Turbine aero-servo-elasticity

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Outline

- Modal dynamics of three-bladed wind turbines
  - Modes at standstill
  - What happens when the turbine is rotating?
    - Methods for analysis
    - Concept of periodic mode shapes
    - Backward and forward whirling
    - Splitting of frequencies
    - Multiple frequencies for single mode

- Closed-loop aero-servo-elastic eigenvalue and frequency-domain analysis
  - HAWCStab2: A linear aero-servo-elastic time-invariant model
  - Example: Analysis and tuning of collective and cyclic pitch controller
  - Outlook: Reduced order models for controllers
Why consider modal dynamics of wind turbines?

Accumulated load spectrum for a 1.4 MW turbine
Talk to your neighbor:
Name and order these modes after frequency

A
B
C
D
E
F
G
H
First 8 standstill mode shapes of 600 kW turbine
WHAT HAPPENS WHEN THE ROTOR IS ROTATING?
Robert P. Coleman’s original system

NACA Wartime Report L-308

Simplification

\[
\begin{align*}
\psi_1 &= \Omega t \\
\psi_2 &= \Omega t + \frac{2\pi}{3} \\
\psi_3 &= \Omega t + \frac{4\pi}{3}
\end{align*}
\]

Figure 1-1.—Simplified mechanical system representing rotor.
Linear equations of motion

\[
\begin{bmatrix}
ml^2 & 0 & 0 & -ml \sin \psi_1(t) & ml \cos \psi_1(t) \\
0 & ml^2 & 0 & -ml \sin \psi_2(t) & ml \cos \psi_2(t) \\
0 & 0 & ml^2 & -ml \sin \psi_3(t) & ml \cos \psi_3(t) \\
-ml \sin \psi_1(t) & -ml \sin \psi_2(t) & -ml \sin \psi_3(t) & M + 3m & 0 \\
ml \cos \psi_1(t) & ml \cos \psi_2(t) & ml \cos \psi_3(t) & 0 & M + 3m \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 \\
\ddot{x}_M \\
\ddot{y}_M \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-\cos \psi_1(t) & -\cos \psi_2(t) & -\cos \psi_3(t) & 0 & 0 \\
-\sin \psi_1(t) & -\sin \psi_2(t) & -\sin \psi_3(t) & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
ml^2 \omega_0^2 & 0 & 0 & 0 & 0 \\
0 & ml^2 \omega_0^2 & 0 & 0 & 0 \\
0 & 0 & ml^2 \omega_0^2 & 0 & 0 \\
ml \Omega^2 \sin \psi_1(t) & ml \Omega^2 \sin \psi_2(t) & ml \Omega^2 \sin \psi_3(t) & k_x & 0 \\
-ml \Omega^2 \cos \psi_1(t) & -ml \Omega^2 \cos \psi_2(t) & -ml \Omega^2 \cos \psi_3(t) & 0 & k_y \\
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
x_M \\
y_M \\
\end{bmatrix}
= 0
Example to be continued ...  

**FIRST SOME THEORY**
**Eigenvalue analysis of rotary systems**

Governing equation of free vibrations with *periodic* system matrices

\[ \mathbf{M}(t)\ddot{\mathbf{x}} + \mathbf{C}(t)\dot{\mathbf{x}} + \mathbf{K}(t)\mathbf{x} = 0 \]

On first order form

\[ \dot{\mathbf{y}} = \mathbf{A}(t)\mathbf{y} \]

where

\[ \mathbf{y} = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} \]

and

\[ \mathbf{A}(t) = \begin{bmatrix} 0 & \mathbf{I} \\
-\mathbf{M}(t)^{-1}\mathbf{K}(t) & -\mathbf{M}(t)^{-1}\mathbf{C}(t) \end{bmatrix} \]

Insertion of solution used in traditional eigenvalue analysis \( \mathbf{y} = \mathbf{Y}e^{\lambda t} \) yields

\[ (\lambda \mathbf{I} - \mathbf{A}(t)) \mathbf{Y} = 0 \]

which can never be fulfilled unless \( \mathbf{Y} = 0 \).

