Active Power Management in Power Distribution Grids: Disturbance Modeling and Rejection

Rasmus Pedersen  Christoffer Sloth  Rafael Wisniewski

Abstract—This paper presents a control strategy for enabling a distribution system operator (DSO) to manage a power distribution grid towards an active power reference. This is accomplished by allowing the DSO to utilize flexibility in production and consumption. The control system consists of a feedforward based on estimates of inflexible consumption profiles, a feedback based on flexible asset dynamics, and a dispatch algorithm for minimizing active power loss. The estimation approach is validated on real consumption data and illustrates the method’s applicability on both a step-ahead and day-ahead scale, which makes it suited for a variety of control scenarios. Simulations demonstrate the control system’s ability to track an active power reference and show a 3% reduction in active power loss, compared to a strategy of uniformly dispatching the power between assets.

I. INTRODUCTION

The ongoing introduction of renewable resources, especially into the distribution grids, are turning a well-behaved system into a highly stochastic and dynamic system, challenging the distribution system operators (DSOs) [1]. Therefore, the DSOs are demanding new control and state estimation solutions, enabling them to maintain a high quality of power delivery [2]. Further, the volatile production and consumption may increase active power loss, a cost carried by the DSO. In this work we provide the DSOs with a combined estimation and control solution, enabling them to manage the active power in a distribution system, while minimizing the active power loss.

Power system state estimation has been addressed in several publications, such as [3] where the consumption patterns of loads are forecast using periodic models to describe their seasonal to daily behavior. Typically, these methods have been used to create strong statistical tools that could be used for long term planning of grid reinforcements, upgrade of transformer stations, etc. [4], [5]. However, these approaches have not been tailored for implementation in an online dynamic control scenario, with step ahead predictions and sampling time down to a sub-minute level. A popular and well-studied approach to solving the loss minimization problem is to run an optimal power flow (OPF). The OPF is a scheduling problem, which is inherently non-convex and therefore difficult to solve, making it ill-suited for fast dynamic control. In [6] the OPF problem is solved using a heuristic method based on particle swarm optimization. Similarly in [7] a genetic algorithm is applied to the problem, where also switching of discrete components are considered. The problems with these methods are that they can not guarantee a global system optimum and they can be computationally intensive. Therefore, different approaches have been taken to relax the problem as in [8], where a low rank solution is found by semi-definite programming or as in [9] where convex approximations are combined with stochastic optimization techniques to minimize active power loss. Another approach taken in [10], is to solve the loss minimization problem by distributing the effort among the flexible assets in the grid and then apply an appropriate communication strategy to ensure convergence, see also [11] for perspectives on the communication strategies for power loss minimization in residential micro grids. This illustrates that there are numerous approaches to solving the loss minimization problem. In this work a simple, fast to compute, and easily implementable algorithm, suited for online implementation is utilized.

This paper extends the use of periodic models to estimate consumption profiles of inflexible loads to allow a distribution grid to follow a power reference. The presented method uses linear harmonic oscillators in combination with a Kalman filter to estimate the load state. The reason for this approach is twofold: first, we will obtain a model of the inflexible load, which can be seen as a disturbance and thus used for feedforward. Secondly, the method allows for implementation in a dynamic control setup, using standard methods. The presented disturbance estimation method is general and its application is not limited to the control approach taken in this work, it could e.g., be used in a model predictive control setting or for offline scheduling purposes such as solving the OPF problem. To relax the OPF problem, we apply assumptions, such as voltages being measured at each bus. Thereby, the problem becomes convex and standard optimization techniques can be applied. The control approach detailed here should not be seen as a replacement for OPF scheduling, but more as an extra functionality capable of handling fast dynamics, ensuring that the schedule is actually followed in real time. The main contribution is the derivation of a simple to implement dynamic control and estimation system, enabling a distribution grid to follow a power reference, while minimizing active power loss.

