Rate Regions for Coordination of Decode-and-Forward Relays and Direct Users

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Abstract—Recently, the ideas of wireless network coding (NC) has significantly enriched the area of wireless cooperation/relaying. They bring substantial gains in spectral efficiency mainly in scenarios with two–way relaying. Inspired by the ideas of wireless NC, recently we have proposed techniques for coordinated direct/relay (CDR) transmissions. Leveraging on the fact that the interference can be subsequently canceled, these techniques embrace the interference among the communication flows to/from direct and relayed users. Hence, by allowing simultaneous transmissions, spectral efficiency is increased. In our prior work, we have proposed CDR with Decode–and–Forward (DF) relay in two scenarios. In this paper, we extend the two existing regenerative CDR schemes and proposed for the other two scenarios such that all schemes benefit from the aforementioned principle of containing the interference. The parameters in the schemes are optimized to have the largest rate region or the highest sum-rate. Numerical results show that DF CDR is better than the reference scheme and almost better than AF CDR.

Index Terms—Cooperative communications, relaying, analog network coding, interference cancellation, a priori information.

I. INTRODUCTION

Recently there have been extensive studies on cooperative, relay–based transmission schemes for extending cellular coverage or increasing diversity. Several basic relaying transmission techniques have been introduced, such as amplify-and-forward (AF) [1], decode-and-forward (DF) [2] and compress-and-forward (CF) [3]. These transmission techniques have been applied in one-, two- or multi-way relaying scenarios.

In particular, two–way relaying scenarios [4], [5], [6] have attracted a lot of attention, since it has been demonstrated that in these scenarios one can apply techniques based on NC in order to obtain a significant throughput gain. There are two basic principles used in designing throughput–efficient schemes with wireless NC (1) aggregation of communication flows - NC operates by having the flows sent/processed jointly; (2) intentional cancelable interference: flows are allowed to interfere over the wireless channel, knowing a priori that the interference can be cancelled by the destination.

Using these insights, in [7] we have proposed schemes, depicted on Figs. 1 and 2 for traffic scenarios that are more general than the usual two–way relaying. These schemes are termed coordinated direct/relay (CDR) transmissions. In the scheme on Fig. 1, termed $S_1$, U receives downlink traffic from the BS, while V sends uplink traffic to the BS. For the scheme $S_1$ (Fig. 2, these traffic patterns are inverted), in the first step the BS transmits to the relay RS. In the second step, RS transmits to U and simultaneously V transmits to the BS. The reception of V’s signal at BS is interfered by the transmission of RS; however, since BS knows the signal of RS a priori, it can cancel it and get a “clean” message from V. Enabling such simultaneous transmissions improves the spectral efficiency. In scheme $S_2$, in the first step BS sends to V and simultaneously U sends to RS, such that RS receives interference of these two signals, such as in analog NC for two–way relaying. But, unlike two–way relaying, the signal sent by RS in the second step need only be decoded at BS, but not at U. This makes the link RS-U irrelevant and, as we will see later, deflecting the traffic to go BS-V instead of BS-RS-U, and combining it with the traffic U-RS-BS, can give advantages in the sum–rate.

We have considered RS that uses Amplify-and-Forward (AF) in [8] and proposed two schemes of Decode-and-Forward (DF) for two CDR scenarios in [9]. In this paper we extend the two existing schemes and proposed DF CDR schemes for the other two CDR scenarios such that in all schemes, a station uses the information about the interference to cancel it and decode the desired signal. The choice of the duration of different phases in the schemes $S_1$, $S_2$, $S_3$ and $S_4$ is subject to optimization. The optimization objective is the rate region and the sum–rate for each of the respective schemes.

The rest of the paper is organized as follows. Section II introduces the system model. The DF reference and CDR schemes are described in section III. Section IV shows and discusses some numerical results. Section V concludes the paper.

