Structured LPV Control of Wind Turbines

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Agenda

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Main challenges for the application of wind turbine control:

- Known parameter-dependencies (gain-scheduling);
- Unknown parameter variations (robustness);
- Faults (reconfiguration);
- Simple implementation.

Fig. 1: Block diagram of the controller structures. The black boxes are common to the LPV controllers, while the red dashed box illustrates the fault diagnosis system required by the AFTC.

- Prevent catastrophic failures and faults from deteriorating other parts of the wind turbine, by early fault detection and accommodation.
- Reduce maintenance costs by providing remote diagnostic details and avoiding replacement of functional parts, by applying condition-based maintenance instead of time-based maintenance.
- Increase energy production when a fault has occurred by means of fault-tolerant control.

This chapter gives an overview of the most common faults that can be modelled as varying parameters. For a clear exposure, the fault-tolerant controller is designed to cope with the simple case of a single fault: altered dynamics of the hydraulic pitch system due to low hydraulic pressure. The fault is a gradual fault affecting the control actions of the turbine. The method used also applies to fast parameter variations, i.e. abrupt faults in the extreme case [12].

Realizing advanced gain-scheduled controllers can in practice be difficult and may lead to numerical challenges [20, 19]. Several plant and controller matrices must be stored on the controller memory. Moreover, matrix factorizations and inversions are among the operations that must be done online by the controller at each sampling time [4, 6].

The synthesis procedures presented in this chapter are serious candidates for solving a majority of practical wind turbine control problems, provided a sufficiently good model of the wind turbine is available. We believe that the resulting controller can also be easily implemented in practice due to the following reasons:

1. Structured controller: the controller structure can be chosen arbitrarily. Decentralized of any order, dynamic (full or reduced-order) output feedback, static output, and full state feedback are among the possible structures. This is in line with the current control philosophy within wind industry.
2. Low data storage: the required data to be stored in the control computer memory is only the controller matrices, and scalar functions of the scheduling variables representing plant nonlinearities (basis functions).
Linear parameter-varying (LPV) modeling and control for practical wind turbine control problems.

\[
\text{minimize } \| T_{z \rightarrow w}(\theta, \alpha, K) \|_{i,2}
\]

where

- $\mathcal{K}$ represents a structural constraint on the controller;
- $\theta$ is a vector of time-varying scheduling parameters;
- $\alpha$ is a vector of uncertain parameters.
Wind Turbine LPV Model
Nominal Model

- BEM aerodynamics (static);
- Flexible two-mass drive-train;
- Fore-aft tower translation (first bending mode);
- Second-order pitch system;
- First order torque delay.

Wind Turbine LPV Model

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Linearized torque and thrust equations,

\[ Q_a(t) \approx \bar{Q}_{\theta_{op}} + \frac{\partial Q_a}{\partial V} \bigg|_{\theta_{op}} \hat{V}(t) + \frac{\partial Q_a}{\partial \Omega_r} \bigg|_{\theta_{op}} \hat{\Omega}_r(t) + \frac{\partial Q_a}{\partial \beta} \bigg|_{\theta_{op}} \hat{\beta}(t) \]

\[ T_a(t) \approx \bar{T}_{\theta_{op}} + \frac{\partial T_a}{\partial V} \bigg|_{\theta_{op}} \hat{V}(t) + \frac{\partial T_a}{\partial \Omega_r} \bigg|_{\theta_{op}} \hat{\Omega}_r(t) + \frac{\partial T_a}{\partial \beta} \bigg|_{\theta_{op}} \hat{\beta}(t) \]
Wind Turbine LPV Model
Aerodynamic Uncertainty

Simplification of the aerodynamic phenomena:
- Blade Element Momentum (BEM) codes;
- Neglected dynamics (e.g. dynamic inflow);
- Deviations from the operating points.

\[
\frac{\partial Q}{\partial \beta}(\theta, \alpha) := \left. \frac{\partial Q}{\partial \beta} \right|_{\theta_{op}} + f_l(\alpha), \quad f_l(\alpha) := a_l + b_l \alpha
\]

where \(a_l, b_l\) characterizes the additive uncertainty for the \(l\)-th aerodynamic gain and \(\alpha\) is an uncertainty parameter.

\[\Lambda = \{\alpha : \alpha \leq \alpha \leq \bar{\alpha}\}.\]
Failures that gradually change system’s dynamics:

- Bias and proportional error in sensors: pitch angle, generator speed;
- Offset of the generated torque due to an offset in the internal power converter control loops;
- Reduction in conversion efficiency;
- Altered dynamics of pitch system (Pressure drop, pump wear, high air content in the oil);

