In this chapter the mobile wireless radio channel and one implementation of the channel is discussed. The channel is modeled as a frequency selective WSSUS channel described by a delay-Doppler power spectrum. The conclusion of this chapter is a discrete time/frequency covariance function for use in the receiver structure.

In the design and implementation of a receiver for wireless communications, it is important to have an understanding of the wireless radio channel, since it influences the received signal significantly. The derived channel model in this study is also used for verification. However, since a model of the wireless radio channel is a simplification, it does not necessarily follow that a receiver derived with the mind on one specific channel model works properly, when it is tested using a real wireless radio channel.

The channel is modeled with the parameters from the LTE downlink SISO OFDM system based on the signal model derived in Chap. ?? blurred. The carrier frequencies for downlink have been allocated in ten bands in the area 0.8 GHz – 2.7 GHz [Dahlman et al. (2007)] page 14. A typical downlink is 4.5 MHz (5 MHz spectrum allocated), 15 kHz subcarrier spacing and based on the block sizes for the LTE downlink we chose 120 subcarriers and 56 OFDM symbols [Dahlman et al. (2007)] page 59. Realistically, the environment plays an important role in the behavior of the channel. The channel has different characteristics depending on if it is indoor, outdoor, urban, hilly etc. A generic channel model based on a specific delay-Doppler power spectrum is derived. The model chosen is power delay profile of the COST 207 typical urban model power delay profile and Jakes’ Doppler spectrum (also

1The COST 207 model is an average from extensive collected data from different locations in Europe. The model considers four different scenarios, each one characterized by a specific profile: typical urban
known as Clarke’s spectrum). The advantage of this model is its simplicity and its generality. The channel model is first described in continuous time. From this model a discrete time/frequency model is developed to describe the channel in each subcarrier. The channels between base-stations are assumed uncorrelated, which is a realistic assumption since they are not normally in physical proximity.

1.1 An Uncorrelated Scattered Model of the Multi-path Channel

Because the radio waves propagate along different paths from the transmitter to the receiver, the channel is modeled as a set of multi-paths with different characteristics. Different effects occur in the channel, which are (i) path loss, exponentially decaying with the distance the wave travelled, (ii) short-term fading, due to small fluctuations in the channel caused by phase changes in each signal contribution, and (iii) shadow fading, which is due to physical changes in the channel. Only the short-term fading is considered in this model. The statistical properties of short-term fading is approximated to Rayleigh fading. For fading with one strong comp-
1.1. AN UNCORRELATED SCATTERED MODEL OF THE MULTI-PATH CHANNEL

ponent (line of sight), Rician fading gives a better approximation. As we only consider the small scale fading, we assume that the stochastic process behind the time/frequency description of the channel is wide sense stationary (WSS), and that the auto-correlation for the delay/Doppler function is zero in all instances, where \( v_i \neq v \) and \( \tau_i \neq \tau \), which means uncorrelated scattering (SU). The channel model showing these characteristics is thus described as the WSSUS channel. We describe the channel as a tap delay line model, given by a time-variant channel impulse response [Tse and Viswanath(2005) page 41] and [Molisch(2005) page 121], which using a base-band representation is described as

\[
(1.1) \quad h(t, \tau) = \sum_{i=1}^{N_{\text{taps}}} \varphi_i(t) \delta(\tau - \tau_i(t)) = \sum_{i=1}^{N_{\text{taps}}} \alpha_i(t) \exp(-j2\pi f_c \tau_i(t)) \delta(\tau - \tau_i(t)),
\]

where \( \varphi_i(t) \) is the \( i \)th tap and \( \delta(\tau) \) is the Kronecker delta function, \( \tau_i(t) \) is the time-variant \( i \)th delay, \( f_c \) is the carrier frequency, \( \alpha_i(t) \) is the time-variant complex amplitude of the \( i \)th tap and \( N_{\text{taps}} \) is the number of taps in the model. The amplitude of each tap \( \alpha_i(t) \) is modelled as a zero-mean complex Gaussian process, thus \( |h(t, \tau)| \) is Rayleigh distributed.