II. SYSTEM MODEL

We consider a scenario with one base station (BS), one relay (RS), and two users (U and V), see Fig. 1. All transmissions have a unit power and normalized bandwidth of 1 Hz. Each of the complex channels $h_i, i \in \{1, 2, 3, 4\}$, is reciprocal, known at the receiver. All the channels are known at BS.

In the scenario, BS sends messages $s_1$ to U and $s_4$ to V and receives messages $s_3$ from U and $s_2$ from V. Note that the example on Fig. 1 does not show traffic patterns that involve $x_3$ and $x_4$, but they are used on Fig. 2. We assume that the data to/from each user is infinitely backlogged so that there is always data to transmit [10]. In each scheme, depending on the channel status, message $s_{1i}, i \in \{1, \ldots, 4\}$ can be divided into sub-messages $s_{1i,1}$ and $s_{1i,2}$. If message $s_{1i,1}$ or $s_{1i,2}$ is sent from BS, U or V, it is encoded to symbol string $x_{i,1}$ or $x_{i,2}$ respectively. If it is sent from RS we have the symbol
string $x_i^R$, $x_{i,1}^R$ or $x_{i,2}^R$ respectively. Denoting $|s|$ and $|x|$ as the number of bits and number of symbols of message $s$ and symbol string $x$ respectively, we have $|s_i| = |s_{i,1}| + |s_{i,2}|$ and $|x_i| = |x_{i,1}| + |x_{i,2}|$. Because there are many cases of channels are considered, we combine some similar cases and describe the schemes in the combined one. Therefore, if $s_{i,1}$ and $x_{i,2}$ are not mentioned in a scheme, it means that $|s_{i,2}| = 0$, $|x_{i,2}| = 0$ and $s_{i,1} = s_i$, $x_{i,1} = x_i$.

The direct channel BS–U is assumed weak and U gets the information from BS only through the decoded/forwarded signal from RS. If in slot $k$, the reception of $x$ at node $m$ is additionally interfered by $w$, then the received signal is $y_m[k] = h_{i,x} + h_{j,w} + z_m[k]$, $k \in \{1, 2, 3\}$, $m \in \{B, R, U, V\}$ where $z_m[k] \sim CN(0, \sigma^2)$ is Additive White Gaussian Noise (AWGN). Denoting the capacity function as $C(\gamma) = \log_2(1 + \gamma)$, we can write the capacity of such a transmission as $C_{i,j} = C \left( \frac{|h_i|^2}{|h_i|^2 + \sigma^2} \right) = C \left( \frac{g_i}{g_i + 1} \right)$ with $g_i = \frac{|h_i|^2}{\sigma^2}$. In case there is no interfering signal the capacity is $C_i = \log_2(1 + g_i)$. The receiver jointly decodes $x$ and $w$, the maximal sum rate for these two signals is $C_{ij} = \log_2(1 + g_i + g_j)$. It is straightforward to see that $C_{ij} = C_{ji}, C_{ij} = C_j + C_{i,1} - 3$.

In each scheme, the total time length is $2N$ symbols. $R_i^U$ and $R_i^V$, $i \in \{E, S_1, S_2, S_3, S_4\}$ are maximal rates for U and V respectively in scheme $i$. $E$ denotes the reference scheme, all schemes will be described in the next part. The sum-rate is therefore estimated as $R_i^U = R_i^U + R_i^V = \frac{1}{N} (D_i^U + D_i^V)$, where $D_i^U$, $D_i^V$ represent the corresponding number of bits. The transmission for the direct user has a duration of $\lambda N$ symbols. In the following part, we analyze the choice of $\lambda$ with respect to the optimization of the sum-rate.