The paper is organized as follows. First in Sec. II the power system control architecture is introduced. Then in Sec. III, the control system structure is given followed by estimator and controller design. In Sec. IV, the proposed control method is illustrated with numerical simulation examples. Finally in Sec. V, we provide conclusions on the work.
II. SYSTEM ARCHITECTURE

In this section an overview of the electrical distribution grid structure is given and the role of the distribution grid controller (DGC) is detailed. The setup is shown in Fig. 1 and illustrates the physical layout along with communication topology.

The idea in this architecture is that the DGC receives flexibility information and state information from assets, and measurements from inflexible entities. Asset flexibility can be in form of a wind turbine’s ability to derate active power production or a load’s ability to increase consumption. Inflexible loads are seen as disturbances into the system, power production or a load’s ability to increase consumption. Asset flexibility information and state information from assets, and illustrates the physical layout along with communication controller (DGC) is detailed. The setup is shown in Fig. 1 and illustrates the physical layout along with communication controller (DGC) is detailed. The setup is shown in Fig. 1

In the following sections, we develop the combined estimation and control system for managing the distribution grid towards an active power reference.

III. CONTROL SYSTEM DESIGN

In this section, the distribution grid control system is introduced, followed by a detailed description of the load estimation, flexible assets, feedback control law, and dispatch algorithm. The controlled system consists of the electrical grid, flexible assets, and inflexible loads illustrated in Fig. 2.

A. Load Estimation

The inflexible (uncontrollable) load profiles from consumers can be seen as a disturbance affecting the control system performance. Knowledge of the disturbance behavior can be incorporated into the control strategy in different ways, e.g., using model predictive control (MPC) or the internal model principle. Obtaining a model of the uncontrollable part of a dynamic system is extremely valuable for the following reason: *A disturbance can be completely rejected, only if the controller contains a model of it* [12].

Let \( \mathcal{L} = \{1, \ldots, S\} \) be the set of loads. Each load is modeled by \( p_i \in \mathbb{R} \) linear oscillators described by the following dynamic system

\[
\begin{align*}
    x_{l,i}(t) &= A_{l,i}x_{l,i}(t) + w_i(t), \quad i \in \mathcal{L}, \\
    y_{l,i}(t) &= C_{l,i}x_{l,i}(t) + v_i(t),
\end{align*}
\]

where \( x_{l,i}(t) \in \mathbb{R}^{2p_i+1} \) is the state vector with the last entry describing a bias term, \( w_i(t) \in \mathbb{R}^{2p_i+1} \) is the process noise, \( v_i(t) \in \mathbb{R} \) is the measurement noise, both assumed to be zero mean Gaussian distributed, \( A_{l,i} \in \mathbb{R}^{2p_i+1 \times 2p_i+1} \) is a block diagonal matrix, where each block is on the skew symmetric form

\[
    A_{l,i} = \text{diag} \left( \begin{bmatrix} 0 & \beta_{1,i} & \cdots & 0 \\ -\beta_{1,i} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \right),
\]

with \( \beta_{j,i} \in \mathbb{R} \) describing the frequency of a single oscillator, \( j = 1, \ldots, p_i \), and \( \text{diag}(X,Y,\ldots) \) is a block diagonal matrix with \( X,Y,\ldots \) as diagonal blocks. It should be noted that the values of \( \beta_{j,i} \) represent the harmonics that the model should capture. \( C_{l,i} \in \mathbb{R}^{2p_i+1} \) is the output matrix given by

\[
    C_{l,i} = [\alpha_{11,i} \quad \alpha_{12,i} \quad \cdots \quad \alpha_{p_i,1,i} \quad \alpha_{p_i,2,i}],
\]

with \( \alpha_{11,i}, \alpha_{12,i}, \ldots, \alpha_{p_i,1,i}, \alpha_{p_i,2,i} \in \mathbb{R} \) being the amplitudes associated to the \( p_i \) oscillators.

The load models can be combined with a Kalman filter to estimate the load states, which can be used for feedforward in the control algorithm. The effectiveness of estimating one aggregated residential load (\( \mathcal{L} = \{1\} \)) using the described model and a Kalman filter is demonstrated in Fig. 3. The figure shows a 14 day period of 200 households, based on data from Denmark [13]. The model used in the estimator consist of two harmonic oscillators with frequencies; \( \beta_{1,1} = \frac{2\pi}{24\text{ hour}} \) and \( \beta_{2,1} = \frac{2\pi}{12\text{ hour}} \), respectively. By choosing these harmonics the bi-daily peaks in the morning and evening are captured. The convergence of the filter is dependent on its tuning, in this case it takes a couple of days. However, Kalman filter tuning can be automated [14].