---

**Warning:** The so-called snapshot eigenvalue analysis computing periodic azimuth dependent eigenvalues \( \lambda = \lambda(t) \) is nonsense.
Seek a transformation to obtain time-invariance

Seek a periodic matrix for transformation to new coordinates

\[ \mathbf{y} = \mathbf{L}(t)\mathbf{z} \text{, where } \mathbf{L}(t + T) = \mathbf{L}(t) \]

Insertion into the first order equation \( \dot{\mathbf{y}} = \mathbf{A}(t)\mathbf{y} \) yields

\[
\dot{\mathbf{L}}(t)\mathbf{z} + \mathbf{L}(t)\dot{\mathbf{z}} = \mathbf{A}(t)\mathbf{L}(t)\mathbf{z}
\]

\[
\uparrow
\]

\[
\dot{\mathbf{z}} = \mathbf{L}^{-1}(t) \left( \mathbf{A}(t)\mathbf{L}(t) - \dot{\mathbf{L}}(t) \right) \mathbf{z}
\]

Time invariant matrix \( \mathbf{A}_L \)

Modal expansion of response using eigensolutions \( \mathbf{v}_{L,k} e^{\lambda_k t} \text{ of } \mathbf{A}_L \)

\[
\mathbf{y} = \mathbf{L}(t) \left( \sum_{k=1}^{N} \mathbf{v}_{L,k} e^{\lambda_k t} \right) = \sum_{k=1}^{N} \mathbf{L}(t) \mathbf{v}_{L,k} e^{\lambda_k t}
\]

Periodic mode shapes
Floquet analysis of rotary systems (1)

First order equations

\[ \dot{y} = A(t)y \]

where the \( N \times N \) system matrix is \( T \)-periodic

\[ A(t + T) = A(t) \quad \text{with} \quad T = \frac{2\pi}{\Omega} \]

Definition of fundamental matrix of \( N \) linearly independent solutions

\[ \Phi(t) = \begin{bmatrix} \phi_1(t) & \phi_2(t) & \ldots & \phi_N(t) \end{bmatrix}, \quad \dot{\Phi}(t) = A(t)\Phi(t) \quad \forall t \in [0; T] \]

obtained by numerical integration with \( N \) linearly independent initial conditions

\[ \Phi(0) = \begin{bmatrix} \phi_1(0) & \phi_2(0) & \ldots & \phi_N(0) \end{bmatrix} \quad \text{e.g.} \quad \Phi(0) = I \]

Shift of the time \( t \) by one period to \( t + T \) yields

\[ \dot{\Phi}(t + T) = A(t + T)\Phi(t + T) = A(t)\Phi(t + T) \]

which shows that \( \Phi(t + T) \) also is a fundamental solution matrix.
Floquet analysis of rotary systems (2)

Only $N$ linearly independent solutions can exist in a $N$-state linear system, hence the latter can be constructed as a linear combination of the former:

$$\Phi(t + T) = \Phi(t)C$$

whereby the so-called monodromy matrix is derived as

$$C = \Phi(0)^{-1}\Phi(T)$$

A constant non-singular matrix is then defined as

$$C = e^{TR}$$

whereby the fundamental solution matrix can be reconstructed as

$$\Phi(t) = Q(t)e^{iR}$$

where $Q(t)$ is a periodic matrix

$$Q(t + T) = \Phi(t + T)e^{-(t+T)R} = \Phi(t)e^{TR}e^{-(t+T)R} = \Phi(t)e^{-iR} = Q(t)$$

Stability analysis: Hence, the exponential decay or growth of vibrations are given by the real part of the eigenvalues of $R$. 
Lyapunov-Floquet (L-F) transformation

For any constant non-singular matrix $\mathbf{R}$, a transformation is defined as

$$ y = \mathbf{L}(t)z, \quad \mathbf{L}(t) = \Phi(t)e^{-t\mathbf{R}}\Phi^{-1}(0)\mathbf{L}(0) $$

From previous slide, the transformed system equation is derived as

$$ \dot{z} = \mathbf{L}^{-1}(t) \left( \mathbf{A}(t)\mathbf{L}(t) - \dot{\mathbf{L}}(t) \right) z $$

The time derivative of the transformation matrix is

$$ \dot{\mathbf{L}}(t) = \left( \dot{\Phi}(t)e^{-t\mathbf{R}} - \Phi(t)\mathbf{R}e^{-t\mathbf{R}} \right) \Phi^{-1}(0)\mathbf{L}(0) $$

$$ = \left( \mathbf{A}(t)\Phi(t)e^{-t\mathbf{R}} - \Phi(t)\mathbf{R}e^{-t\mathbf{R}} \right) \Phi^{-1}(0)\mathbf{L}(0) $$

$$ = \mathbf{A}(t)\mathbf{L}(t) - \Phi(t)\mathbf{R}e^{-t\mathbf{R}}\Phi^{-1}(0)\mathbf{L}(0) $$