III. REFERENCE AND CDR SCHEMES

CDR scheme 1 denoted as $S_1$ delivers two messages $s_1$ and $s_2$. CDR schemes $S_2, S_3$ and $S_4$ deliver messages pairs $(s_3, s_4)$, $(s_1, s_4)$ and $(s_2, s_3)$ respectively. CDR schemes combine the transmissions of the two messages in such a way that the information about the interference is exploited as much as possible while reference schemes use orthogonal transmissions by multiplexing them in time. However, since the transmit power of all nodes are the same and all channels are reciprocal, 4 reference schemes which are corresponding to 4 CDR schemes have the same rates.

A. Reference Scheme

First, BS encodes $s_1$ to $x_1$ with rate $R_1$ and transmits it to RS $y_B[1] = h_1 x_1 + z_B[1]$. Second, RS decodes $x_1$ to $s_1$, re-encodes it to $x_1^R$ with rate $R_i^R$ and transmits it to U (see Fig. 1) $y_U[2] = h_2 x_1^R + z_U[2]$. Third, V encodes $s_2$ to $x_2$ with rate $R_2$ and transmits it to BS $y_B[3] = h_3 x_2 + z_B[3]$. Since the V–BS transmission’s length is pre-defined as $\lambda N$ symbols and all transmissions are performed separately, the total time length for U is therefore $(2 - \lambda) N$. We denote the number of symbols in the RS–U transmission as $\mu N$. The rates $R_1, R_i^R$ and $R_2$ are selected as the maximal rates over the corresponding channels $R_1 = C_1, R_i^R = C_2$ and $R_2 = C_3$. The maximal data sent through the BS–RS, RS–U and V–BS transmissions are respectively $D_i^U = (2 - \lambda - \mu) N C_1, D_i^U = \mu N C_2, D_i^V = \lambda N C_3$. The total data transmitted for two users is $D_i^U = \min(D_i^U, D_i^U) + D_i^V$. Since $D_i^V$ does not depend on $\mu$, $D_i^U$ is a decreasing function and $D_i^V$ is an increasing function of $\mu$, in order to get maximal $D_i^U$, $\mu$ is selected such that $D_i^U = D_i^V$. Solving this equation we have the optimal $\mu = D_i^V = 2(\lambda - \lambda) N C_1 + C_3$ and $D_i^V = \lambda N C_3$. The sum-rate is $R_i^U = (2 - \lambda) N C_1 + C_3 + \lambda N C_3$.

B. CDR Scheme 1

First, BS transmits $x_1$ to RS (see Fig. 1) $y_B[1] = h_1 x_1 + z_B[1]$. Second, RS decodes $x_1$ to $s_1$, divides it into two sub-messages, re-encodes them to $x_{1,1}^R, x_{1,2}^R$ and transmits $x_{1,1}^R$ to U. In the meantime and similarly, V transmits $x_{2,1}$ to BS $y_B[2] = h_2 x_{1,1}^R + h_1 x_{2,1} + z_B[2]$. $y_U[2] = h_1 x_{1,1}^R + h_1 x_{2,1} + z_U[2]$. Third, if $\mu \geq \lambda$, RS transmits $x_{1,2}^R$ to U interference-free $y_U[3] = h_1 x_{1,2} + z_U[3]$. If $\mu < \lambda$, V transmits $x_{2,2}$ to BS interference-free $y_B[3] = h_3 x_{2,2} + z_B[3]$.