From [Bello(1963)] we use the notation to express the description of the channel in the different domains and the connection between them, see Fig. 1.2. The time-variant transfer function is the Fourier transform of the impulse response (1.1) from the delay \( \tau \) domain to the frequency domain \( f \)

\[
(1.2) \quad H(t, f) = \int_{-\infty}^{\infty} d\tau \ h(t, \tau) \exp(-j2\pi f \tau).
\]

A statistical description of the channel is obtained by the correlation function in time and frequency, given by the auto-correlation function of \( H(t, f) \) over time \( t \) and frequency \( f \) [Molisch(2005)]

\[
(1.3) \quad \langle H^*(t, f)H(t + \Delta t, f + \Delta f) \rangle_{f, t} = R_H(\Delta t, \Delta f),
\]

where \( \langle \cdot \rangle_x \) denotes the expected value with respect to \( x \) and \( \Delta t \) and \( \Delta f \) is the shift in time and frequency respectively. Notice, for the WSS channel the first moment is constant. The time correlation function and the frequency correlation function of

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2Rician fading is a generalization of Rayleigh fading, where one strong component is present.
the channel are obtained from $R_H(\Delta t, \Delta f)$ \cite{Molisch2005} according to

(1.4) \hspace{1cm} R_H(\Delta t, 0) = R_H(\Delta t) \hspace{1cm} (1.5) \hspace{1cm} R_H(0, \Delta f) = R_H(\Delta f)$.

Averring across many channels given many different locations it is assumed that

(1.6) \hspace{1cm} R_H(\Delta t, \Delta f) = R_H(\Delta t)R_H(\Delta f)$.

The assumption is not true for one instantaneous channel, since the time and frequency correlation functions here cannot be assumed independent. Because of the US assumption the second moment of the scattering function is

(1.7) \hspace{1cm} \langle s^*(v_1, \tau_1)s(v_2, \tau_2) \rangle_{v, \tau} = S_H(v_1, \tau_1)\delta(v - v_1)\delta(\tau - \tau_1)$.

The power spectrum is also obtained from the correlation function in (1.3), see Fig. 1.2 by the Fourier transform in the time direction and the inverse Fourier transform in the frequency direction of the correlation function \cite{Durgin2003}

(1.8) \hspace{1cm} S_H(\tau, v) = \int_{-\infty}^{\infty} d\Delta f \exp(j2\pi \Delta f \tau) \int_{-\infty}^{\infty} d\Delta t \hspace{0.1cm} R_H(\Delta t, \Delta f) \exp(-j2\pi \Delta f v)$. 

Fig 1.2: Relationship between channel functions in the WSSUS channel model. The figure shows four system descriptions, and their second order descriptions. The system descriptions are connected as Fourier pairs, similarly this applies for the second order descriptions. The Fourier pairs are marked with $\downarrow$, where $\leftarrow$ represent the Fourier transformed. We only consider the parts of the model marked in black.
1.2. SELECTED CHANNEL MODEL

In this section a channel model is chosen and described on the form $S_H(\tau)S_H(\nu)$ and $R_H[\Delta n]R_H[\Delta l]$, where in the latter expression $R_H[\Delta n]$ and $R_H[\Delta l]$ are discrete-time and frequency correlation functions across subcarriers $l$ and time interval between OFDM symbols $n$, respectively. For one subcarrier $l$ in one OFDM symbol $n$, we assume a frequency flat and time constant channel. We will later see from the behavior of the correlation function $R_H(\Delta t)R_H(\Delta f)$, that this assumption is reasonable. The considered model for the Doppler domain with one transmitter and one receiver is depicted in Fig. 1.3. We assume that all scatters on average are distributed uniformly on a circle around the receiver, with the same average power from all directions. Invoking the central limit theorem, we can assume that the power from one direction is complex Gaussian distributed. If we assume that the receiver moves with a certain speed, the number of components coming from each direction will take the tub form of Jakes’ spectrum [Jakes(1974)]. For the non-spacial channel model, this is thus the Doppler spectrum $S_H(\nu)$, which specifies the average power from different frequency shifts over infinite many different implementations of the channel described.
by the spectrum. Jakes’ spectrum is computed from the definition of the Doppler shift. The Doppler shift $v$ is caused by the velocity $u$ of the receiver