The Kalman filter is a closed-loop method depending on measurements of the output in order to adapt. Therefore, the estimation approach should also be verified in open-loop, i.e., how well does the proposed model perform when estimating...
Fig. 3. Performance of proposed estimation method throughout a 14 day period. The measurement is a residential load, based on data from 200 houses in Denmark. The two last days of the estimation is highlighted and it can be seen that even with a simplistic estimation model, good tracking capabilities are achieved.

Fig. 4. Open-loop performance of the proposed estimation method. The measurement represents an aggregated load of 200 houses, based on real data.

TABLE I

<table>
<thead>
<tr>
<th>Number of Oscillators</th>
<th>Day-ahead Tracking Error (RMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[kW] %</td>
</tr>
<tr>
<td>( p_1 = 2 )</td>
<td>4.16 100</td>
</tr>
<tr>
<td>( p_1 = 6 )</td>
<td>2.83 68</td>
</tr>
<tr>
<td>( p_1 = 20 )</td>
<td>2.91 70</td>
</tr>
<tr>
<td>( p_1 = 50 )</td>
<td>2.96 71</td>
</tr>
</tbody>
</table>

B. Flexible Assets

Let \( \mathcal{A} = \{1, \ldots, m\} \) be the set of flexible assets, modeled as constrained first order systems. The choice of a first order system to model flexible assets is based on the assumption of a well behaving local control loop exhibiting no overshoot. However, the model can easily be extended to capture more complex dynamics. The control model of a flexible asset is then given by

\[
\dot{x}_{a,i}(t) = a_{a,i}x_{a,i}(t) + b_{a,i}u_i(t), \quad i \in \mathcal{A} \tag{5}
\]

\[
y_{a,i}(t) = c_{a,i}x_{a,i}(t), \tag{6}
\]

\[
u_{\text{min},i}(t) \leq u_i(t) \leq u_{\text{max},i}(t), \tag{7}
\]

where \( x_{a,i}(t) \in \mathbb{R} \) is the asset state, \( u_i(t) \in \mathbb{R} \) is the reference signal, \( y_{a,i}(t) \in \mathbb{R} \) is the active power output, \( a_{a,i}, b_{a,i}, c_{a,i} \in \mathbb{R} \) are asset parameters, \( u_{\text{min},i}(t) \in \mathbb{R} \) is the lower bound on available flexibility, and \( u_{\text{max},i}(t) \in \mathbb{R} \) is the upper bound on available flexibility. Further, all assets are assumed to have unity gain, i.e., \( b_{a,i} = -a_{a,i} \) and \( c_{a,i} = 1 \). It should be noted that the bounds can change over time, e.g., the available amount of wind power depends on the wind speed.
C. Feedback Control

Even though the estimation method described in Sec. III-A shows good performance there are still discrepancies between the measured and estimated output. This indicates that we can not rely entirely on feedforward control to achieve good reference tracking. Therefore, a dynamic feedback control law is formulated to handle these discrepancies. All the controllable asset models are combined into one model as follows

$$\dot{x}(t) = A_\text{a}x(t) + B_\text{a}u_\text{ref}(t),$$  \hspace{1cm} (8)
$$y_\text{a}(t) = C_\text{a}x(t),$$  \hspace{1cm} (9)

with

$$A_\text{a} = \text{diag}(a_\text{a,1}, \ldots, a_{\text{a,m}}),$$  \hspace{1cm} (10)
$$B_\text{a} = [b_{\text{a,1}} \ldots b_{\text{a,m}}]^{T},$$  \hspace{1cm} (11)
$$C_\text{a} = [c_{\text{a,1}} \ldots c_{\text{a,m}}],$$  \hspace{1cm} (12)

where $x(t) \in \mathbb{R}^m$ is the controllable system state vector, $A_\text{a} \in \mathbb{R}^{m \times m}$, $B_\text{a} \in \mathbb{R}^m$, and $C_\text{a} \in \mathbb{R}^m$ are parameters, $y_\text{a}(t) \in \mathbb{R}$ is the total active power output of all controllable assets and, $u_\text{ref}(t) \in \mathbb{R}$ is the input to the system. Note that because of the control system structure it is possible to apply model reduction techniques if a large number of flexible assets are available, i.e., the overall system dynamic might be dominated by a subset of the assets.