Insertion of $\mathbf{L}(t)$ and $\dot{\mathbf{L}}(t)$ into the transformed system equation yields

$$ \dot{z} = \left( \Phi(t)e^{-t\mathbf{R}}\Phi^{-1}(0)\mathbf{L}(0) \right)^{-1} \Phi(t)\mathbf{R}e^{-t\mathbf{R}}\Phi^{-1}(0)\mathbf{L}(0)z $$

$$ = \mathbf{L}^{-1}(0)\Phi(0)e^{t\mathbf{R}}\mathbf{R}e^{-t\mathbf{R}}\Phi^{-1}(0)\mathbf{L}(0)z $$

$$ = \mathbf{L}^{-1}(0)\Phi(0)\mathbf{R}\Phi^{-1}(0)\mathbf{L}(0)z $$

constant matrix
L-F transformation – Choice of R

Choice of the constant non-singular matrix $\mathbf{R}$ to make $\mathbf{L}(t)$ $T$-periodic

$$\mathbf{L}(t + T) = \Phi(t + T)e^{-(t+T)\mathbf{R}}\Phi^{-1}(0)\mathbf{L}(0)$$

Choosing $\mathbf{R}$ such that $\mathbf{C} = e^{T\mathbf{R}}$ yields

$$\mathbf{L}(t + T) = \Phi(t)e^{-t\mathbf{R}}\Phi^{-1}(0)\mathbf{L}(0) = \mathbf{L}(t)$$

Matrix $\mathbf{R}$ is computed from the monodromy matrix $\mathbf{C} = \Phi(0)^{-1}\Phi(T)$ as

$$\mathbf{C} = e^{T\mathbf{R}} \iff \mathbf{R} = \frac{1}{T} \ln \mathbf{C}$$

If all eigenvectors of $\mathbf{C}$ are linearly independent then

$$\mathbf{C} = e^{T\mathbf{R}} = \mathbf{V}\mathbf{J}\mathbf{V}^{-1} \iff \mathbf{R} = \frac{1}{T} \mathbf{V}\ln(\mathbf{J})\mathbf{V}^{-1}$$

where $\mathbf{J} = \text{diag}\{\rho_1, \rho_2, \ldots, \rho_N\}$ are the corresponding eigenvalues of $\mathbf{C}$. 
L-F transformation – Frequency indeterminacy

L-F transformed time-invariant system equation

\[ \dot{z} = A_L z \quad \text{with} \quad A_L = L^{-1}(0) \Phi(0) R \Phi^{-1}(0) L(0) \]

If \( C \) has a diagonal Jordan form then \( R \) and therefore \( A_L \) do too:

\[ R = V \Lambda V^{-1} \quad \text{and} \quad A_L = V_L \Lambda V_L^{-1} \]

where

\[ V_L = L^{-1}(0) \Phi(0) V \quad \text{and} \quad \Lambda = \frac{1}{T} \ln(J) = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\} \]

The eigenvalues of \( A_L \) are the complex logarithm of the eigenvalues of \( C \)

\[ \lambda_k = \frac{1}{T} \ln(|\rho_k|) + i \frac{1}{T} (\arg(\rho_k) + j_k 2\pi) = \frac{1}{T} \ln(|\rho_k|) + i \frac{1}{T} \arg(\rho_k) + i j_k \Omega \]

where \( j_k \) is an arbitrary integer leading to the frequency indeterminacy

\[ y = \sum_{k=1}^{N} \Phi(t) e^{-tR} v_k e^{\lambda_k t} = \sum_{k=1}^{N} \Phi(t)v_k e^{-(\lambda_{p,k} + j_k \Omega)t} e^{(\lambda_{p,k} + j_k \Omega)t} \]
Coleman transformation for $B$-bladed rotors with $B > 2$

$$y = B(t)z$$

where $B(t)$ is an analytical function of time:

$$B(t) = \begin{bmatrix}
1 & 1 \cos \psi_1 & 1 \sin \psi_1 & \cdots & 1 \cos(\tilde{B}\psi_1) & 1 \sin(\tilde{B}\psi_1) & -1 & 0 \\
1 & 1 \cos \psi_2 & 1 \sin \psi_2 & \cdots & 1 \cos(\tilde{B}\psi_2) & 1 \sin(\tilde{B}\psi_2) & 1 & 0 \\
1 & 1 \cos \psi_3 & 1 \sin \psi_3 & \cdots & 1 \cos(\tilde{B}\psi_3) & 1 \sin(\tilde{B}\psi_3) & -1 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
1 & 1 \cos \psi_B & 1 \sin \psi_B & \cdots & 1 \cos(\tilde{B}\psi_B) & 1 \sin(\tilde{B}\psi_B) & (-1)^B & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1
\end{bmatrix}$$

where

$$\psi_b = \psi_1(t) + \frac{2\pi}{B} (b-1), \ b = 2, \ldots, B \quad \text{and} \quad \tilde{B} = \begin{cases} 
(B-1)/2 & B \ odd \\
(B-2)/2 & B \ even
\end{cases}$$

