RF Receiver Front-Ends (GPS)

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Contents

Preface

This report is to be used for a 2 mini module course on front-end receivers for GPS. The emphasis is put on explaining functional principles and not on exact models, complicated system analysis etc.

Major emphasis has also been put on visualizing signals to gain insight into the fundamental principles. This without the need for an RF (radio frequency) engineering background.

The author highly recommends that printed versions are made in color. This greatly facilitates the understanding of the illustrations.

As always, the author invites all constructive comments, critique etc.

Aalborg, November 6, 2000 Torben Larsen

Chapter 1

Course information

1.1 Mini module 1

Topics

- Prerequisites; The ideal received signal, signal power and energy, noise and interference, signal-to-noise ratio.
- Front-end functionality; The antenna signal, output signals.
- Receiver components; Amplifiers.

Educational aims

The student must understand and be able to explain:

- the behaviour of the ideal received antenna signal — both the low frequency information carrying signals and the high frequency signal.
- the concept of signal envelope and power envelope.
- average power and energy of a signal.
- some characteristics of noise and illustrate some differently filtered noise signals in time domain.
- the concept of signal-to-noise ratio.
- the functional principle of a front-end.
- the real life antenna signal and the ideal desired front-end output signals.
- the functional principle of an amplifier, its simple models, and illustrate the distortions caused by noise and nonlinearity.

Literature

The present report page 6–26.

Exercises

Exercise 1–3 in the present report on page 44. Written solutions to exercises must be delivered to the lecturer for approval.

1.2 Mini module 2

Topics

- Mixers; Simple models, mixer functional visualization, illustrative example.
- Filters; Simple models, filter examples, MATLAB assistance.
- Quadrature down-converter; Functional working principle.
- Receiver architectures; Direct conversion receivers, super heterodyne receivers, receiver pros and cons.

Educational aims

The student must understand and be able to explain:

- the functional principle of a mixer, its simple models, and illustrate the input-output relation.
- the overall idea of filters and illustrate the input-output relations of some filters when almost white noise is applied at the input.
- the working principle of the quadrature down-converter including input-output relations.
- the functional working principles of the direct conversion receiver and the super heterodyne receiver as well as pros and cons for the two receiver types.

Literature

The present report page 27–43.

Exercises

Exercise 4–5 in the present report on page 44. Written solutions to exercises must be delivered to the lecturer for approval.

Chapter 2

Prerequisites

This section gives gives some of the essential prerequisites to front-end receiver understanding. A very simplistic approach is generally taken where most non-ideal effects are ignored.

2.1 An ideal received signal

Generally, the ideal received signal at the antenna terminals from a single satellite can be written as:

$$s_{ant,ideal}(t) \quad = \quad I(t)\cos(2\pi f_0 t + \phi_0) \; + \; I(t)\sin(2\pi f_0 t + \phi_0) \qquad (2.1)$$

where $I(t)$ and $Q(t)$ are the information carrying low frequency signals, f_0 is the high frequency carrier which enables wireless transmission from the satellite to the receiver, and ϕ_0 is a constant but otherwise arbitrary phase — this phase is omitted in many situations as it has no effect on functionality. Occasionally the type of signal in Equation (2.1) is written as:

$$s_{ant,ideal}(t) \quad = \quad a(t)\cos(2\pi f_0 t + \phi(t) + \phi_0) \qquad (2.2)$$

where $a(t)$ is referred to as the signal envelope or amplitude envelope (a low frequency signal), and $\phi(t)$ is the time varying phase of the signal. The relations between Equations (2.1) and (2.2) are:

$$I(t) \; = \; a(t)\cos(\phi(t)), \quad Q(t) \; = \; -a(t)\sin(\phi(t)) \qquad (2.3)$$

$$a(t) \; = \; \sqrt{I^2(t) + Q^2(t)}, \quad \phi(t) \; = \; -\arctan\left(\frac{Q(t)}{I(t)}\right) \qquad (2.4)$$

It may be worth the time to visually illustrate some of the signals in Equations (2.1)–(2.4). First of all, examples of information carrying $I(t)$ and $Q(t)$ signals are shown in Figure 2.1. As seen from the figure, the $I(t)$ and $Q(t)$ signals are very different versus time. Figure 2.2 shows the corresponding envelope $a(t)$ and phase $\phi(t)$. Again there is both envelope and phase variation versus time. Finally, Figure 2.3 shows the ideal received antenna signal $s_{ant,ideal}(t)$ from Equation (2.1). This is the blue curve in the figure. Also the envelope $a(t)$ (and $-a(t)$) are illustrated in Figure 2.3. This is exactly the envelope of the rapid varying (high frequency) $s_{ant,ideal}(t)$ signal. Simplifying things, it is exactly the task of the receiver front-end to deduce the information carrying signals $I(t)$ and $Q(t)$ from the received $s_{ant,ideal}(t)$ signal (blue curve in Figure 2.3).

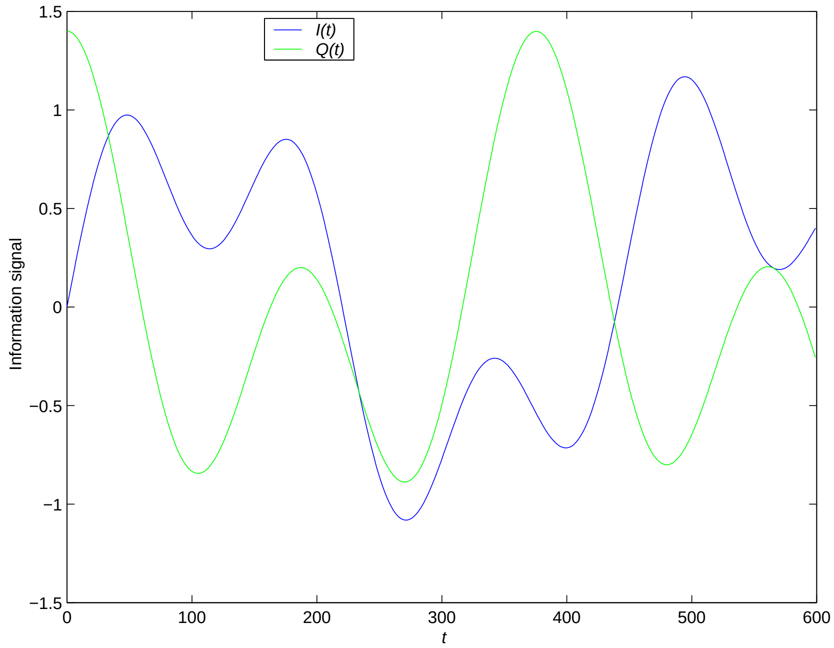

Figure 2.1: Example of information carrying low frequency signals $I(t)$ and $Q(t)$.

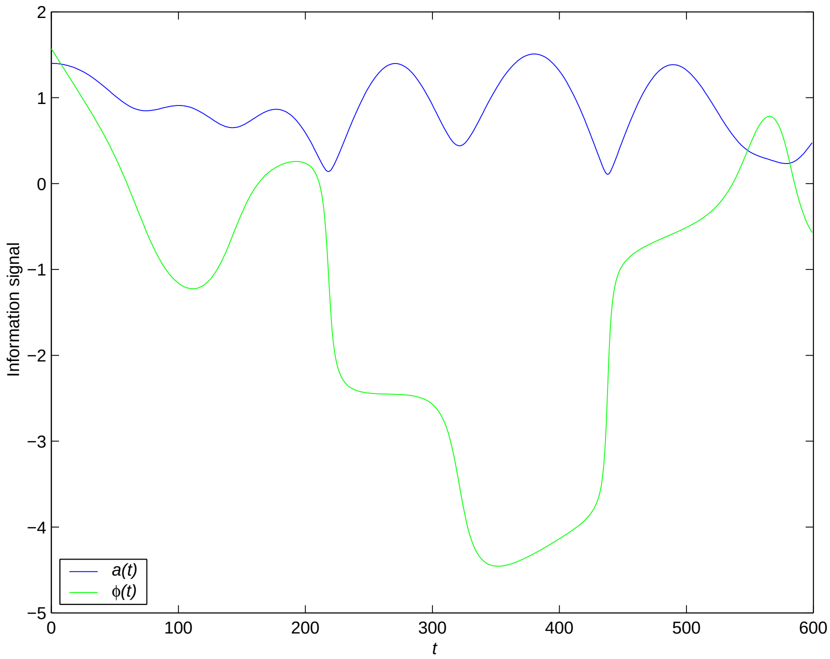

Figure 2.2: The signal envelope $a(t)$ and phase $\phi(t)$ (in radians) versus time corresponding to the $I(t)$ and $Q(t)$ signals in Figure 2.1.

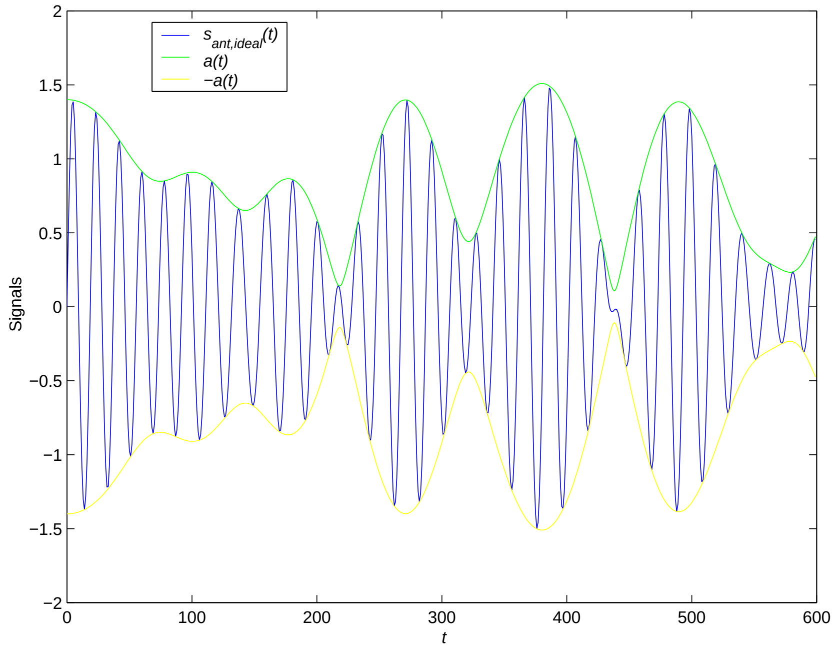

Figure 2.3: Blue curve: The ideal received signal from Equation (2.1) when the $I(t)$ and $Q(t)$ signals in Figure 2.1 are used. Green curve: The signal envelope $a(t)$. Yellow curve: Minus the signal envelope, $-a(t)$.

2.2 Signal power and energy

The instantaneous power of a signal $s(t)$ at time t is given by:

$$P_s(t) = s^2(t) \tag{2.5}$$

The instantaneous power of the signal envelope $a(t)$ is also known as the power envelope of the signal $s(t)$. For the signals in Equations (2.1) and (2.1) this is $P_a(t) = a^2(t)$.

The average power of a signal $s(t)$ in the time interval $t \in [t_1, t_2]$ is given by:

$$P_s = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} s^2(t)\, dt \tag{2.6}$$

It is interesting to note that the average power of the ideal antenna signal $s_{ant,ideal}(t)$ in Equation (2.1) is given by:

$$P_{s_{ant,ideal}} = \frac{1}{2}\{P_I + P_Q\} \tag{2.7}$$

provided the time interval $t_2 - t_1$ is sufficiently large. In Equation (2.7), P_I is the average power of $I(t)$ and P_Q is the average power of $Q(t)$.

As an example, the power of the high frequency ideal antenna signal $s_{ant,ideal}(t)$ from Figure 2.3 is $P_{s_{ant,ideal}} = 0.4536$. The average power determined from Equation (2.7) and Figure 2.1 is 0.4541. This is for a case of 600 time steps which gives a very good agreement.