We define $z(t) = [r(t)^T \ y_\text{a}(t)^T]^T$, with $r(t) \in \mathbb{R}$ being the reference signal in combination with the load estimate, i.e., $r(t) = p_{\text{ref}}(t) - \hat{y}_t$, where $p_{\text{ref}}(t) \in \mathbb{R}$ is the reference signal received from an upper layer, $\hat{y}_t = \sum_{i=1}^{S} y_{i,t}$ is the feedforward signal from the load estimate, and $S \in \mathbb{N}$ is the number of estimated loads. Note that production is seen as positive and load as negative power injection. The dynamic control law is given as

$$\dot{x}_c(t) = A_c x_c(t) + B_c z(t),$$  \hspace{1cm} (13)
$$u_\text{ref}(t) = C_c x_c(t) + D_c z(t),$$  \hspace{1cm} (14)

where $x_c(t) \in \mathbb{R}^m$ is the controller state and $u_\text{ref}(t) \in \mathbb{R}$ is the control output. The feedback control system parameters $A_c$, $B_c$, $C_c$, and $D_c$ are determined using the controllable asset model along with state feedback gain, observer gain, and feedforward gain as follows

$$\dot{x}_c(t) = \underbrace{(A_c + B_c K + LC_c a)}_{A_c} x_c(t) + \underbrace{[M - L] z(t)}_{B_c},$$
$$u_\text{ref}(t) = \underbrace{K \ x_c(t)}_{C_c} + \underbrace{[N \ 0] z(t)}_{D_c},$$  \hspace{1cm} (15)
(16)

where $K \in \mathbb{R}^m$ is the feedback gain, $L \in \mathbb{R}^m$ is the observer gain, $M \in \mathbb{R}^m$ is the observer feedforward gain, and $N \in \mathbb{R}^m$ is the control feedforward gain. Both $M$ and $N$ are determined such that the there is unity gain from reference to output, using the zero assignment method [15, p. 510]. The feedback gain, $K$ and observer gain, $L$ are designed, using pole placement, such that $A_c + B_c K$ and $A_a + LC_a$ are both Hurwitz. It should be noted that robustness in reference tracking can be obtained by adding integral action to the system. This is done by augmenting the control system matrix with an additional state.

D. Dispatch Strategy

The control signal, $u_\text{ref}(t)$, produced by the controller detailed above, is passed through a dispatch algorithm. The objective of the dispatch algorithm is to minimize active power loss while respecting asset constraints. One solution could be to solve the optimal power flow problem using the method outlined in [16]. This method puts restrictions on the electrical grid topology in order to guarantee zero duality gap. The simplistic approach taken in this work relies on assumptions, such as no voltage problems (voltages are close to nominal value), resulting in a convex formulation without restrictions on grid topology.

The active power loss of an electrical grid can be written as

$$J(t) = i^*(t) \text{Re}(Z(\omega)) i(t),$$  \hspace{1cm} (17)

where $i(t) \in \mathbb{C}^n$ is the injected current at each bus, $Z(\omega) \in \mathbb{C}^{n \times n}$ is the bus impedance matrix, $\text{Re}(Z(\omega))$ denotes the real part of $Z(\omega), \omega \in \mathbb{R}$ is the electrical grid frequency, $n \in \mathbb{N}$ is the number of buses in the system, and $x^*$ denotes the conjugate transpose of $x$. See [17, p. 289] for a full derivation of the active power loss formula, and [17, p. 369] for an algorithm to compute $Z(\omega)$. In the following we assume a constant frequency.