The total length of the transmissions for the direct user, which is the V–BS transmissions here, is pre-defined as $\lambda N$ symbols. Denote the number of symbols in the RS–U transmissions as $\mu N$. Since BS and RS cannot transmit and receive at the same time, the BS–RS transmission cannot be performed simultaneously with any other transmission. Because the RS–U and V–BS transmissions do not completely coincide, the length of the BS–RS transmission is thus determined as $(2 - \max(\mu, \lambda)) N$ symbols. Therefore, the messages $s_1$ and $s_2$ are divided and encoded at RS and V respectively such that $|x_{1,1}| = |x_{1,2}| = \min(\mu, \lambda) N$. If $\mu \geq \lambda, |x_{1,1}| = |x_{1,2}| = |x_1^R|$ and $|x_{1,1}| = \lambda - \mu$. If $\mu < \lambda, |x_{1,1}| = |x_{1,2}| = |x_1^R|$ and $|x_{1,2}| = \lambda - \mu$. In the following, we estimate the optimal value of $\mu$ for a pre-defined value of $\lambda$. Since BS knows $x_1$ and therefore $x_{1,1}^R$ and $x_{1,2}$ thus BS cancels the contribution of $x_1$ in the received signal. The total data sent through the BS–RS, RS–U and V–BS transmissions are respectively
Fig. 3. Rate regions of $(R_U, R_V)$ of $S_2$ in slot 1.

$$D_{U_1}^{S_1} = (2 - \max(\mu, \lambda))N \frac{C_1}{2N}, \quad D_{U_2}^{S_1} = \min(\mu, \lambda)N \frac{C_2-4}{2N}, \quad \lambda N C_3.$$ The sum-rate of two users is $R_S = \frac{D_{U_1}^{S_1} + D_{U_2}^{S_1}}{2N}$. We consider two cases:

- **$C_1 < C_5$:** $R_U$ and $R_V$ are selected such that $V$ can decode $x_4$ treating $x_3$ as noise $R_V \leq C_4-4$ and $R_U$ can decode $x_3$. There are two cases to satisfy the second condition:
  - RS decodes $x_3$ treating $x_4$ as noise: $R_U \leq C_2-1$.
  - RS decodes both $x_3$ and $x_4$ according to Multiple Access Channel (MAC) [11]: $R_U \leq C_2, \quad R_V \leq C_1, \quad R_U + R_V \leq C_{1,2}$.

C. CDR Scheme 2

First, $U$ transmits $x_{3,1}$ with rate $R_U$ to RS and BS transmits $x_{4,1}$ with rate $R_U$ in $\min(\mu, \lambda)N$ symbols simultaneously $y_U[1] = h_2 x_{3,1} + h_1 x_{4,1} + z_R[1], \quad y_V[1] = h_4 x_{4,1} + h_4 x_{4,1} + z_V[1]$. Second, $U$ transmits $x_{3,2}$ to RS $y_U[2] = h_2 x_{3,2} + z_R[2]$ or BS transmits $x_{4,2}$ to V $y_V[2] = h_3 x_{4,2} + z_V[2]$ in $|\mu - \lambda|N$ symbols interference-free with maximal rates of the corresponding channels $C_2$ and $C_3$ respectively (see Fig. 2). Third, RS decodes $x_{3,1}$ and $x_{3,2}$, re-encodes and forwards them to BS $y_B[3] = h_1 x_{3,1} + z_B[3]$. Since BS and RS cannot transmit and receive at the same time, the RS–BS transmission cannot be performed simultaneously with any other transmission, it starts only after the first $\max(\mu, \lambda)N$ symbols are finished. Thus $|x_3^R| = |x_{3,1}^R| + |x_{3,2}^R| = (2 - \max(\mu, \lambda))N$. We consider two cases:

- **$C_1 < C_5$:** $R_U$ and $R_V$ are selected such that $V$ can decode $x_4$ treating $x_3$ as noise $R_V \leq C_4-4$ and RS can decode $x_3$. There are two cases to satisfy the second condition:
  - RS decodes $x_3$ treating $x_4$ as noise: $R_U \leq C_2-1$.
  - RS decodes both $x_3$ and $x_4$ according to Multiple Access Channel (MAC) [11]: $R_U \leq C_2, \quad R_V \leq C_1, \quad R_U + R_V \leq C_{1,2}$.