Recall that the variance of a signal $s(t)$ is given by:

$$\text{Var}[s(t)] = R_{ss}(0) = \langle (s(t) - \mu_s)^2 \rangle \tag{2.8}$$

where $\langle \cdot \rangle$ denotes the ensemble average, and μ_s is the ensemble mean of $s(t)$. In far most practical cases it is safe to assume $\mu_s = 0$. The energy of a signal $s(t)$ in the time interval $t \in [t_1, t_2]$ is given by [1]:

$$E_s = \int_{t_1}^{t_2} s^2(t)\, dt \tag{2.9}$$

Thus, the energy is the instantaneous power integrated over the given time interval.

2.3 Noise and interference

In many cases in electronic analysis of random signals, the noise signals are considered to be white noise with a specified spectral density. Say, $n(t)$ is a white noise signal. In this case the auto-correlation is:

$$R_{nn}(\tau) = \sigma^2\, \delta(\tau) \tag{2.10}$$

where δ is the Dirac delta-function (a generalized function). In the spectral domain this is:

$$S_{nn}(f) = \int_{-\infty}^{\infty} R_{nn}(\tau) \exp[-j2\pi f\tau] \, d\tau \qquad (2.11)$$

$$= \sigma^2 \qquad (2.12)$$

Equation (2.11) is known as the Fourier transform of $R_{nn}(\tau)$. The result of this equation is the frequency content of the variable $R_{nn}(\tau)$. As seen from Equation (2.12), the spectral density is constant at all frequencies. This is not true for any real world noise source as it would imply infinite power when integrating over frequency [2]. However, many analysis situations can be greatly simplified by using white noise — and the end result may be very close to reality as some band limiting is usually present anyway.

In many cases the noise is band limited — see Figure 2.4. Note from Equation (2.10) that the average power (or variance) is $R_{nn}(0) = \sigma^2\delta(0)$ which is a generalized function. This indicates that the average power in indefinite.

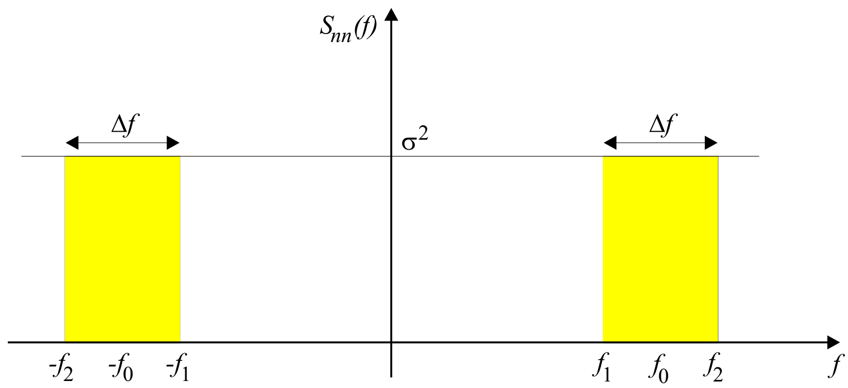

Figure 2.4: Illustration of pure white noise with spectral density σ^2, and of band limited white noise marked with yellow. The total average power in the two yellow areas is $2\sigma^2\Delta f$.

In many practical cases an ideal 'brick wall' filtered white noise source is used. This has the power spectral density as shown in yellow in Figure 2.4. It can be shown that the auto-correlation function of this type of noise is [3]:

$$R_{nn}(\tau) = \frac{2\sigma^2}{\pi\tau} \sin(\pi\tau\Delta f) \cos(2\pi f_0\tau) \qquad (2.13)$$

Note that the average power is $P_n = R_{nn}(0) = 2\sigma^2\Delta f$ quite as expected which makes Figure 2.4 agree with Equation (2.13).

Examples of almost white noise filtered by different types of filters are shown in Figure 2.5. As seen from the figure, the filtering has a large impact on the time domain waveforms. Just to give an idea on the average power (or variance) of the signals in Figure 2.5, the values have been calculated as:

- No filtering: $P_{blue\,curve} = 1.0023$.

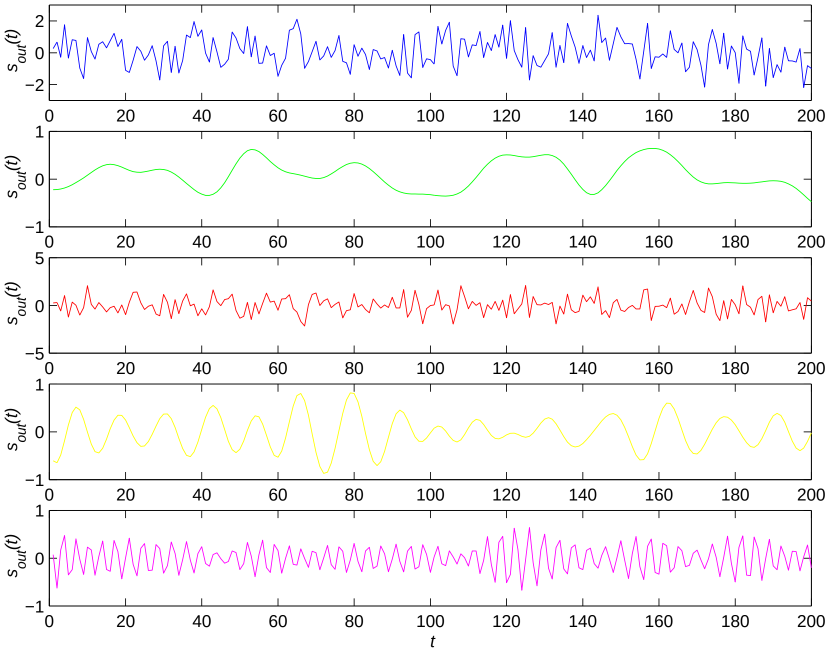

Figure 2.5: Examples of (pseudo) white Gaussian noise. Blue curve: Non-limited noise. Green curve: Noise filtered by 5MHz low pass filter. Red curve: Noise filtered by 5MHz high pass filter. Yellow curve: Noise filtered by 5–10MHz band pass filter. Magenta curve: Noise filtered by 25–30MHz band pass filter.

- 5MHz low pass filter: $P_{green\ curve} = 0.0999$.
- 5MHz high pass filter: $P_{red\ curve} = 0.9024$.
- 5–10MHz band pass filter: $P_{yellow\ curve} = 0.1023$.
- 25–30MHz band pass filter: $P_{magenta\ curve} = 0.1041$.

A few words about the above results. The input noise to the filters was generated by a pseudo random number generator. The noise spectrum of this approaches white noise (but is not). The average power (or variance) of the input noise signal was 1. This agrees well with the output result in case of no filtering. Adding the values for 5MHz low pass filtering with that of 5MHz high pass filtering also gives a total power (or variance) close to 1. The two band pass filters have the same bandwidth and also very close to the same average power. This is to be expected as the pseudo random number generator gives close to white noise spectral properties.

2.4 Signal-to-noise ratio

The signal-to-noise ratio (*SNR*) is very important to evaluate signal quality. *SNR* describes how much more power there is in the desired signal than in the noise influencing the signal. *SNR* is given by:

$$SNR = \frac{P_s}{P_n} \tag{2.14}$$

In far most cases *SNR* is given in 'dB' which is:

$$SNR_{dB} = 10 \cdot \log_{10}\left(\frac{P_s}{P_n}\right) \tag{2.15}$$

Figures 2.6–2.8 illustrate the relation between signal-to-noise ratio and signal quality. The three examples show curves for $SNR = 5, 10, 20$dB. The desired input signal is the same as shown in Figure 2.3. A noise signal with correct average power (or variance) is added to the desired signal. This forms the output signal with a given *SNR* value.

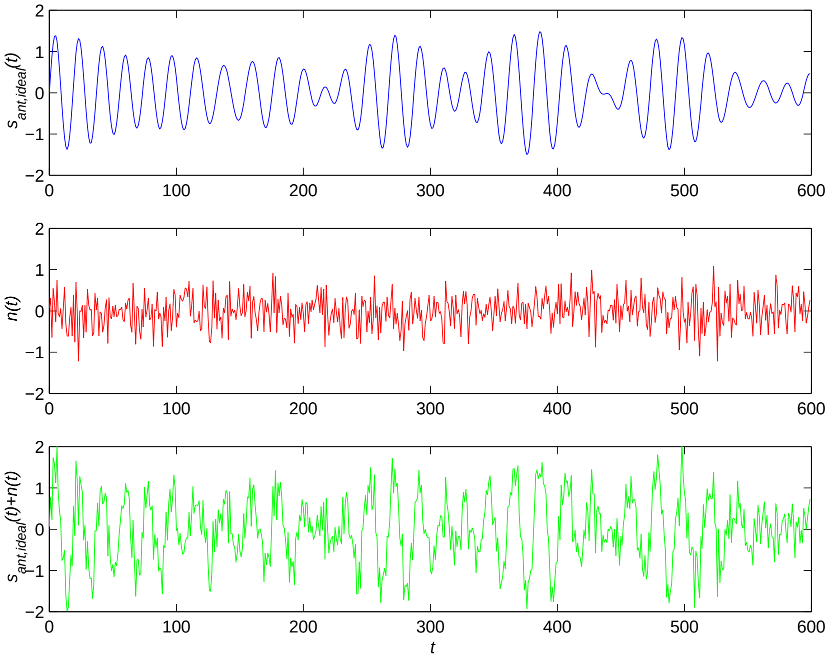

Figure 2.6: Examples of signal and noise which when combined has $SNR = 5$dB. Blue curve: The high frequency ideal antenna signal $s_{ant,ideal}(t)$ (same as Figure 2.3) — the average power is $P_s = 4.54 \cdot 10^{-1}$. Red curve: Zero mean Gaussian noise signal $n(t)$ with average power (or variance) $P_n = 1.43 \cdot 10^{-1}$. Green curve: The added signal $s_{ant,ideal}(t) + n(t)$ which has a $SNR = 5$dB.

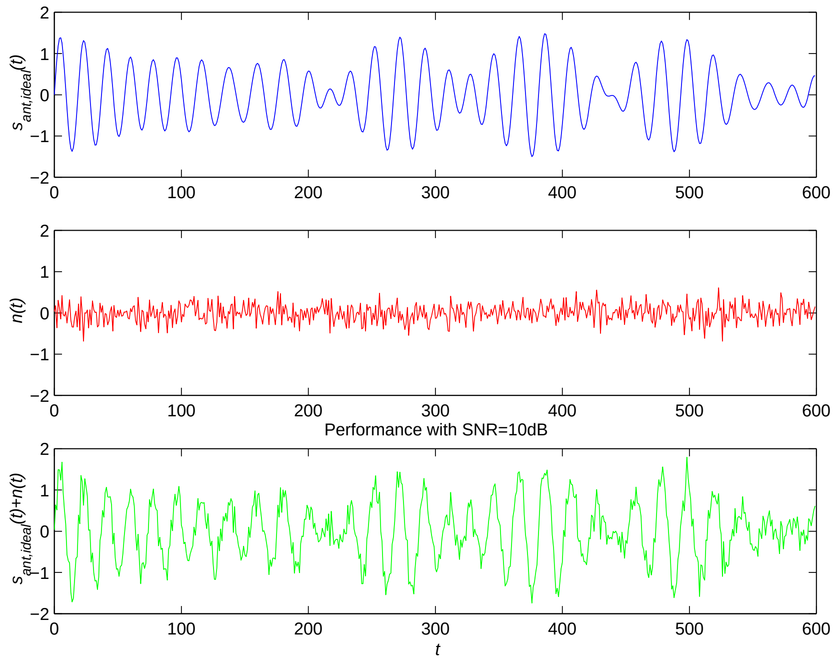

Figure 2.7: Examples of signal and noise which when combined has $SNR = 10\text{dB}$. Blue curve: The high frequency ideal antenna signal $s_{ant,ideal}(t)$ (same as Figure 2.3) — the average power is $P_s = 4.54 \cdot 10^{-1}$. Red curve: Zero mean Gaussian noise signal $n(t)$ with average power (or variance) $P_n = 4.54 \cdot 10^{-2}$. Green curve: The added signal $s_{ant,ideal}(t) + n(t)$ which has a $SNR = 10\text{dB}$.