As shown later, the new multi-blade coordinates of the Coleman transformation will describe the rotor motion in the ground fixed frame of reference.
Coleman transformed equation

\[ \dot{z} = \left( B^{-1}(t)A(t)B(t) - B^{-1}(t)\dot{B}(t) \right) z \]

Time-invariant matrix if the rotor is isotropic, i.e., the blades are identical and purely azimuth dependent coupling to surroundings.

The Coleman transformation is a special case of the L-F transformation.

Advantage: Frequencies are determined and defined in the ground fixed frame:

\[ y = \sum_{k=1}^{N} B(t)v_k e^{\lambda_k t} \]

BACK TO THE EXAMPLE
**Rotor motion described in ground fixed frame**

Coleman transformation into multi-blade coordinates

\[
\begin{align*}
\theta_1 &= a_0 + a_1 \cos \psi_1 + b_1 \sin \psi_1 \\
\theta_2 &= a_0 + a_1 \cos \psi_2 + b_1 \sin \psi_2 \\
\theta_3 &= a_0 + a_1 \cos \psi_3 + b_1 \sin \psi_3
\end{align*}
\]

Center of gravity for all three rotor blades of equal mass

\[
\begin{bmatrix} x_{cg} \\
y_{cg} \end{bmatrix} = l \sum_{k=1}^{3} \begin{bmatrix} \cos(\psi_k + \theta_k) \\
\sin(\psi_k + \theta_k) \end{bmatrix}
\]

Assume \( \theta_k \ll 1 \)

\[
\approx l \sum_{k=1}^{3} \begin{bmatrix} 
\cos(\psi_k) - \sin(\psi_k) \theta_k \\
\sin(\psi_k) + \cos(\psi_k) \theta_k 
\end{bmatrix}
\]

\[
- l \sum_{k=1}^{3} \theta_k \begin{bmatrix} -\sin \psi_k \\
\cos \psi_k \end{bmatrix} - \frac{3}{2} l \begin{bmatrix} -b_1 \\
a_1 \end{bmatrix}
\]
Original linear equations of motion

\[
\begin{pmatrix}
    ml^2 & 0 & 0 & -ml \sin \psi_1(t) & ml \cos \psi_1(t) \\
    0 & ml^2 & 0 & -ml \sin \psi_2(t) & ml \cos \psi_2(t) \\
    0 & 0 & ml^2 & -ml \sin \psi_3(t) & ml \cos \psi_3(t) \\
    -ml \sin \psi_1(t) & -ml \sin \psi_2(t) & -ml \sin \psi_3(t) & M + 3m & 0 \\
    ml \cos \psi_1(t) & ml \cos \psi_2(t) & ml \cos \psi_3(t) & 0 & M + 3m
\end{pmatrix}
\begin{pmatrix}
    \ddot{\theta}_1 \\
    \ddot{\theta}_2 \\
    \ddot{\theta}_3 \\
    \ddot{x}_M \\
    \ddot{y}_M
\end{pmatrix}
= 
\begin{pmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    0
\end{pmatrix}
\]
Coleman transformed equations of motion

\[
\begin{bmatrix}
ml^2 & 0 & 0 & 0 & 0 \\
0 & ml^2 & 0 & 0 & ml \\
0 & 0 & ml^2 & -ml & 0 \\
0 & 0 & -3ml/2 & M + 3m & 0 \\
0 & 3ml/2 & 0 & 0 & M + 3m \\
\end{bmatrix}
\begin{bmatrix}
\ddot{a}_0 \\
\ddot{a}_1 \\
\ddot{b}_1 \\
\ddot{x}_M \\
\ddot{y}_M \\
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2ml^2\Omega & 0 & 0 \\
0 & -2ml^2\Omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{a}_0 \\
\dot{a}_1 \\
\dot{b}_1 \\
\dot{x}_M \\
\dot{y}_M \\
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
ml^2\omega_0^2 & 0 & 0 & 0 & 0 \\
0 & ml^2\omega_0^2 - ml^2\Omega^2 & 0 & 0 & 0 \\
0 & 0 & ml^2\omega_0^2 - ml^2\Omega^2 & 0 & 0 \\
0 & 0 & 0 & k_x & 0 \\
0 & 0 & 0 & 0 & k_y \\
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
b_1 \\
x_M \\
y_M \\
\end{bmatrix} = 0
\]
Campbell diagram (frequency - rotor speed)

\[ m = 1 \quad l = 1 \quad \omega_0 = 2\pi \quad M = 30 \quad k_x = 30\pi \quad k_y = 30\pi \]
Campbell diagram for asymmetric support

\[ m = 1 \quad l = 1 \quad \omega_0 = 2\pi \quad M = 30 \quad k_x = 30\pi \quad k_y = 200\pi \]
Modal dynamics of 3-bladed turbines during operation
Coleman transformation into multi-blade coordinates