Example: Pitch system

Damping ratio and natural frequency from their nominal values $\zeta_0$ and $\omega_{n,0}$ to their faulty values $\zeta_f$ and $\omega_{n,f}$. Convex combination of the vertices of the parameter sets,

$$
\ddot{\beta}(t) = -2\zeta(\theta_f)\omega_n(\theta_f)\dot{\beta}(t) - \omega^2_n(\theta_f)\beta(t) + \omega^2_n(\theta_f)\beta_{ref}(t)
$$

$$
\omega^2_n(\theta_f) = (1 - \theta_f)\omega^2_{n,0} + \theta_f\omega^2_{n,lp}
$$

$$
-2\zeta(\theta_f)\omega_n(\theta_f) = -2(1 - \theta_f)\zeta_0\omega_{n,0} - 2\theta_f\zeta_{lp}\omega_{n,lp}
$$

where $\theta_f \in [0, 1]$ is an scheduled indicator for the fault.
Discrete-time LPV system obtained by discretization (Bilinear) of continuous-time counterpart,

\[
\begin{align*}
x(k+1) &= A(\theta, \alpha)x(k) + B_w(\theta, \alpha)w(k) + B_u(\theta, \alpha)u(k) \\
z(k) &= C_z(\theta, \alpha)x(k) + D_{zw}(\theta, \alpha)w(k) + D_{zu}(\theta, \alpha)u(k) \\
y(k) &= C_y(\theta, \alpha)x(k) + D_{yw}(\theta, \alpha)w(k).
\end{align*}
\]

Affine in scalar functions \(\rho_i(\theta)\) known as basis functions and \(\theta_{f,m}\).

\[
\begin{align*}
\begin{bmatrix}
A & B_w & B_u \\
C_z & D_{zw} & D_{zu} \\
C_y & D_{yw} & 0
\end{bmatrix}_0 + \sum_i \begin{bmatrix}
A & B_w & B_u \\
C_z & D_{zw} & D_{zu} \\
C_y & D_{yw} & 0
\end{bmatrix}_i (\rho_i(\theta) + f_i(\alpha)) \\
+ \sum_m \begin{bmatrix}
A & B_w & B_u \\
C_z & D_{zw} & D_{zu} \\
C_y & D_{yw} & 0
\end{bmatrix}_m \theta_{f,m}, \quad i = 1, \ldots, n_{\rho}, \quad m = 1, \ldots, n_{\theta_f}.
\end{align*}
\]
The aerodynamic gains are natural candidates for $\rho_i(\theta)$,

$$
\begin{align*}
\rho_1(\theta) & := \frac{1}{J_r} \frac{\partial Q_a}{\partial \Omega} \bigg|_{\theta_{op}} \\
\rho_2(\theta) & := \frac{1}{J_r} \frac{\partial Q_a}{\partial V} \bigg|_{\theta_{op}} \\
\rho_3(\theta) & := \frac{1}{J_r} \frac{\partial Q_a}{\partial \beta} \bigg|_{\theta_{op}} \\
\rho_4(\theta) & := \frac{1}{M_t} \frac{\partial T_a}{\partial \Omega} \bigg|_{\theta_{op}} \\
\rho_5(\theta) & := \frac{1}{M_t} \frac{\partial T_a}{\partial V} \bigg|_{\theta_{op}} \\
\rho_6(\theta) & := \frac{1}{M_t} \frac{\partial T_a}{\partial \beta} \bigg|_{\theta_{op}}
\end{align*}
$$

where the division by $J_r$ and $M_t$ is adopted to improve numerical conditioning.
System and Controller Description

LPV Controller

The LPV controller has the form,

\[ x_c(k + 1) = A_c(\theta)x_c(k) + B_c(\theta)y(k) \]
\[ u(k) = C_c(\theta)x_c(k) + D_c(\theta)y(k), \]

Controller matrices are continuous functions of \( \theta \) with similar type of dependence,

\[
A_c(\theta) = A_{c,0} + \sum_{i=1}^{n_\theta} \rho_i(\theta)A_{c,i} + \sum_{i=1}^{n_\theta_f} \theta_{f,i}A_{c,n_\rho+i},
\]

\[
B_c(\theta) = B_{c,0} + \sum_{i=1}^{n_\theta} \rho_i(\theta)B_{c,i} + \sum_{i=1}^{n_\theta_f} \theta_{f,i}B_{c,n_\rho+i},
\]

\[
C_c(\theta) = C_{c,0} + \sum_{i=1}^{n_\theta} \rho_i(\theta)C_{c,i} + \sum_{i=1}^{n_\theta_f} \theta_{f,i}C_{c,n_\rho+i},
\]

\[
D_c(\theta) = D_{c,0} + \sum_{i=1}^{n_\theta} \rho_i(\theta)D_{c,i} + \sum_{i=1}^{n_\theta_f} \theta_{f,i}D_{c,n_\rho+i}.
\]
The controller matrices can be represented in a compact way,

\[ K(\theta) := \begin{bmatrix} D_c(\theta) & C_c(\theta) \\ B_c(\theta) & A_c(\theta) \end{bmatrix}. \]

The interconnection of system and controller leads to the following closed-loop LPV system denoted \( S_{cl} \),

\[
S_{cl} : \quad x_{cl}(k + 1) = A(\theta, \alpha, K(\theta))x_{cl}(k) + B(\theta, \alpha, K(\theta))w(k) \\
z(k) = C(\theta, \alpha, K(\theta))x_{cl}(k) + D(\theta, \alpha, K(\theta))w(k).
\]