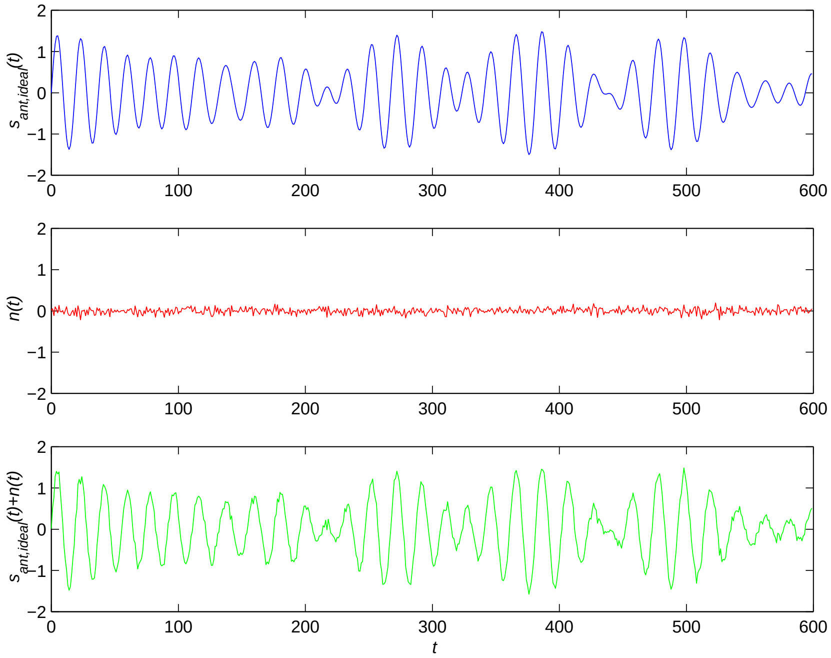

Figure 2.8: Examples of signal and noise which when combined has $SNR = 20\text{dB}$. Blue curve: The high frequency ideal antenna signal $s_{ant,ideal}(t)$ (same as Figure 2.3) — the average power is $P_s = 4.54 \cdot 10^{-1}$. Red curve: Zero mean Gaussian noise signal $n(t)$ with average power (or variance) $P_n = 4.54 \cdot 10^{-3}$. Green curve: The added signal $s_{ant,ideal}(t) + n(t)$ which has a $SNR = 20\text{dB}$.

Chapter 3

Front-end functionality

First consider the illustration of a receiver in Figure 3.1. The RF receiver system consists of an antenna, an RF (radio frequency) front-end, and signal processing. The outcome of all this is eventually the position and accompanying information.

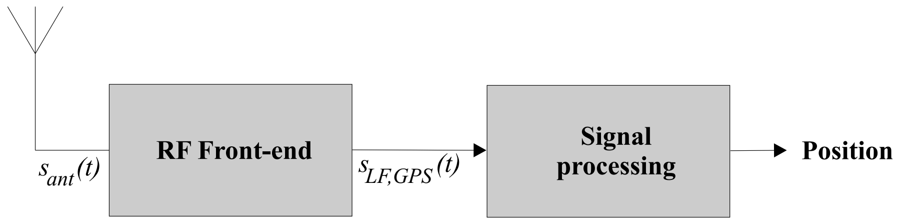

Figure 3.1: Illustration of the receiver system including (i) antenna, (ii) RF frontend, and (iii) signal processing. The outcome of the system should be the position of the receiver.

The purpose of the main components of Figure 3.1 are explained in the following.

The antenna picks up the electromagnetic signal in the air and translates this to an electrical signal. This electrical signal contains the GPS signals (L1 and L2 bands) as well as many other signals irrelevant to GPS. The GPS transmitter has put the information around a high frequency — 1.57542GHz for L1 band and 1.22760GHz for L2 band [5]. Generally, the antenna signal is also very weak.

The purpose of the RF frontend is thus to:

- Move the information of the GPS signal around the high frequency to the same information located at a low frequency. This enables the post signal processing required to strip off the interesting GPS information.

- Select the GPS band(s) and deselect all other irrelevant bands. This is necessary as the irrelevant bands may distort the desired GPS signal when the signal processing is made in the frontend.

- Amplify the very weak incoming GPS signal to a level which can be post processed digitally. While doing this, the frontend must contribute with as litte disturbance as possible.

While performing the above three tasks the RF front-end in itself must distort the GPS signal as little as possible. However, the RF frontend is not ideal and it will contribute to degrade the signal quality. The design target is thus to minimize this signal degradation while using little power end keeping cost at a reasonable level.

3.1 The antenna signal

The signal received at the antenna terminals generally contains all sorts of information. Far most of it is not relevant to the given application. Figure 3.2 shows a very simplified illustration of a small part of the frequency spectrum. It is so that ITU (International Telecommunication Union) which is part of United Nations distributes frequency bands to different applications.

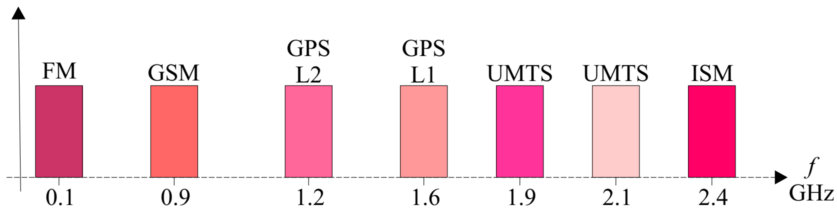

Figure 3.2: Very simplified illustration of selected applications and approximate frequency locations. Nothing is drawn to scale! FM is the radio broadcast stations, GSM and UMTS are for mobile communication, ISM is for e.g. short range communications and microwave ovens, and GPS is for the Global Positioning System.

A few characteristics of the signal(s) received at the antenna terminals are:

- Generally, the signal picked up by the antenna is the desired signal (in this case GPS L1 or L2 bands) and a lot of irrelevant signals.
- Generally, the level of the desired signal is very, very small. Some or many of the adjacent signals may be large in amplitude depending on e.g. the given application and location of the GPS terminal.

Mathematically, the signal received at the antenna terminals is:

$$s_{ant}(t) \quad = \quad \sum_{n=1}^{N} \Big\{ I_n(t)\cos(2\pi f_0 t + \phi_0) \; + \; Q_n(t)\sin(2\pi f_0 t + \phi_0) \Big\} \; + \; W(t) \; + \; N(t) \tag{3.1}$$

where $I_n(t)$ and $Q_n(t)$ are the in-phase and quadrature signals, respectively, from satellite number n of a total of N satellites, f_0 is the RF carrier frequency, and ϕ_0 is an arbitrary but constant initial phase. Thus, the sum part of this equation is a sum of RF signals from each GPS satellite, $W(t)$ is an interference signal describing the signals from other frequency bands and applications, and $N(t)$ is the unavoidable noise caused by solar spots, background sky noise etc. [4]. Although $N(t)$ and $W(t)$ are both distortions to the desired signal they are usually separated in two contributions. Note that the signals $I_n(t)$ and $Q_n(t)$ are the GPS information carrying signals. Average power level may vary over time due to e.g. changing satellite positions, changing receiver position, and multipath fading.

3.2 Output signals

If things were all ideal the RF front-end receiver should separate the two in-phase and quadrature-phase signals as:

$$I(t) = k \sum_{n=1}^{N} I_n(t) \tag{3.2}$$

$$Q(t) = k \sum_{n=1}^{N} Q_n(t) \tag{3.3}$$

where k is an amplifying factor ($k > 1$). The RF front-end is not able to separate the signals from each satellite. This is a task for the post signal processing utilizing the a priori known correlation properties of the signals. Thus in the ideal world, the signals above are the best obtainable. This is illustrated in Figure 3.3.

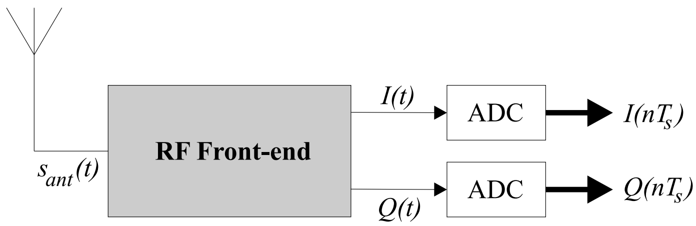

Figure 3.3: Illustration of the front-end including (i) antenna, (ii) RF frontend, and (iii) analog-to-digital (ADC) conversion.

Chapter 4

Receiver components

To facilitate separation of the interesting signals $I(t)$ and $Q(t)$ from Equations (3.2) and (3.2), a number of essential receiver components are available. The most important of these are discussed in this chapter. Emphasis is on explaining functionality and simple but yet reasonably realistic models of the components.

4.1 Amplifiers

Amplifiers are essential building blocks of practically all electronic equipment. The task of the amplifier is to amplify the signal and at the same time distort as little as possible. The functional block symbol of an amplifier is shown in Figure 4.1.

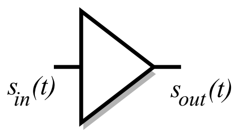

Figure 4.1: Functional block symbol of an amplifier.

4.1.1 Simple models

Ideally an amplifier ... well ... amplifies the input signal. Thus the input-output relation of signals is:

$$s_{out}(t) \quad = \quad a_1 s_{in}(t) \tag{4.1}$$

where a_1 is a real constant and $a_1 > 1$ to make sense of “amplification”. In the real world the input-output relationship is more like:

$$s_{out}(t) \quad = \quad \sum_{m=1}^{M} a_m s_{in}^m(t) + n(t) \tag{4.2}$$

where $a_1, \ldots, a_M$ are real constants and $n(t)$ is a noise signal. There are two non-ideal contributions in Equation (4.2):

- When $a_2, \ldots, a_M \neq 0$ nonlinear distortion is added to the ideal signal $a_1 s_{in}(t)$.
- A noise signal $n(t)$ is added. This occurs as the electronic components generate noise mainly caused by thermal agitation of imperfect conductors.

4.1.2 An illustrative example — nonlinear effect

Consider a simple example where the input-output relation is:

$$s_{out}(t) \quad = \quad a_1 s_{in}(t) + a_3 s_{in}^3(t) \tag{4.3}$$

This type of nonlinear input-output relation can be used for many practical types of amplifiers. For this example $a_1 = 1$ and $a_3 = -0.3$. The input-output relation is shown in Figure 4.2.

