Problem 4.1 (Problem 6.13 in Shanmugan)

The conditional pdfs corresponding to two hypotheses are given:

\[ f_{Y|H_0}(y|H_0) = \frac{1}{2} \exp \left( -\frac{y}{2} \right), \quad 0 < y \]
\[ f_{Y|H_1}(y|H_1) = \frac{1}{4} \exp \left( -\frac{y}{4} \right), \quad 0 < y \]

Suppose we want to test these hypotheses based on two independent samples \( Y_1 \) and \( Y_2 \). Assume equally likely priors, that is \( P(H_0) = P(H_1) = \frac{1}{2} \).

a. Derive the MAP decision rule for the test.

b. Calculate \( P_M = P(D_0|H_1) \) and \( P_F = P(D_1|H_0) \).

Problem 4.2 (Problem 6.14 in Shanmugan)

The signaling waveforms used in a binary communication system are

\[ s_1(t) = \begin{cases} 4 \sin(2\pi f_0 t), & 0 \leq t \leq T \\ 0, & \text{elsewhere} \end{cases} \]
\[ s_0(t) = -s_1(t), \]

\[ P[x(t) = s_1(t)] = P[x(t) = s_0(t)] = \frac{1}{2} \]

where \( T = 1\text{ms} \) is the signal duration and \( f_0 = 10/T \).

Assume that the signal is corrupted by zero-mean additive white Gaussian noise with power spectral density

\[ S_{WW}(f) = 10^{-3} \text{ W/Hz} \]

a. Find the decision rule that minimizes the average probability of error \( P_e \).

b. Find the value of \( P_e \).