D. CDR Scheme 3

The transmissions are conducted in the following steps (Fig. 4): First, BS transmits $x_1$ to RS in $(2 - \max(\mu, \lambda))N$ symbols $y_U[1] = h_1 x_1 + z_R[1]$. Second, RS and BS transmits $x_{1,1}^R$ with rate $R_U$ and $x_{1,1}$ with rate $R_V$ respectively and simultaneously in $\min(\mu, \lambda)N$ symbols $y_U[2] = h_2 x_{1,1}^R + z_R[2], \quad y_V[2] = h_5 x_{1,1} + h_3 x_{4,1} + z_V[2]$. Third, RS transmits $x_{1,2}^R$ in $(\mu - \lambda)_N$ symbols $y_U[3] = h_2 x_{1,2}^R + z_U[3]$ if $\mu \geq \lambda$ and BS transmits $x_{4,2}$ in $(\lambda - \mu)_N$ symbols $y_V[3] = h_5 x_{4,2}^R + h_3 x_{4,2} + z_V[3]$ if $\mu < \lambda$. Note that when $\mu \geq \lambda$, $|x_{4,2}| = 0$ and $x_{4,1} = x_4$.
and when $\mu < \lambda$, $|x_{1,2}| = 0$ and $x_{1,1}^R = x_i^R$. We consider two cases:

- $C_1 < C_3$: $R_U$ and $R_V$ are selected such that $U$ can decode $x_{1,1}$ ($R_U \leq C_2$, since the BS-U channel is zero) and $V$ can decode $x_{i,4}$. There are two cases to satisfy the second condition:
  - $V$ decodes $x_{i,1}$ treating $x_{1,1}^R$ as noise: $R_V \leq C_{3,5}$.
  - $V$ decodes both $x_{1,1}^R$ and $x_{i,1}$: $R_U \leq C_5$, $R_V \leq C_3$, $R_U + R_V \leq C_{5,5}$.

The data transmitted in BS–RS, RS–U, BS–V transmissions and the total data transmitted for two users are respectively $D_{S_1}^U = (2 - \max(\mu, \lambda))C_1N$, $D_{S_2}^V = (\mu - \min(\mu, \lambda))C_2N + \min(\mu, \lambda)R_U N$, $D_{S_3}^O = (\lambda - \min(\mu, \lambda))C_3N + \min(\mu, \lambda)R_V N$ and $D_{S_4}^V = \min(D_{S_1}^U, D_{S_2}^V) + D_{S_3}^O$. Similarly to Scheme 2, Fig. 5 demonstrates the rate region of $(R_U, R_V)$. It has different shapes corresponding to different values of $C_2$.

- $C_1 \geq C_3$: Here RS and V can decode $x_{i,1}$. Using the information about $s_1$, the interference in slot 2 at V can be completely canceled. Therefore in slot 2, BS can transmit $x_{i,1}$ to V with the maximal rate $R_V = C_3$ while RS can transmit $x_{i,1}$ to U with the maximal rate $R_U = C_2$. The data transmitted in RS–U, BS–V transmissions are different from the previous case $D_{U_2}^S = \mu C_2N$, $D_{V_2}^S = \lambda C_3N$.

The sum-rate of $S_3$ is $R_{S_3}^U = \frac{D_{S_3}^O}{2N}$. Again combing two symbol string with different codebooks can be used here at V to decode its desired signal.

### E. CDR Scheme 4

The transmissions are conducted in the following steps (Fig. 6): First, U and V transmits $x_{i,1}$ with rate $R_U$ and $x_{i,2}$ with rate $R_V$ respectively and simultaneously in $\min(\mu, \lambda)N$ symbols $y_{i,1} = h_2x_{i,1} + h_5z_{i,1}$, $y_{i,2} = h_3x_{i,1} + z_{i,1}$. Second, U transmits $x_{i,2}$ in $\mu N$ symbols $y_{i,2} = h_2x_{i,2} + z_{i,2}$ if $\mu \geq \lambda$ and V transmits $x_{i,2}$ in $\lambda N$ symbols $y_{i,2} = h_3x_{i,2} + z_{i,2}$ if $\mu < \lambda$. Third, RS transmits $x_{i,0}^R$ to BS in $(2 - \max(\mu, \lambda))N$ symbols $y_{i,0}^R = h_1x_{i,0}^R + z_{i,0}^R$. Note that when $\mu \geq \lambda$, $x_{i,2,1} = 0$ and $x_{i,1} = x_{i,2}$ and when $\mu < \lambda$, $x_{i,2,1} = 0$ and $x_{i,1} = x_{i,3}$. $x_{1,1}^R$ is decoded.