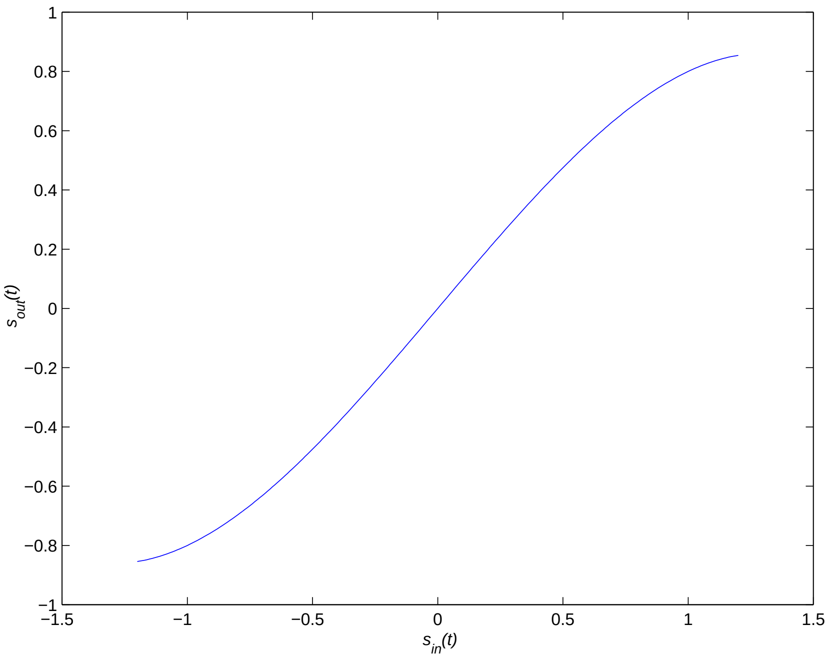

Figure 4.2: Example of nonlinear input-output relation for an amplifier.

As can be seen from Figure 4.2 the input-output relation is very close to linear for small input signals. For large inputs the amplitude compresses which causes distortion. The chosen input signal to the nonlinear amplifier is shown in Figure 4.3. As seen there is substantial amplitude variation.

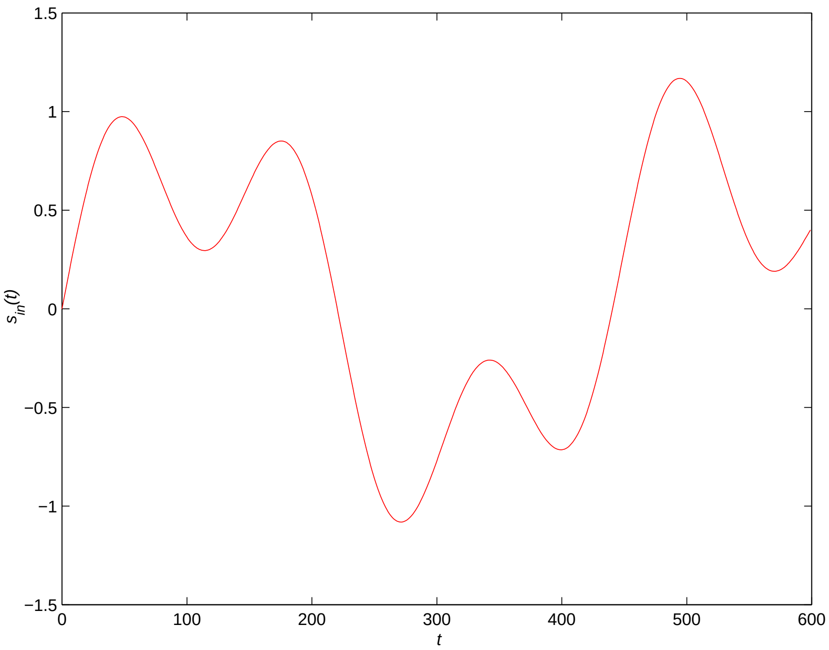

Figure 4.3: Example of input signal to an amplifier.

Figure 4.4 shows the output from the amplifier. Both the ideal linear amplifier response and the nonlinear distorted output are shown. As is seen clearly from Figure 4.4 the nonlinear amplifier is capable of tracking the signal at low signal level. However, at large levels the output signal level is too small, and thus distortion occurs. This type of signal distortion depends heavily on the input signal as seen from the figure

4.1.3 An illustrative example — noise effect

Consider a simple example where the input-output relation is:

$$s_{out}(t) \quad = \quad a_1 s_{in}(t) + n(t) \tag{4.4}$$

It is far from uncommon to have Gaussian noise in electronic circuits. So assume for this example that $n(t)$ is pseudo random Gaussian noise with a mean of 0 and a standard deviation of 0.1. The noise signal versus time is shown in Figure 4.5. The signal amplification factor $a_1 = 1$

The chosen input signal to the nonlinear amplifier is shown in Figure 4.6. As seen there is substantial amplitude variation.

Figure 4.7 shows the output from the amplifier. Both the ideal noise free amplifier response and the noisy output are shown. As is seen clearly from Figure 4.7 the noisy amplifier distorts the signal no matter what the input signal level is. Thus it is a fundamentally different type of signal distortion than in nonlinear amplifiers.

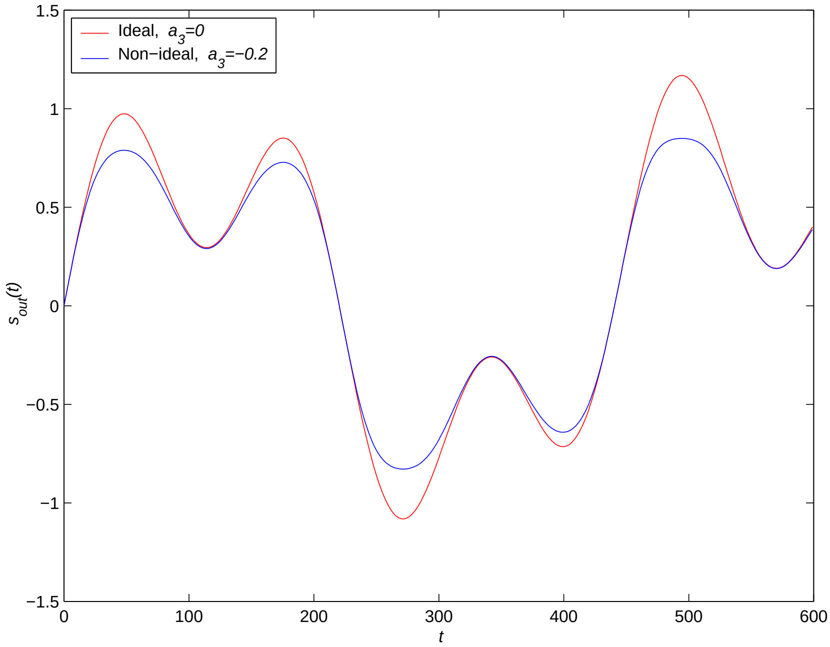

Figure 4.4: The output from the amplifier. The red curve shows the ideal amplifier output with $a_1 = 1$ and $a_3 = 0$. The blue curve shows the non-ideal amplifier output with nonlinear distortion with $a_1 = 1$ and $a_3 = -0.2$.

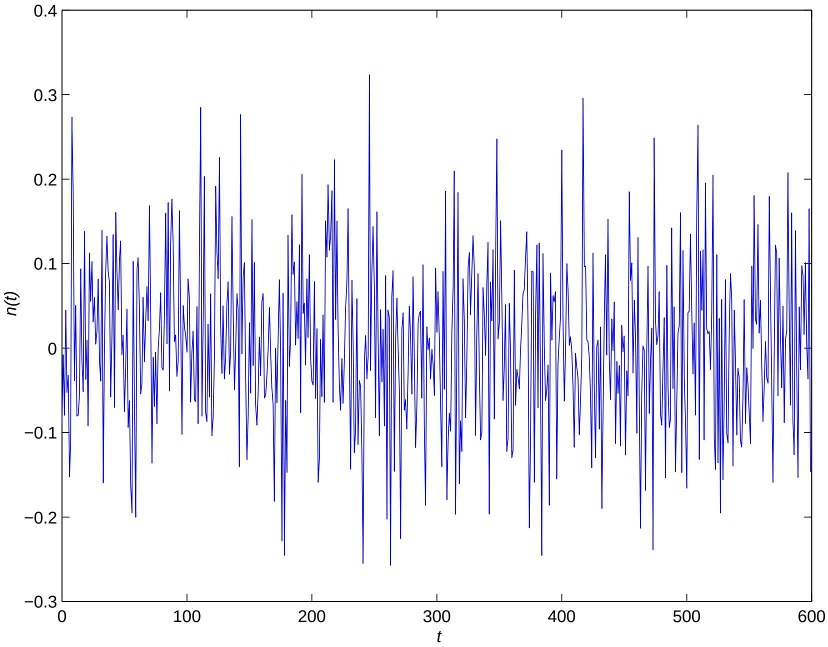

Figure 4.5: Example of noise signal added by noisy amplifier.

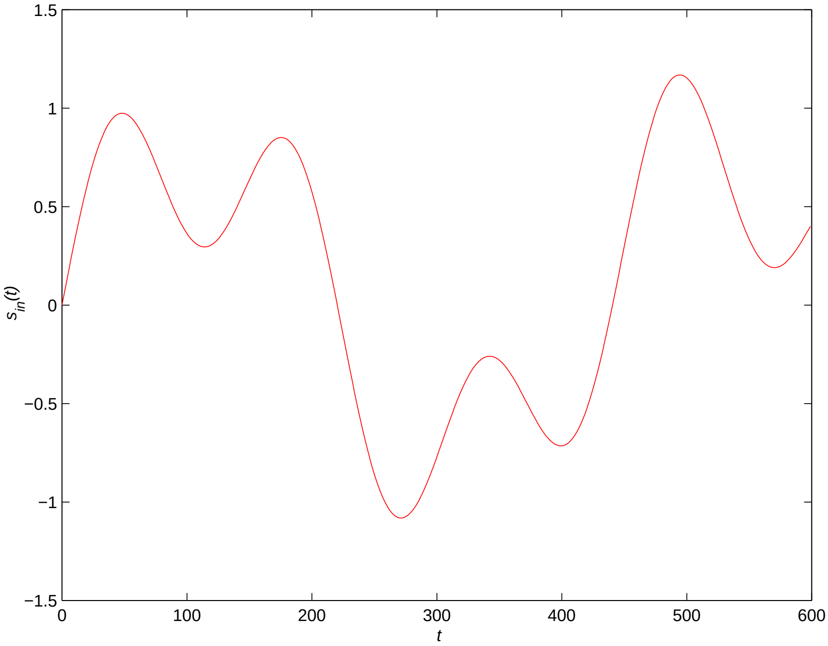

Figure 4.6: Example of input signal to a amplifier.

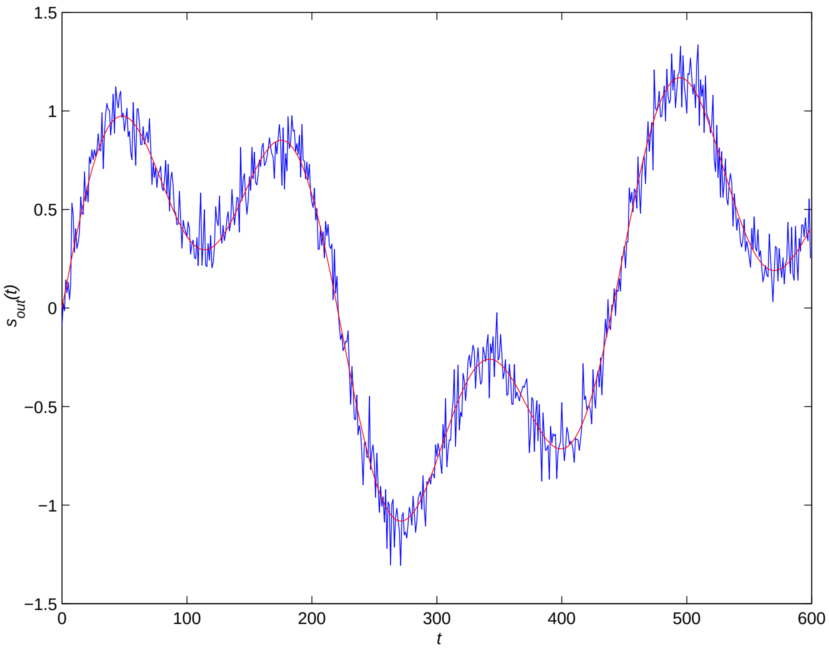

Figure 4.7: Output signal from the noisy amplifier. Red curve: Ideal noise free amplifier output with $n(t) = 0$. Blue curve: Output from the noisy amplifier.