$$v = \frac{f_c u}{c_0} \cos(\phi) \text{ Hz},$$

where $c_0 = 3 \cdot 10^8 \text{ m/s}$ is the speed of the impacting radio wave, and $f_c$ is the carrier frequency and $\phi$ is the azimuth of arrival of the radio wave. The Doppler shifts from $\phi$ and $-\phi$ is the same, we thus limit the probability density function (pdf) to $p(\phi) = U(0, \pi)$. Thus for each frequency in the Doppler dispersion is given

$$f = f_c \left( 1 - \frac{|u|}{c_0} \cos(\phi) \right),$$

where $u = |u| \cos(\phi)$. Transforming this uniform distribution into the Doppler domain we get

$$p(v) = \left| \frac{d\phi}{dv} \right| U(0, \pi).$$

If we consider the Doppler spectrum as the pdf in (1.11) and by realizing that the maximum Doppler shift $v_{\text{max}}$ is caused by $\phi = 0$ or $\phi = \pi$, we thus by simple differentiation and insertion find the average energy from each Doppler shift

$$S_H(v) = \begin{cases} \frac{1}{\pi v_{\text{max}}} \sqrt{1 - \left( \frac{v}{v_{\text{max}}} \right)^2} & -v_{\text{max}} \leq v \leq v_{\text{max}}, \\ 0 & \text{elsewhere}, \end{cases}$$

The spectrum can be interpreted such that infinite many contributions, which are very small, come from $0^\circ$ and $180^\circ$, which creates two singularities in $v_{\text{max}}$ and $-v_{\text{max}}$.

The power delay profile (pdp) $S_H(\tau)$ is generated by a set of delayed paths. For the tap delay line model, each cluster has an exponential profile. The pdp is typically different depending on the environment. To simplify the model, $S_H(\tau)$ is approximated by an exponential decaying function [Molisch(2005) page 121], the physical explanation of this phenomenon are still not solved, however this has been verified by extensive measurements campaign [Molisch(2005)]. For the indoor scenario, models explaining have been made [Steinböck(2008)], where it was shown that this behavior can be explained by reverberation effects in the environment. A simple description of the pdp profile is given as

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3For LTE $f_c$ is in the range of 1 GHz.

4This is obvious a purely theoretic phenomenon.
1.2. SELECTED CHANNEL MODEL

Fig 1.4: THE DELAY-DOPPLER POWER SPECTRUM $S_H(\tau, \nu)$, WHICH CORRESPONDS TO AN AVERAGE OVER MANY CHANNEL REALIZATIONS. FOR ONE LOCALIZED CHANNEL THE DELAY-DOPPLER SPECTRUM CANNOT FACTORIZE ACCORDING $S_H(\tau, \nu) = S_H(\tau)S_H(\nu)$ BECAUSE THE DELAY AND THE DOPPLER FREQUENCY IS NOT INDEPENDENT. IN THE PLOT THE DELAY-SPREAD IS $\sigma_\tau = 1 \mu$s.

(1.13) 
$$S_H(\tau) = \begin{cases} \frac{1}{\sigma_\tau} \exp\left(-\frac{\tau}{\sigma_\tau}\right) & 0 \leq \tau \\ 0 & 0 > \tau, \end{cases}$$

where $\sigma_\tau$ is the delay spread. The delay spread is defined as the square root of the second moment of the pdp. The delay-Doppler power spectrum $S_H(\tau, \nu)$ is then defined from the disjoint specters (1.12) and (1.13) as

(1.14) 
$$S_H(\tau, \nu) = \begin{cases} \frac{\exp\left(-\frac{\tau}{\sigma_\tau}\right)}{\sigma_\tau\nu_{\max} \sqrt{1 - \left(\frac{\nu}{\nu_{\max}}\right)^2}} & 0 \leq \tau \text{ and } -\nu_{\max} \leq \nu \leq \nu_{\max} \\ 0 & \text{else}. \end{cases}$$