The rate regions of $(R_U, R_V)$ of $S_4$ in slot 1.

$R_U$ and $R_V$ are selected such that BS can decode $x_{2,1}$ ($R_V \leq C_3$, since the U-BS channel is zero) and RS can decode $x_{3,1}$. There are two cases to satisfy the second condition:

- RS decodes $x_{3,1}$ treating $x_{2,1}$ as noise: $R_U \leq C_{2,5}$.
- RS decodes both $x_{3,1}$ and $x_{2,1}$: $R_U \leq C_2$, $R_V \leq C_5$, $R_U + R_V \leq C_{2,5}$.

The rate region of $(R_U, R_V)$ is demonstrated in Fig. 7. We have $D_{U_1}^S = \min(\mu, \lambda)R_U N + (\mu - \min(\mu, \lambda))C_2N$, $D_{U_2}^V = (2 - \max(\mu, \lambda))C_1N$, $D_{V_2}^S = \min(\mu, \lambda)R_V N + (\lambda - \min(\mu, \lambda))C_3N$ and $D_{S_4}^V = \min(D_{U_1}^S, D_{U_2}^V) + D_{V_2}^S$. The sum-rate of $S_4$ is $R_{S_4}^V = \frac{D_{S_4}^V}{2N}$.

### IV. Numerical Results

Fig. 8 shows the rate regions $(R_i, R_j)$, $i \in \{S_1, S_2, S_3, S_4\}$ of different schemes, where $R_{ij}, j \in \{U, V\}$, is the rate delivered to user $j$ in scheme $i$. The simulation is conducted in case of channel $\gamma = [\gamma_1 \gamma_2 \gamma_3 \gamma_4 \gamma_5] = [15 10 13 - 10 0]dB$. The simulation result of rate regions is achieved by calculating the rate pair $(R_{ij}, R_{ij})$ for all values of $\lambda, \mu$ and $R_U, R_V$ which are selected such that satisfying the conditions in each scheme with resolution $\Delta \lambda = \Delta \mu = 0.1$ and $\Delta R_U = \Delta R_V = 0.2$. The reference scheme has the most contained rate region since it does not exploit the information about the interference as all of the CDR schemes do. CDR scheme 1 has best rate region (high $R_U$ and not low $R_V$ because the only limiting factor in this scheme is the interference from V to U over the inter-user channel,
In this paper, we propose and analyze the Coordinated transmissions to Direct and Relayed user in a wireless cellular network with relays using Decode-and-Forward. The durations of the transmissions for the direct and relayed users as well as the rates of simultaneous transmissions are optimized to have the best rate region and the maximal sum-rate. We compare the quality of the proposed schemes with their version of Amplify-and-Forward as well as the conventional scheme. Numerical results show that the proposed schemes almost provide better rates and sum-rate than the AF and reference schemes.

V. CONCLUSION

In this paper, we propose and analyze the Coordinated transmissions to Direct and Relayed user in a wireless cellular network with relays using Decode-and-Forward. The durations of the transmissions for the direct and relayed users as well as the rates of simultaneous transmissions are optimized to have the best rate region and the maximal sum-rate. We compare the quality of the proposed schemes with their version of Amplify-and-Forward as well as the conventional scheme. Numerical results show that the proposed schemes almost provide better rates and sum-rate than the AF and reference schemes.

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