4.2 Mixers

Mixers are ideally multipliers. These functional blocks are used to translate high frequencies to low frequencies (and vice versa). The functional block symbol of a mixer is shown in Figure 4.8.

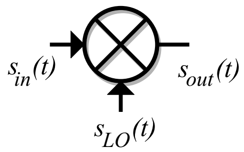

Figure 4.8: Functional block symbol of a mixer.

4.2.1 Simple models

Ideally a mixer multiplies two signal — one signal is the information carrying input signal and the other is a local oscillator (LO) signal. Thus the input-output relation of signals is:

$$s_{out}(t) \quad = \quad a_{1,1}\, s_{in}(t)\, s_{LO}(t) \tag{4.5}$$

where a_1 is a real constant. In the real world the input-output relationship is more like:

$$s_{out}(t) \quad = \quad \sum_{m=0}^{M}\sum_{n=0}^{N} a_{m,n}\, s_{in}^{m}(t)\, s_{LO}^{n}(t) \;+\; n(t) \tag{4.6}$$

where $a_{m,n}$ are real constants and $n(t)$ is a noise signal. Equation (4.6) is identical to the ideal case in Equation (4.5) when all $a_{m,n} = 0$ except $a_{1,1}$, and $n(t) = 0$. There are two non-ideal contributions in Equation (4.6):

- When $a_{m,n} \neq 0$ for one or more m,n except $m = n = 1$ nonlinear distortion is added to the ideal signal $a_{1,1}\, s_{in}(t)\, s_{LO}(t)$.
- A noise signal $n(t)$ is added. This occurs as the electronic components generate noise mainly caused by thermal agitation of imperfect conductors.

4.2.2 Mixer functional visualization

The following tries to visualize signals when processed by a mixer functional block. First, assume that the input signal to a mixer is given by:

$$s_{in}(t) \quad = \quad I(t)\cos(2\pi f_0 t) \tag{4.7}$$

The information carrying low frequency signal $I(t)$ is as shown in Figure 4.9. As seen from the figure, there is plenty of signal variation versus time. As seen from Equation

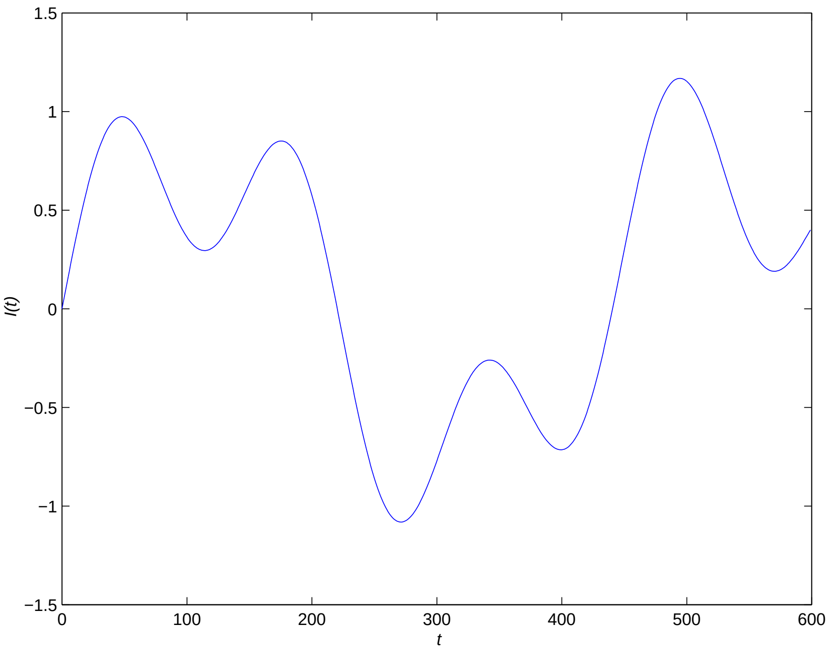

Figure 4.9: Information carrying signal $I(t)$.

(4.7) this low frequency signal must be multiplied by a rapid changing cos-function. This is shown in Figure 4.10.

As seen from Figure 4.10 the low frequency signal $I(t)$ is the envelope of the mixer high frequency input signal $s_{in}(t)$. The green curve in Figure 4.10 is included to emphasize this relation — otherwise it has no importance. The local oscillator signal which is the other input to the mixer is shown in Figure 4.11.

The output from the mixer according to Equation (4.5) is shown in Figure 4.12. Quite as expected a cos-signal with twice the frequency of the input appears in the output signal $s_{out}(t)$. By low pass filtering this high frequency cos-signal vanishes. The result is an exact copy of the low frequency information carrying signal $I(t)$. Thus, the mixer has removed the high frequency content and preserved the information carrying signal. A result of this filtering is that half the signal level is 'lost'. The desired output signal after filtering is $I(t)/2$. But this can be corrected by subsequent amplification.

4.2.3 An illustrative example — nonlinear effect

Consider a simple example where the input-output relation is:

$$s_{out}(t) \quad = \quad a_{1,1}s_{in}(t)s_{LO}(t) + a_{2,0}s_{in}^2(t) \tag{4.8}$$

As the constant $a_{2,0} \neq 0$ this corresponds to that the input signal also has a nonlinear contribution to the output. This type of nonlinear input-output relation can be used to

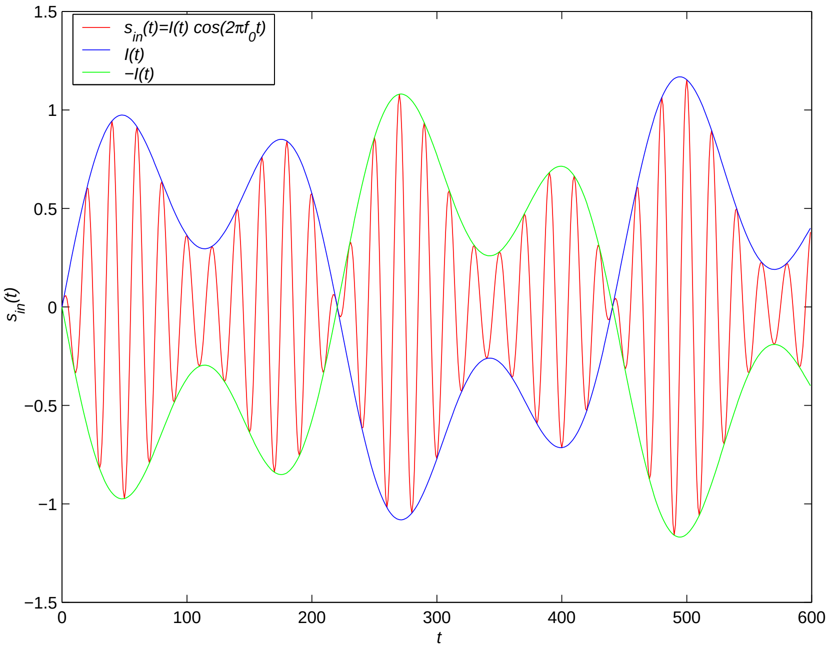

Figure 4.10: Blue curve: Low frequency information carrying input $I(t)$. Green curve: Negative of low frequency information carrying input $-I(t)$. Red curve: Input signal to mixer block s_{in} from Equation (4.7).

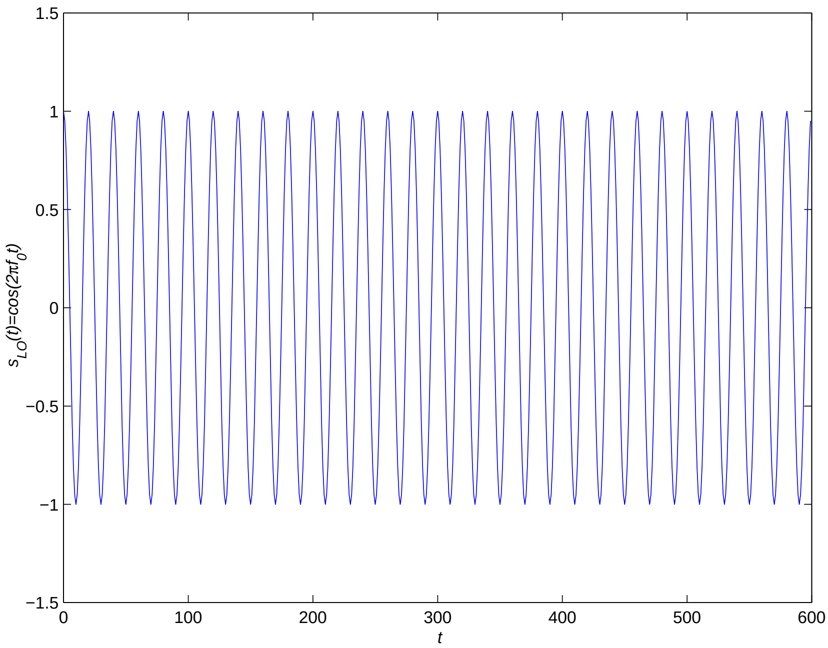

Figure 4.11: Local oscillator $s_{LO}(t)$ input to mixer.

describe what is known as second order distortion for many practical types of mixers. Second order as $s_{in}(t)$ is raised to the power of 2. For this example $a_{1,1} = 1$ and $a_{2,0} = 0.2$.

When the input signal level is small it is to expect that the nonlinear distortion is small as seen from Equation (4.8). The chosen input signal to the nonlinear amplifier is shown in Figure 4.10. As seen there is substantial amplitude variation. The chosen local oscillator signal is shown in Figure 4.11.

Figure 4.13 shows the output from the mixer. Both the ideal and distorted mixer envelope outputs response and the true unfiltered mixer output is shown. As is seen clearly from Figure 4.13 the non-ideal mixer is capable of tracking the signal at low signal levels. However, at large levels the output signal level is distorted. This type of signal distortion depends heavily on the input signal as seen from the figure

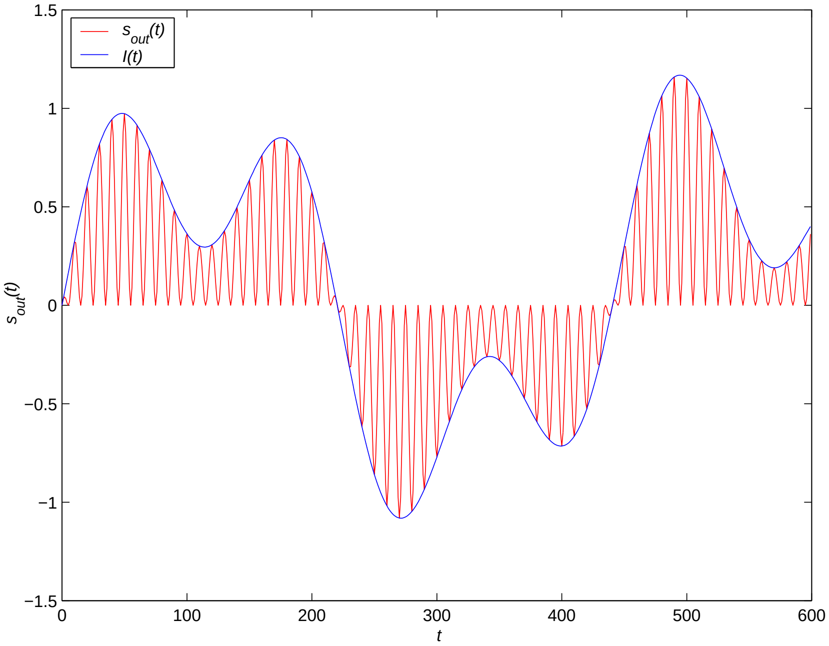

Figure 4.12: Output from mixer $s_{out}(t)$ (red curve). Input information carrying signal $I(t)$ (blue curve).