The power delay profile (pdp) is depicted in Fig. 1.4 From the power spectrum $S_H(\tau, \nu)$, we find the autocorrelation function $R_H(\Delta t, \Delta f)$. The problem is considered disjoint, so first the autocorrelation function $R_H(\Delta t)$ is derived from the Doppler spectrum (1.12) by the inverse Fourier transform [Tse and Viswanath(2005)]

(1.15) 
$$R_H(\Delta t) = \int_{-\infty}^{\infty} d\tau S_H(\nu) \exp(j2\pi\nu\Delta t).$$

By solving the integral, we get the second moment of the time transfer function
**CHAPTER 1. MODEL OF THE WIRELESS RADIO CHANNEL**

Time \[\Delta t\] [s]

\[R_H(\Delta t) = J_0(2\pi\Delta t\nu_{\text{max}}),\]

where \(J_0(\cdot)\) is the Bessel function of the zeroth order, given by

\[J_0(2\pi\Delta t\nu_{\text{max}}) = \frac{1}{\pi} \int_0^\pi \phi \exp(j2\pi\Delta t\nu_{\text{max}}\cos\phi).\]

The auto-correlation function is depicted in Fig. 1.5. The discrete function \(R_H[\Delta n]\) is obtained letting \(N_s\Delta n = \Delta t\), where \(N_s\) is the time between two OFDM symbols, thus

\[R_H[\Delta n] = J_0(2\pi N_s\Delta n\nu_{\text{max}}).\]

By the Fourier transform of the power delay profile \(S_H(\tau)\), we find the time correlation function \(R_H(\Delta f)\) of the channel as

\[R_H(\Delta f) = \frac{1}{\sigma_\tau} \int_0^\infty \tau \exp\left(\frac{-\tau}{\sigma_\tau}\right) \exp(-j2\pi\Delta f\tau) \Rightarrow R_H(\Delta f) = \frac{1}{1 + j2\pi f\sigma_\tau},\]

which has the characteristic of a low-pass filter, see Fig. 1.6. The width of the low-pass filter is proportional to the delay spread \(\sigma_\tau\). The auto-correlation function for the subcarriers \(l\) is thus a discrete function given by

\[R_H[\Delta l] = \frac{1}{1 + j2\pi\sigma_\tau L_{\text{sp}}\Delta l},\]

where \(L_{\text{sp}}\) is the subcarrier spacing. The time/frequency correlation function is

\[R_H[\Delta n, \Delta l] = \frac{1}{\pi \left(1 + j2\pi\sigma_\tau L_{\text{sp}}\Delta l\right)} \int_0^\pi \phi \exp(j2\pi N_s\Delta n\nu_{\text{max}}\cos\phi).\]
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Fig 1.6: The autocorrelation function $R_H(\Delta f)$ with a delay spread $\sigma_\tau = 6 \mu s$. The coherence bandwidth is $B_c = 0.2$ MHz.

Thus in the selected channel model we end up with a description, where the channel is characterized by two parameters; the delay spread $\sigma_\tau$ and the maximum Doppler frequency $\nu_{\text{max}}$, notice that this is not equal to the Doppler spread. For the COST 207 typical urban channel model (which has the considered exponential profile) the delay spread is approx. 1 $\mu$s ($-30$ dB on 7 $\mu$s, \textsuperscript{[Molisch(2005)]}). The approximate coherence bandwidth is thus $B_c \approx \frac{1}{\Delta f} = 0.2$ MHz \textsuperscript{[Durgin(2003)]}, see also Fig. 1.6. For the general case a linear relationship exist between the delay spread and the coherence bandwidth, from \textsuperscript{[Fleury(1996)]} the uncertainty relation between the coherence bandwidth and the delay spread was stated.

(1.22) \[ B_c \gtrsim \frac{1}{2\pi \sigma_\tau}. \]

Over the coherence bandwidth the channel is assumed constant.\footnote{The coherence bandwidth is considered here as a $-3$ dB limit.} Thus across a small number of neighboring subcarriers with a spacing of 15 kHz, the channel frequency response can still be assumed flat. However over the whole bandwidth of 1.8 MHz the channel is frequency selective.
References