\( \theta \) ranges over a hyperrectangle denoted \( \Theta \),

\[ \Theta = \{ \theta : \underline{\theta}_i \leq \theta_i \leq \overline{\theta}_i, \ i = 1, \ldots, n_{\theta} \}. \]

Rate of variation \( \Delta \theta = \theta(k + 1) - \theta(k) \) belongs to a hypercube denoted \( \mathcal{V} \),

\[ \mathcal{V} = \{ \Delta \theta : |\Delta \theta_i| \leq v_i, \ i = 1, \ldots, n_{\theta} \}. \]
Proposition (L₂-gain)

If there exist $K(\theta)$, $\mathcal{P}(\theta, \alpha) = \mathcal{P}(\theta, \alpha)^T$ and $Q(\theta)$ satisfying,

$$\begin{bmatrix}
  r^2\mathcal{P}(\theta + \Delta \theta, \alpha) & A(\theta, \alpha, K(\theta))Q(\theta) & B(\theta, \alpha, K(\theta)) & 0 \\
  * & -\mathcal{P}(\theta, \alpha) + Q(\theta)^T + * & 0 & Q(\theta)^T C(\theta, \alpha, K(\theta))^T \\
  * & * & \gamma I & D(\theta, \alpha, K(\theta))^T \\
  * & * & * & \gamma I
\end{bmatrix} > 0$$

with $r = 1$, $\forall (\theta, \Delta \theta, \alpha) \in \Theta \times \mathcal{V} \times \Lambda$, then the system $S_{cl}$ is exponentially stable and $\| T_{zw}(\theta, \alpha) \|_2 < \gamma$. 

Iterative LMI Algorithm
Induced $\mathcal{L}_2$-gain
Iterative LMI Algorithm
Lyapunov and Slack Matrices

The Lyapunov and slack variables are here defined affine functions of the basis functions,

\[
P(\theta, \alpha) = P_0 + \sum_{i=1}^{n_\theta} (\rho_i(\theta) + f_i(\alpha)) P_i + \sum_{i=1}^{n_\theta_f} \theta_{f,i} P_{n_\theta + i}
\]

\[
Q(\theta) = Q_0 + \sum_{i=1}^{n_\theta} \rho_i(\theta) Q_i + \sum_{i=1}^{n_\theta_f} \theta_{f,i} Q_{n_\rho + i}
\]

The Lyapunov function at \(\theta^+ := \theta + \Delta \theta\) can be described as,

\[
P(\theta^+, \alpha) = P_0 + \sum_{i=1}^{n_\theta} (\rho_i(\theta^+) + f_i(\alpha)) P_i + \sum_{i=1}^{n_\theta_f} \left(\theta^+_{f,i}\right) P_{n_\theta+i}.
\]

Conveniently, the basis functions at \(\theta^+\) are approximated by a linear function of \(\rho(\theta)\) and \(\Delta \theta\),

\[
\rho_i(\theta^+) := \rho_i(\theta) + \frac{\partial \rho_i(\theta)}{\partial \theta} \Delta \theta,
\]
Iterative LMI Algorithm

Iteration Scheme

- Sequence of LMI problems: $Q(\theta)^{\{j\}} = P(\theta)^{\{j-1\}}$;
- Gridded parameter space subset denoted $\Theta_g \subset \Theta$. LMI checked at $\Theta_g \times \text{Vert}(\mathcal{V}) \times \text{Vert}(\Lambda)$ at each iteration;
- Minimization of performance level $\gamma$;
- Feasibility phase creates a convergent sequence $r_j$ that tries to tend 1.
Numerical Example
Fault-Tolerant PI LPV Control for High Wind Speeds
Considering $G_p$ augmented with the integrator filter as the plant for synthesis purposes, the LPV controller structure reduces to a parameter-dependent static output feedback of the form,

$$K(\theta) = D_{c,0} + \sum_{i=1}^{6} \rho_i(\theta) D_{c,i} + \theta_f D_{c,7},$$

$$D_{c,n} := \begin{bmatrix} D_{p,n} & D_{i,n} & D_{q,n} \end{bmatrix}, \quad n = 0, 1, \ldots, 7.$$  

Initial $K(\theta)$ based on analytical pole placement (tower fore aft mode neglected).

$$k_p(\theta) = \frac{2\xi\omega (J_r + N_g^2 J_g) - N_g \frac{\partial Q_g}{\partial \Omega_g} + \rho_1(\theta)}{-N_g \rho_3(\theta)},$$

$$k_i(\theta) = \frac{\omega^2 (1 + \xi^2) (J_r + N_g^2 J_g)}{-N_g \rho_3(\theta)}.$$  

The tower feedback gain of the initial controller is $k_q(\theta) = 0$, meaning no active tower damping.
Numerical Example
Fault-Tolerant PI LPV Control for High Wind Speeds

Figure: Evolution of performance level $\gamma$ and controller gains $k_p, k_i, k_\dot{q}$ during the iterative LMI synthesis. Controller gains computed at $\theta_{op} = 15 \text{ m/s}, \theta_f = 0$. 
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PI and Tower Feedback Gains

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(d) 

(e) 

(f) 

(g)
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(h)

(i)

(j)
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