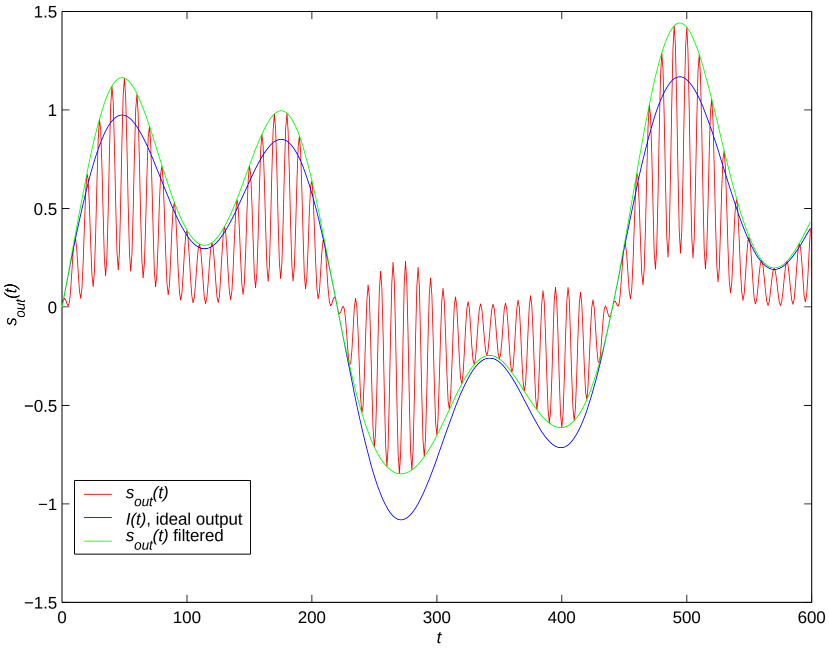

Figure 4.13: The output from the mixer. Red curve: The distorted mixer output. Blue curve: The ideal mixer output envelope signal without distortion. Green curve: The distorted mixer output envelope.

4.3 Filters

Filters play a major role in all kinds of electronic equipment. Filters are basically used to select or omit certain frequency bands. The functional block symbol of a filter is shown in Figure 4.14.

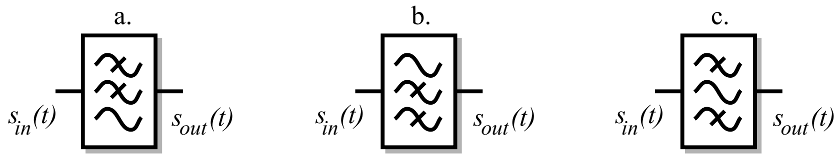

Figure 4.14: Functional block symbols of filters. a: Low pass filter, b: High pass filter, c: band pass filter.

4.3.1 Simple models

In many aspects, a filter is just some special kind of amplifier. The characteristic thing is that the amplification is frequency dependent. This means that one frequency may be amplified by a factor of 1 whereas another frequency may be amplified by a factor of e.g. 0.2.

Filters may be described both in time domain and in frequency domain. In time domain the input-output relation is:

$$v_o(t) \quad = \quad \int_{-\infty}^{\infty} h(t-\tau)\, v_i(\tau)\, d\tau \tag{4.9}$$

In RF front-end analysis and design it is common practice to describe filters in the frequency domain via a transfer function. The input-output relation is:

$$v_o(f) \quad = \quad H(f)\, v_i(f) \tag{4.10}$$

where $H(f)$ is the transfer function. At a given f, $H(f)$ is generally a complex number. The presence of an imaginary part implies that the filter changes the phase. Said differently, the time delay for a signal moving through a filter may not be the same for all frequency components. In some cases this phase variation may be a problem which must be accounted for. But for the sake of simplicity the following only considers the magnitude part of the transfer function — that is $|H(f)|$.

Non-ideal filters may give both nonlinear distortion and noise. However, in most cases only noise is included in analysis of RF front-ends as it is predominant. Thus, a frequency domain model including noise can be described by:

$$v_o(f) \quad = \quad H(f)\, v_i(f) \; + \; n(f) \tag{4.11}$$

where $n(f)$ is a frequency domain representation of the noise signal.

4.3.2 A few filter examples

First, take a look at Figure 4.15 which illustrates the effect of filtering on a close to white noise input. As seen from the figure, the different filters have large impact on the output time domain waveforms of the signals. As an example, a low pass filter will remove fast changes in the input signal. Or say that a band pass filter is included around L1 frequency. In this case the GSM bands, L2 band etc. can be attenuated. If L1 contains the information of interest, it is essential to remove unwanted signals. This is the exact purpose of filters.

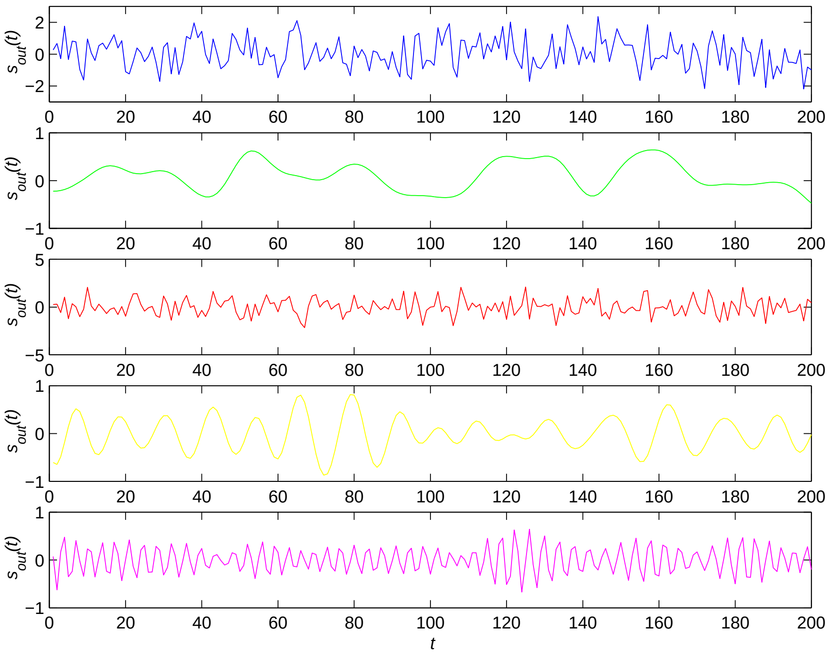

Figure 4.15: Examples of (pseudo) white Gaussian noise. Blue curve: Non-limited noise. Green curve: Noise filtered by 5MHz low pass filter. Red curve: Noise filtered by 5MHz high pass filter. Yellow curve: Noise filtered by 5–10MHz band pass filter. Magenta curve: Noise filtered by 25–30MHz band pass filter. (Copy of Figure 2.5).

The magnitude of the transfer functions used for Figure 4.15 is shown in Figure 4.16. The colors used in the two figures refer to directly comparable situations.

4.3.3 MATLAB assistance

The indispensable MATLAB programming language contains some very useful filter tools — in particular in the Digital Signal Processing toolbox. Very little knowledge is needed to utilize these routines. Direct filter synthesis based on readily available information can be found in e.g. functions `butter`, `besself`, `ellip`, `cheby1`,

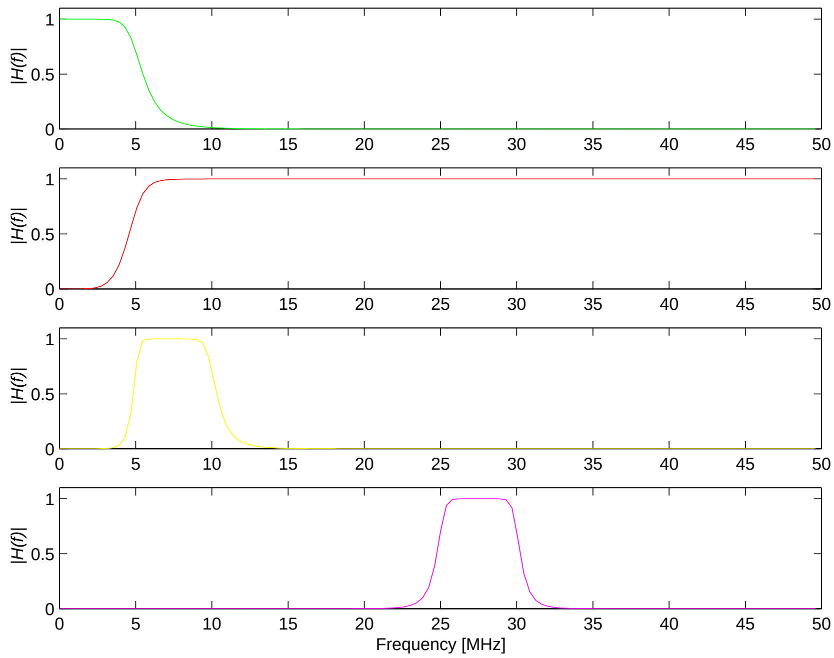

Figure 4.16: Transfer functions of filters. Green curve: 5MHz low pass filter. Red curve: 5MHz high pass filter. Yellow curve: 5–10MHz band pass filter. Magenta curve: 25–30MHz band pass filter.

and cheby2. Once the filter is designed, the routine filter can be used to process time domain input samples through the filter.

The following MATLAB code gives a small example of the filtering function in MATLAB's Digital Signal Processing toolbox.

```
clear all;                  % Clear all variables

L = 100000;                 % Number of data samples
fs = 100E6;                 % Sample rate
OF = 6;                     % Order of filter
n = randn(1,L);             % Gaussian noise, variance 1

DF = [5E6 7E6]/(fs/2);      % Normalized frequencies
[b,a] = butter(OF,DF);      % Design filter
y = filter(b,a,n);          % Process n data by filter

figure(1); clf;             % Make figure window
subplot(3,1,1);             % Create subplot 1
plot(n(L-199:L),'b');       % Plot the last part of n
ylabel('{\it n(t)}');       % Put on y-label title
subplot(3,1,2);             % Create subplot 2
plot(y(L-199:L),'r');       % Plot the last part of y
ylabel('{\it y(t)}');       % Put on y-label title
subplot(3,1,3);             % Create subplot 3
plot(y(1:200),'g');         % Plot the first part of y
ylabel('{\it y(t)}');       % Put on y-label title
xlabel('{\it t}');          % Put on x-label title
```

The key part of the code is the three lines beginning with DF = ... This line sets up lower and higher frequencies normalized to half the sample rate. The sample rate f_s is simply $1/T_s$ where T_s is the time distance between two adjacent time samples. The sample rate in itself is not that important, but is must be much larger than the filter frequencies. The sampling rate in this case is 100MHz and the low and high frequencies are 5MHz and 7MHz, respectively. This indicates that the filter is a band pass filter.

The call to butter then designs a filter of type Butterworth. This filter type has little in-band variation at the expense of a rather slow attenuation of out of band signals. Other types of filters are available such as besself and ellip. The returned parameters a and b describe the transfer function of the filter corresponding to $H(f)$ in Equation (4.11).

The call to filter then filters the input data vector n by the transfer function specified by parameters a and b to yield the output signal y. The input signal n is zero mean Gaussian noise with variance 1.

