditions. As it appears from Fig. 5, there is a very good agreement between the IE path loss predictions and UTD path loss predictions for a smooth wedge.

V. THE IE MODEL APPLIED TO PATH LOSS MEASUREMENTS

In Northern Jutland near Aalborg in Denmark, measurements at the frequencies 143.9 MHz, 435 MHz, 970 MHz, and 1.9 GHz have been performed. Five profiles with a path length of 6–11 km were selected. The sought characteristics of the profiles were that the roads were fairly straight over some kilometers in terms of lateral variations, so that the 2-D assumptions would be approximated. In the vertical plane there were no restrictions. The height variations are of the order 20–50 meters.

Along the profiles there are a few buildings and some small forests. The path loss predictions are based on digitized terrain profiles with a horizontal resolution of 50 m. The measurements are performed in the autumn when there still were leaves on the trees. The measurements were carried out with transmitted power of 10 W and a transmitter antenna gain of 8 dBi. The receiver antenna is a λ/4 monopole on top of a van. Field strength is recorded for each 0.4 m and a mean value is derived by averaging all 25 values for each 10 m. Most of the short-term fading would thus be averaged out.

In Figs. 6–9 a typical profile “Hjørringvej” is presented with measurements, IE path loss predictions, and Hata model path loss predictions for all four frequencies. The measurements are carried out with a transmitter height of 10.4 m and the receiving antenna is 2.4 m above the profile. Although the Hata model is based on urban measurements, it does allow for propagation in rural areas, and since it is widely used for planning purposes, it seemed relevant to use it for comparison.

The profile Hjørringvej includes three classes of land cover. The first 6 km is characterized by farmland with a few farms and trees along the road. From 6–9 km the profile is more characterized as a loosely built-up area with small houses and a few trees. The last 2 km of the profile is characterized by farmland with no houses and farms and only very few trees.

In Fig. 6, the results for the lowest frequency 143.9 MHz are presented. At this frequency, the agreement between the measurements and the IE path loss predictions is excellent. The Hata path loss predictions are slightly above the measurements. In Fig. 7 the results for the frequency 435 MHz are presented. At this frequency the agreement between the measurements and the IE path loss predictions are also excellent, and the Hata path loss predictions are very good for the mean value.

At the higher frequency 970 MHz in Fig. 8, the agreement seems to be fairly good in the area 0–8 km, while for the last 3 km the accuracy decreases. The lack of accuracy could be caused by the loosely built-up area which dominates the profile from 6–9 km and could contribute off-path scatter. The Hata path loss predictions make a good fit to the gross average.

At the highest frequency, 1900 MHz in Fig. 9, the agreement with the IE path loss predictions seems to be fairly good in the area 0–9 km. For the last 2 km the accuracy decreases. Here as with 970 MHz the lack of accuracy could be caused by the built-up area. The Hata path loss predictions make again a good fit to the measurements.

In Figs. 10–11, the STD of the difference between the path loss predictions and the measurements and the numerical value of the mean difference between the path loss predictions and
the measurements are illustrated as functions of the frequency. The values are found by averaging over all 5 profiles. The STD of the IE path loss predictions is smoothly growing from approximately 3 dB at 144 MHz to approximately 9 dB at 1900 MHz. For the Hata model, it is the same smoothly growing curve but 2–3 dB higher. The physics behind these systematic trends is unknown, possibly local roughness. Similar curves for the mean error show no such systematic trends. The numerical mean errors for the IE model are 2–3 dB at the lower frequencies, 144–435 MHz, and 5–6 dB at the higher frequencies, 970–1900 MHz.

VI. DISCUSSION

A propagation model based on an integral equation has been derived. To simplify the calculations, a number of assumptions have been built into the method. The surface is assumed to be smooth and perfectly reflecting, corresponding to a reflection coefficient of minus one for vertical polarization. This is known to be a reasonable approximation for grazing incidence independent of the ground constants. This assumption translates in the program to assuming a soft surface or a perfect magnetic conductor.

A further approximation is to neglect scattering from the side. This is mostly significant when considering wideband systems, where looking in the time domain, the shape of the impulse response is best understood by including side scatter [11]. In the present case, we are using narrow band signals for determining the total power, and the side scatter is assumed to be negligible.

Likewise, back scattering is neglected. This assumption has been tested numerically for the smooth surfaces, and the contribution to the integral from regions behind the receiver and transmitter has been shown to be negligible, so it is not
really a new assumption. It has the important impact on the numerical calculations that only summations are carried out. At each point, the new value of the surface current is found from the incident field and from the previous values of the surface currents, only implying forward scatter from known currents. Still, the method is slow because of the quadratic dependence on size and frequency. For each new point the summation must start from the beginning. We expect that a number of time-saving measures may be developed, but this has not been the goal of this paper.

It is interesting to observe the agreement with measurement over undulating terrain for a number of sites and frequencies. At the two lower frequencies, 144 and 435 MHz, both the mean error and the STD are quite satisfactory, which we interpret in such a way that the basic assumptions of smoothness and negligible side scatter are fulfilled. At the higher frequencies, 970 and 1900 MHz, the deviations are larger, although still acceptable from a planning point of view. It should also be recalled that the landscape is heavily undersampled.

REFERENCES