The values of the currents $i(t)$ are typically not known, but can be expressed by the complex power and voltage at each bus

$$i_k(t) = \frac{s_k(t)}{V_k(t)}, \quad k = 1, \ldots, n, \quad (18)$$

where $V_k(t) \in \mathbb{C}$ is the voltage at bus $k$, $s_k(t) = p_k(t) + j q_k(t)$ is the complex power at bus $k$, with $p_k, q_k \in \mathbb{R}$ being the active and reactive power component, respectively. If the bus voltages are measured or assumed constant, they can be combined with the grid impedance matrix as follows

$$B(t) = V^*(t) \text{Re}(Z) V(t),$$  \hspace{1cm} (19)

resulting in the following expression for active power loss

$$J(t) = s^*(t) B(t) s(t),$$  \hspace{1cm} (20)

where $s(t) = [s_1(t) \ldots s_n(t)]^T$. Now (17) is formulated with respect to the complex power at each bus. With the estimate of power consumption at load busses (both active and reactive), along with assumed voltage measurements, the active power dispatch optimization problem, to be solved at time $t$, can be formulated as

$$\begin{align*}
\min_{u_1(t), \ldots, u_m(t)} & \quad J(t) \\
\text{s.t.} & \quad \sum_{i=1}^{m} u_i(t) = u_\text{ref}(t) \\
& \quad u_{\min,i}(t) \leq u_i(t) \leq u_{\max,i}(t), \\
& \quad i \in \mathcal{A}
\end{align*}$$
with variables $u_1, \ldots, u_m$ and data $u_{\text{min},1}(t), \ldots, u_{\text{min},m}(t)$, $u_{\text{max},1}(t), \ldots, u_{\text{max},m}(t)$, $q_1, \ldots, q_m$, $v_1(t), \ldots, v_m(t)$, $\bar{p}(t)$, $\bar{q}(t)$, and $Z(\omega)$. It should be noted that the decision variables $u_i$ represent active power injection/consumption at flexible asset busses. The cost function stated in (22) can be shown to be convex in the real and imaginary parts of the complex power, and with the constraints also being convex, it is a convex programming problem. The problem is similar to the one formulated in [18], but with no capacity constraints on lines, and without the voltages as variables. What should be noted from this formulation is that it is a simple problem, which can be solved fast for a large number of decision variables, $m$, and thus easily implementable. However, it relies on the flexible assets having almost the same dynamics to guarantee an optimal operating point at each time instance $t$, i.e., in steady state the optimum will be reached. To handle differences in asset dynamics these can be included in the optimization problem in a receding horizon control strategy [19]. The caveat to adding asset dynamics in this way is an increase in problem complexity and thus computation and deployment time.

IV. SIMULATION EXAMPLE

The proposed control system is evaluated on a realistic simulation scenario with complex asset and electrical grid models, using the power systems simulation toolbox DiSC [13]. Thereby, the simplifications and assumptions made in the control design process are tested for robustness against parameter deviations, unmodeled dynamics, etc..

We consider a distribution grid with seven busses, three inflexible residential loads, and two flexible production units (wind power plant (WPP) and solar power plant (SPP)). The DGC receives a power reference signal and dispatches this to the two production units. The setup is depicted in Fig. 5, and the parameters are summarized in TABLE II.

![Fig. 5. Distribution grid used in the simulation example. The distribution grid controller receives a power reference, $p_{\text{ref}}$, power measurements from the load busses, $p_{\text{load}}$, and current state along with flexibility information from the production units, $p_{\text{max}}$, $u_{\text{min}}$, $u_{\text{max}}$. The controller then dispatches reference signals to the two production units, $u$.](image)