The result of running the above MATLAB code is shown in Figure 4.17. The blue curve shows the input signal to the filter. As seen previously this Gaussian noise signal has the expected content of very many frequencies — a wide band input with a close to

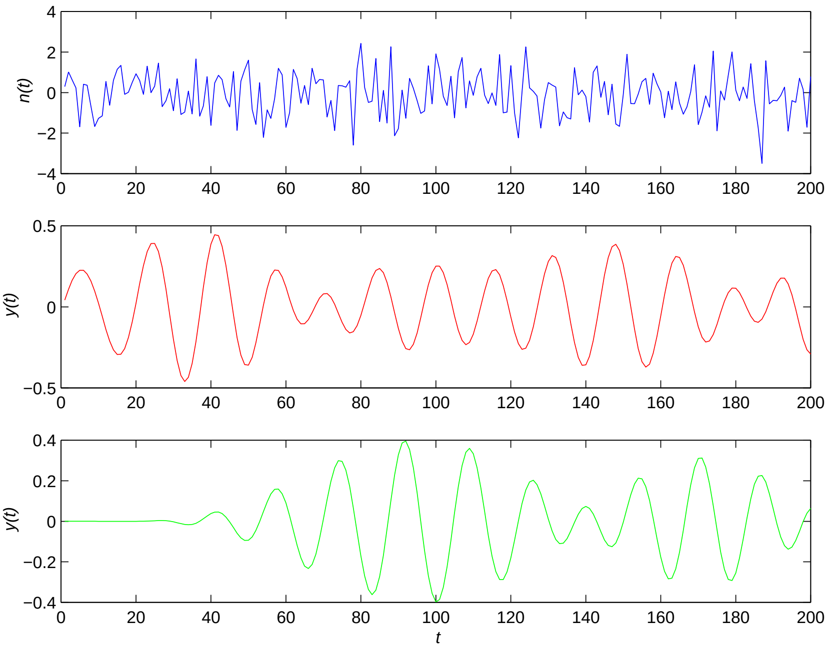

Figure 4.17: Input to and output from designed Butterworth filter. Blue curve: Input Gaussian noise signal with variance 1. Red curve: Output from the filter well into the data vector. Green curve: Output from the filter at the beginning of the data vector.

constant power spectral density (as seen by the vertical black line in Figure 2.4). The curve shows the last 200 data values in the noise vector n. The red curve shows the last 200 output data values from the filter. The green curve shows the first 200 output data values from the filter. As seen from the figure, it takes some time before the output appears. This is a time lag and start up phenomena of the filter. The duration of this varies with filter type and order, but having a delay is unavoidable. Generally, this is no problem as long as it is kept in mind.

4.4 Quadrature down-converter

The quadrature down-converter is a very essential building block of all modern communication equipment. The block diagram of a quadrature down-converter is shown in Figure 4.18 as well as the functional block symbol of a quadrature down-converter (QDC).

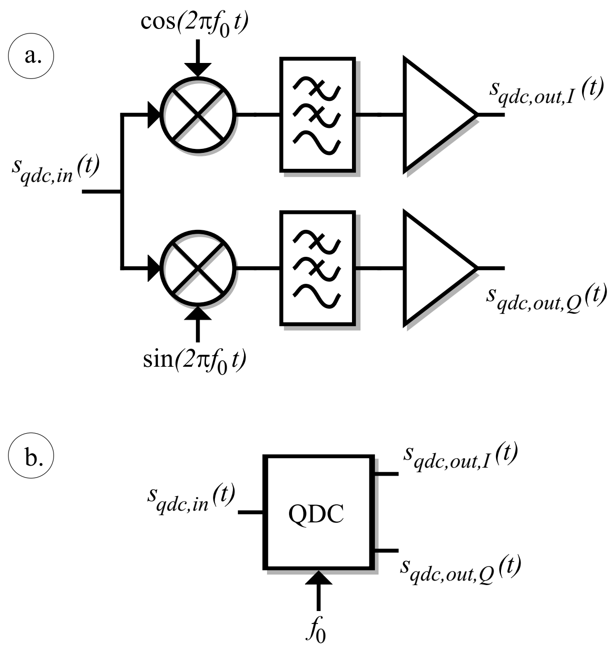

Figure 4.18: The quadrature down-converter. a: Block diagram. b: Functional block symbol.

4.4.1 Functional working principle

To facilitate the understanding of the working principle of the quadrature down-converter the signal processing by each component is performed in the following. Ideal noise free and fully signal linear components are used for all blocks.

First of all, the input signal is a modulated signal described by:

$$s_{qdm,in}(t) \quad = \quad I(t)\cos(2\pi f_0 t) + Q(t)\sin(2\pi f_0 t) \tag{4.12}$$

Consider the output $s_{qdm,out,I}(t)$ signal. This is the upper path of Figure 4.18a. Refer to this path as the I-path. The ideal functionality of the mixer is to multiply the two incoming signals. Thereby, the output signal of the I-path mixer is:

$$\begin{aligned} s_{out,mix,I}(t) &= s_{qdm,in}(t)\cos(2\pi f_0 t) & (4.13)\\ &= \Big\{ I(t)\cos(2\pi f_0 t) + Q(t)\sin(2\pi f_0 t) \Big\} \cos(2\pi f_0 t) & (4.14)\\ &= I(t)\cos^2(2\pi f_0 t) + Q(t)\sin(2\pi f_0 t)\cos(2\pi f_0 t) & (4.15)\\ &= \frac{1}{2}I(t) + \frac{1}{2}I(t)\cos(2\pi 2 f_0 t) + \frac{1}{2}Q(t)\sin(2\pi 2 f_0 t) & (4.16) \end{aligned}$$

where it is assumed that the mixer gain is unity (this gives no loss of generality). Some interesting points can be made from Equation (4.16). First of all, observe that the information carrying signal $I(t)$ appears directly — no high frequency carrier is included in the first term of Equation (4.16). Secondly, observe that the latter two terms in Equation (4.16) appears around a carrier frequency of $2f_0$ — thus twice the carrier frequency. The signal $s_{out,mix,I}(t)$ does not directly give the desired $I(t)$ or $Q(t)$ signal — but it is close.

The next functional block which the signal sees is the low pass filter. Suppose the filter is designed such that: (i) it lets the low frequency information carrying signals $I(t)$ and $Q(t)$ pass, and (ii) it effectively attenuates frequencies at and above f_0 — actually $2f_0$ suffices but some distortion mechanisms will require an efficient attenuation at f_0. Further, a low bandwidth is needed to reduce noise and interference in general. Thus, the output from the low pass filter is:

$$s_{out,filter,I}(t) \quad = \quad \frac{1}{2}I(t) \tag{4.17}$$

Finally, there is a need to increase the signal level. In general the average power at the quadrature down-converter input is very small, and it is absolutely essential for the following signal processing to have a high signal level. The output of the amplifier is:

$$s_{qdc,out,I}(t) \quad = \quad \frac{k}{2}I(t) \tag{4.18}$$

where k is the amplifier gain. As the input signal to the amplifier is a low frequency signal, the amplifier can be designed to amplify only low frequency signals. This is much simpler than to design high frequency amplifiers.

Next, consider the output $s_{qdm,out,Q}(t)$ signal. This is the lower path of Figure 4.18a. Refer to this path as the Q-path. It can easily be shown that the output signal from this path is:

$$s_{qdc,out,Q}(t) \quad = \quad \frac{k}{2}Q(t) \tag{4.19}$$

when the same principles for filtering etc. as in the I-path are used.

Chapter 5

Receiver architectures

This chapter essentially covers two key types of receiver architectures — the direct conversion receiver and the super heterodyne receiver. Both types of receivers count on the quadrature down-converter to extract the information carrying $I(t)$ and $Q(t)$ signals.

5.1 Direct conversion receivers

The direct conversion receiver is in principle very close to being a quadrature down-converter. The block diagram of a direct conversion receiver is shown in Figure 5.1.

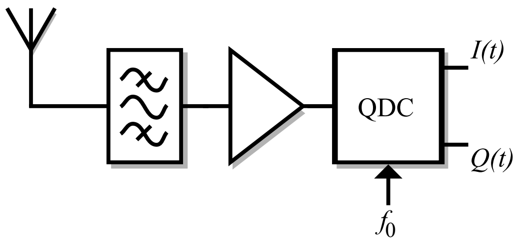

Figure 5.1: Block diagram of a direct conversion receiver.

The frequency f_0 applied to the quadrature down-converter (QDC) is directly the high carrier frequency — 1.57542GHz for L1 and 1.22760GHz for L2. There is not much more to the functional principles than for a quadrature down-converter.

As seen from Figure 5.1 the antenna signal first meets a band pass filter. Due to practical reasons (size, cost, performance etc.) the bandwidth of this filter is usually much larger than the information bandwidth. As L1 and L2 frequencies are separated by more than 300MHz is is likely that two different filters are used. To receive L1 and L2 simultaneously, two receivers are most likely needed. If either L1 or L2 are received, one receiver is sufficient where the first band pass filter can be switched between two is likely. One band pass filter passes L1 at 1.57542GHz and another band pass filter passes L2 at 1.22760GHz. The purpose of the filter is to attenuate disturbing signals and to a lesser extend to reduce the average noise power level.

Next, the signal is amplified to increase the very low received average power. The output signal from the amplifier is essentially (besides some scaling) the same as the input signal to the quadrature down-converter as in Equation (4.12). Thus the $I(t)$ and $Q(t)$ output as indicated in Figure 5.1 are correct.

5.2 Super heterodyne receivers

The super heterodyne receiver has been around for many years and is still very popular for many communications receivers [6].

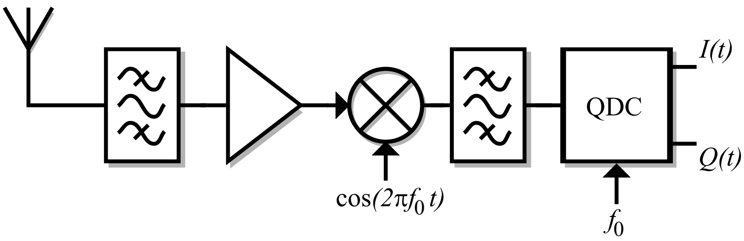

Figure 5.2: Block diagram of a super heterodyne receiver.

The working principle is as follows:
The signal received by the antenna is given by:

$$s_{ant}(t) \quad = \quad I(t)\cos(2\pi f_0 t) + Q(t)\sin(2\pi f_0 t) \tag{5.1}$$

where $I(t)$ and $Q(t)$ contain the addition of the information carrying in-phase and quadrature signals from all satellites, respectively. In addition to this there is also some noise and interference. For the sake of just showing the working principle, these two signals are omitted.

The antenna signals is first filtered in a band pass filter. Due to practical reasons (size, cost, performance etc.) the bandwidth of this filter is usually much larger than the information bandwidth. As L1 and L2 frequencies are separated by more than 300MHz is is likely that two different filters are used. To receive L1 and L2 simultaneously, two receivers are most likely needed. If either L1 or L2 are received, one receiver is sufficient where the first band pass filter can be switched between two is likely. One band pass filter passes L1 at 1.57542GHz and another band pass filter passes L2 at 1.22760GHz. The purpose of the filter is to attenuate disturbing signals and to a lesser extend to reduce the average noise power level. The output signal from the filter can described by:

$$s_{out,filt,1}(t) \quad = \quad a_{filt,1}\, I(t)\cos(2\pi f_0 t) + a_{filt,1}\, Q(t)\sin(2\pi f_0 t) \tag{5.2}$$

where $a_{filt,1}$ is the amplification factor. In real world electronics this is, however, attenuation — i.e. $0 < a_{filt,1} < 1$. So essentially the filter does not distort the information signals but it reduces the power of the noise and interference disturbances.

Then the signal is amplified. It is likely that the gain of the amplifier can be changed to some extend. This is because the average power of the desired signal may vary over time and because the disturbing signals may overload the amplifier if its gain is to high. The antenna filter described above is not sufficient to remove all environmental disturbances. The output signal from the amplifier is thus:

$$s_{out,amp}(t) = a_{amp}\, a_{filt,1}\, I(t)\cos(2\pi f_0 t) + a_{amp}\, a_{filt,1}\, Q(t)\sin(2\pi f_0 t) \tag{5.3}$$

The remaining noise and interference is thus amplified as well as the desired signal. The amplifier is unfortunately not able to distinguish between fried and enemy.

Next, the signal is processed by a mixer where the local oscillator signal is $\cos(2\pi f_1 t)$ where it is assumed that $f_0 > f_1$. The output from the mixer is thus:

$$\begin{aligned} s_{out,mix}(t) &= s_{out,amp}(t)\cos(2\pi f_1 t) && (5.4)\\ &= a_{amp}\, a_{filt,1}\left\{ I(t)\cos(2\pi f_0 t) + Q(t)\sin(2\pi f_0 t)\right\}\cos(2\pi f_1 t) && (5.5)\\ &= \frac{a_{af1}}{2}\left\{ I(t)\cos(2\pi(f_0 - f_1)t) + Q(t)\sin(2\pi(f_0 - f_1)t)\right\}\\ &\quad + \frac{a_{af1}}{2}\left\{ I(t)\cos(2\pi(f_0 + f_1)t) + Q(t)\sin(2\pi(f_0 + f_1)t)\right\} && (5.6) \end{aligned}$$

where $a_{af1} = a_{amp}\, a_{filt,1}$. Thus, the information carrying signals have been moved from a carrier frequency of f_0 to the same information at two new carrier frequencies $f_0 - f_1$ and $f_0 + f_1$. The information carrying signals are undisturbed (besides the non-ideal effects of the mixer) but moved to new carrier frequencies. As one of the tasks of the receiver is to move information located at high frequencies to low frequencies, the signal with the carrier frequency $f_0 - f_1$ is usually the desired signal. This frequency is called the intermediate frequency of a super heterodyne receiver. The frequency $f_0 - f_1$ is typically chosen close to 10% of the carrier frequency f_0. So for L1 and L2 bands this is in the $120 - 150$MHz range. The specific choice is heavily influenced by commercial availability of cheap and high performing filters.

Next, the signal from the mixer is filtered by a band pass filter. The task of the filter is to select the signal at the carrier frequency $f_0 - f_1$ and to exclude out of band distortion and noise. The output signal from this second filter in the receiver is thus:

$$s_{out,filt2}(t) = \frac{a_{af1f2}}{2}\left\{ I(t)\cos(2\pi(f_0 - f_1)t) + Q(t)\sin(2\pi(f_0 - f_1)t)\right\} \tag{5.7}$$

where $a_{af1f2} = a_{af1}a_{f2}$ with a_{f2} being the amplification constant of the filter.

Choosing the carrier frequency to the quadrature down-converter (QDC) it is known from page 38 that the output from the upper and lower paths in the QDC is:

$$s_{qdc,out,I}(t) = \frac{1}{4} k a_{af1f2} I(t) \tag{5.8}$$

$$s_{qdc,out,Q}(t) = \frac{1}{4} k a_{af1f2} Q(t) \tag{5.9}$$

Thus, as indicated in the block diagram of the super heterodyne receiver in Figure 5.2, the upper path gives $I(t)$ and the lower path gives $Q(t)$.

5.3 Receiver pros and cons

The following compares the architectures of the direct conversion receiver in Figure 5.1 and the super heterodyne receiver in Figure 5.2. As seen from the figures the direct conversion receiver is conceptually simpler than the super heterodyne receiver. At least in block count.

Characteristics of the direct conversion receiver:

- The great advantage of this receiver is simplicity and that it is well suited to integrated chip fabrication. This is because there are essentially no high quality filtering in this architecture.
- A problem is that there is not that much amplification. Amplification without pre-filtering often leads to nonlinear distortion. Further, as amplification is sparse the ability of this type of receiver to receive very weak signals is not the best — this is known as poor sensitivity.

Characteristics of the super heterodyne receiver:

- The great advantage of this receiver is good ability to receive very weak signals — good sensitivity. This is enabled through the use of high quality filtering. Further, the filtering means that nonlinear distortion generated in the receiver is generally sparse.
- The main disadvantages is complexity and that it is very difficult if not impossible to implement on an integrated chip.

In the real world, high quality receivers can be made both using the direct conversion architecture and the super heterodyne architecture. There is, however, a clear trend towards the direct conversion receiver. The relative ease of integrated circuit design is the key driving factor in this. Advanced adaptive schemes have been developed to reduce the shortcomings. And now it is possible to use direct conversion receivers even for high performance applications such as the mobile communication system GSM. On the other hand, if ultimate performance is the key issue, it is unlikely that the super heterodyne receiver will loose the battle.

Chapter 6

Exercises

6.1 Exercise 1

Prove that Equation (2.7) is correct. Then make a small `MATLAB` m-file to illustrate the formula. You can do this by generating two $I(t)$ and $Q(t)$ signals. Multiply these signals by the appropriate carriers to form the high frequency signal. Then compare the high frequency calculated power with the result obtained from Equation (2.7).

6.2 Exercise 2

Say you have a signal which in `MATLAB` code is:

```
t = [0:599];
it = 0.43*sin(t/size(t,2)*2*pi*4.1) + ...
   0.73*sin(t/size(t,2)*2*pi*sqrt(2));
qt = 0.82*cos(t/size(t,2)*2*pi*3.3) + ...
   0.56*cos(t/size(t,2)*2*pi*sqrt(2+0.3));
s_ant_ideal = it.*cos(2*pi*30*t/size(t,2)) + ...
   qt.*sin(2*pi*30*t/size(t,2));
```

where `it` is the $I(t)$ signal, `qt` is the $Q(t)$ signal, and `s_ant_ideal` is the $s_{ant,ideal}(t)$ signal. Then answer the following questions:

1. What is the average power of $I(t)$ and $Q(t)$ evaluated in the full time interval?
2. What is the average power of the ideal antenna signal $s_{ant,ideal}(t)$ evaluated in the full time interval?
3. Plot the power envelope of the signal.
4. What is the energy of the $I(t)$, $Q(t)$, and $s_{ant,ideal}(t)$ signals evaluated in the full time interval?

6.3 Exercise 3

Say you have a signal which in `MATLAB` code is:

```
t = [0:599];
it = 0.5*sin(t/size(t,2)*2*pi*4) + ...
   0.8*sin(t/size(t,2)*2*pi*sqrt(2));
qt = 0.8*cos(t/size(t,2)*2*pi*3.2) + ...
   0.6*cos(t/size(t,2)*2*pi*sqrt(2+0.5));
s_ant_ideal = it.*cos(2*pi*30*t/size(t,2)) + ...
   qt.*sin(2*pi*30*t/size(t,2));
```

where `it` is the $I(t)$ signal, `qt` is the $Q(t)$ signal, and `s_ant_ideal` is the $s_{ant,ideal}(t)$ signal. The task is then to add noise signals $n(t)$ to the $s_{ant,ideal}(t)$ such that *SNR* equal to 7.5dB, 12.5dB, and 17.5dB. Your noise signal must be designed from a zero mean Gaussian pseudo random noise generator. Then answer the following questions:

1. What is the average power of $I(t)$, $Q(t)$, and $s_{ant,ideal}(t)$?
2. What average power (or variance) of the noise is needed to obtain the three required *SNR* values?
3. Plot $s_{ant,ideal}(t)$, $n(t)$, and $s_{ant,ideal}(t) + n(t)$ for all three required *SNR* values.

6.4 Exercise 4

The idea of this exercise is to use MATLAB to simulate a direct conversion receiver and to gain insight into some important receiver principles. A few general points: (i) The sampling rate is chosen to $f_s = 2^{21} = 2.097152$MHz, (ii) the data length is $L = 2^{20} = 1,048,576$ samples

- First of all, two information carrying $I(t)$ and $Q(t)$ signals must be generated.
  - Use a pseudo random number generators with variance $\sigma^2 = 1$ as the starting point. *Hint*: Use the command `randn('seed',0);` (seed initialization of generator) to obtain results which can directly be compared to the instructors.
  - These signals have very wide bandwidths which are not needed. Filter each $I(t)$ and $Q(t)$ signal by a 10*th* order low pass Butterworth filter with a cut off frequency at $B = 25$kHz.
  - Plot the last 500 data points in the $I(t)$ and $Q(t)$ signals and compare.
  - Plot the power spectral densities (PSD's) of the $I(t)$ and $Q(t)$ signals on a logarithmic scale and in the $0 - 50$kHz range. *Hint*: Use the MATLAB functions:

```
[PI,FI] = PSD(I,1024,fs,[],256);
plot(FI/1E3,10*log10(PI));
```

where `PI` gives the PSD and `FI` is the corresponding frequency vector for PSD analysis of the time domain signal `I`.

- Determine the average powers (or variance) of the $I(t)$ and $Q(t)$ signals. *Hint*: With an input variance σ^2, a sample frequency f_s and a low pass bandwidth B the output variance should be reasonably close to $2\sigma^2 B/f_s$.

- Now the two information carrying input signals are available. This leaves a few tasks:
  - Calculate the ideal antenna signal $s_{ant,ideal}(t)$ when the carrier frequency is 125kHz. Plot the power spectral density, and calculate the average power of $s_{ant,ideal}(t)$. Do these results agree with your theoretical expectations?
  - Plot the last 250 samples of the $s_{ant,ideal}(t)$ as well as the amplitude envelope.
  - Add a bit of noise $n(t)$ using a Gaussian pseudo random number generator with a standard deviation equal to 0.03. This resulting signal $s_{ant}(t) = s_{ant,ideal}(t) + n(t)$. Plot the power spectral density of this signal and compare with the noise free $s_{ant,ideal}(t)$ signal. Comment on the result.

- Then apply the antenna filter — as the frequencies used in this simulation are quite low, a low pass filter is used in stead of a band pass filter. The cut off frequency is 850kHz — this frequency must be high in order not to deteriorate the signal. Use a 1*st* order Butterworth filter to do the job. Plot the output power spectral density in the frequency range from 0 to 1MHz. Comment on the result. What is now the average power of the signal?

- To simplify matters assume that the amplifier gain is 1 and that the amplifier is noise free and perfectly linear.

- Then finally feed the signal to the quadrature down-converter. The mixing signal in the *I*-path is $\cos(2\pi f_0 t)$ and in the *Q*-path it is $\sin(2\pi f_0 t)$. Then filter each signal using a 3*rd* order low pass Butterworth filter with a cut off frequency of 50kHz. The amplifier following the low pass filter amplifies with a factor of 1 and is noise free and perfectly linear.

- Plot the down converted *I*- and *Q*-signals along the ideal input *I*- and *Q*-signals. Comment on the result.

- The true output *I*- and *Q*-signals are delayed due to filter lag and the signal level is not directly comparable to the input signal. Use a lag of 14 samples and correct (calculate it!) the amplitude to ease comparison. Plot the input and output signals for *I* and *Q* and compare the results. *Hint*: Make sure the average power (or variance) of the two signals are scaled to the same value.

6.5 Exercise 5

Prove that the output signal from the Q-path in Figure 4.18 is given by Equation (4.19).

Bibliography

[1] S. Haykin, *Communication Systems*, John Wiley and Sons, 1983.

[2] J. Engberg and T. Larsen, *Noise Theory of Linear and Nonlinear Circuits*, John Wiley and Sons, 1995.

[3] W. B. Davenport and W. L. Root, *An Introduction to the Theory of Random Signals and Noise*, IEEE Press, New York, USA, 1987.

[4] R. Pettai, *Noise in Receiving Systems*, John Wiley and Sons, 1984.

[5] Bradford W. Parkinson and James J. Spilker Jr. (eds.), *Global Positioning System, Theory and Applications - volume I*, Published by the American Institute of Aeronautics and Astronautics, 1996.

[6] U. L. Rohde and T. T. N. Bucher, *Communications Receivers — Principles and design*, McGraw-Hill Book Company, 1988.