**TABLE II**

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind power plant</td>
<td>Total rated power</td>
<td>2.4 MW</td>
</tr>
<tr>
<td>PV system</td>
<td>Total rated power</td>
<td>2.4 MW</td>
</tr>
<tr>
<td>Transformer</td>
<td>Impedance</td>
<td>3+j13 Ω</td>
</tr>
<tr>
<td>Line $B_2-B_3$</td>
<td>Impedance</td>
<td>1+j1 Ω</td>
</tr>
<tr>
<td>Line $B_2-B_4$</td>
<td>Impedance</td>
<td>3.25+j2.25 Ω</td>
</tr>
<tr>
<td>Line $B_3-B_5$</td>
<td>Impedance</td>
<td>0.65+j0.45 Ω</td>
</tr>
<tr>
<td>Line $B_5-B_6$</td>
<td>Impedance</td>
<td>0.13+j0.06 Ω</td>
</tr>
<tr>
<td>Line $B_5-B_7$</td>
<td>Impedance</td>
<td>3.25+j3 Ω</td>
</tr>
<tr>
<td>Line $B_6-B_7$</td>
<td>Impedance</td>
<td>0.5+j0.5 Ω</td>
</tr>
<tr>
<td>Load $L_1$</td>
<td>Number of houses</td>
<td>2000</td>
</tr>
<tr>
<td>Load $L_2$</td>
<td>Number of houses</td>
<td>2000</td>
</tr>
<tr>
<td>Load $L_3$</td>
<td>Number of houses</td>
<td>2000</td>
</tr>
</tbody>
</table>

We consider a windy and cloudy day in June. The power reference received has been arbitrarily chosen to illustrate the system’s tracking ability. During eight hours in the midday the reference is set to zero, i.e., the grid should not import or export active power. This could be caused by frequency control issues detected on the transmission level or other external factors. The DGC’s reference tracking ability is illustrated in Fig. 6 and the active power production of the WPP and SPP are shown in Fig. 7. Further, the proposed dispatch algorithm is compared to a strategy of uniformly distributing the power reference between the two assets. This comparison is summarized in TABLE III.

A number of interesting results are observed. Under the developed control system the distribution grid is capable of following the reference signal with a root mean square error (RMSE) of only 94.8 kW during one day. Moreover, during the eight hours of no import/export the system evolves closely around the referenced value, indicating that the distribution grid can offer services such as frequency regulation, to upper layers. From Fig. 7 it is seen that the proposed dispatch lowers the production from the WPP and increases the production from the SPP, during the eight hours of no import/export. This makes sense as the system should only provide power for the three loads which are placed closer to the SPP, seen from an electrical point of view. The proposed dispatch strategy decreases the power loss with 3 %. However, this number is expected to increase if the control system is applied to a larger distribution grid.

<table>
<thead>
<tr>
<th>Method</th>
<th>Tracking Error (RMS) [kW]</th>
<th>Power Loss [MWh]</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uni. Disp.</td>
<td>94.0</td>
<td>100.0</td>
<td>0.33</td>
</tr>
<tr>
<td>Prop. Disp.</td>
<td>94.8</td>
<td>100.8</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>97.0</td>
</tr>
</tbody>
</table>
Fig. 6. The control system’s reference tracking ability. The fluctuations around the reference are caused by clouds passing over the SPP and WPP operating below rated wind speed.

Fig. 7. Available and actual power production of the WPP (top) and SPP (bottom).

V. CONCLUSION

In this work, we presented a control system allowing a distribution system operator to manage a power distribution grid, towards an active power reference. The control system consisted of three main components; an inflexible load estimator based on linear harmonic oscillators, a feedback control law for managing the flexible assets, and a dispatch algorithm for minimizing active power loss. The outlined estimation procedure was validated on real consumption data and showed good estimation capability, both on a step-ahead and day-ahead scale. Through simulations on a 7 bus distribution grid with two flexible assets it was shown that the designed feedback control in combination with the dispatch algorithm provided excellent tracking ability with a root mean square error of approximately 94 kW, even with the stochastic nature of the flexible assets. Further, the proposed dispatch algorithm was compared with a strategy of evenly distributing the reference between the assets. It was possible to decrease the loss with 3%, even for a small distribution grid. The developed control system was simple in nature, allowing for rapid implementation and deployment.

ACKNOWLEDGMENTS

The research leading to these results has received funding from the European Community’s Seventh Framework Programme (FP7/2007-2013) under grant agreement n° 318023 for the SmartC2Net project. Further information is available at www.SmartC2Net.eu.

REFERENCES