Physical blade coordinates described by multi-blade coordinates

\[ q_k(t) = a_0(t) \quad | \quad a_1(t) \cos \psi_k \quad | \quad b_1(t) \sin \psi_k \]

\[ = a_0(t) + a_1(t) \cos \left( \Omega t + \frac{2\pi}{3} (k - 1) \right) + b_1(t) \sin \left( \Omega t + \frac{2\pi}{3} (k - 1) \right) \]

All blade coordinates transformed to the ground fixed frame
Modal response measured on blade

The eigenvalue solution $\mathbf{v} e^{\lambda t} = \mathbf{v} e^{(\alpha + i\omega)t}$ for the multi-blade coordinates

\[
\begin{align*}
a_0(t) &= e^{\alpha t} (\text{Re}\{v_{a_0}\} \cos(\omega t) - \text{Im}\{v_{a_0}\} \sin(\omega t)) \\
a_1(t) &= e^{\alpha t} (\text{Re}\{v_{a_1}\} \cos(\omega t) - \text{Im}\{v_{a_1}\} \sin(\omega t)) \\
b_1(t) &= e^{\alpha t} (\text{Re}\{v_{b_1}\} \cos(\omega t) - \text{Im}\{v_{b_1}\} \sin(\omega t))
\end{align*}
\]

Insertion into Coleman transformation yields response in the rotor frame:

\[
q_k = e^{\alpha t} (A_0 \cos(\omega t + \phi_0) + \frac{1}{2} A_{BW} \cos \left( (\omega + \Omega) t + \frac{2\pi}{3} (k - 1) + \phi_{BW} \right) + \frac{1}{2} A_{FW} \cos \left( (\omega - \Omega) t - \frac{2\pi}{3} (k - 1) + \phi_{FW} \right)
\]

where

\[
\begin{align*}
A_0 &= \frac{1}{2} \sqrt{\text{Re}\{v_{a_0}\}^2 + \text{Im}\{v_{a_0}\}^2} \\
A_{BW} &= \frac{1}{2} \sqrt{(\text{Re}\{v_{b_1}\} - \text{Im}\{v_{a_1}\})^2 + (\text{Im}\{v_{b_1}\} + \text{Re}\{v_{a_1}\})^2} \\
A_{FW} &= \frac{1}{2} \sqrt{(\text{Re}\{v_{b_1}\} + \text{Im}\{v_{a_1}\})^2 + (\text{Re}\{v_{a_1}\} - \text{Im}\{v_{b_1}\})^2}
\end{align*}
\]
Frequency response measured on blade

Harmonic excitation in a point on the ground fixed structure $f_z = f_0 e^{i\omega t}$

$$\dot{z} = A_B z + f_0 e^{i\omega t} \Rightarrow z = \frac{f_0}{i\omega I - A_B} e^{i\omega t} = Z(\omega) e^{i\omega t}$$

Insertion into $y = B(t)z$ yields response for the DOFs $y_k$ of blade $k$:

$$y_k = A_0(\omega) \cos(\omega t + \phi_0) + \frac{1}{2} A_{BW}(\omega) \cos \left( (\omega + \Omega) t + \frac{2\pi}{3} (k - 1) + \phi_{BW} \right) + \frac{1}{2} A_{FW}(\omega) \cos \left( (\omega - \Omega) t - \frac{2\pi}{3} (k - 1) + \phi_{FW} \right)$$

where

$$A_0(\omega) = \frac{1}{2} \sqrt{\text{Re}\{Z_{a_0}\}^2 + \text{Im}\{Z_{a_0}\}^2}$$

$$A_{BW}(\omega) = \frac{1}{2} \sqrt{(\text{Re}\{Z_{b_1}\} - \text{Im}\{Z_{a_1}\})^2 + (\text{Im}\{Z_{b_1}\} + \text{Re}\{Z_{a_1}\})^2}$$

$$A_{FW}(\omega) = \frac{1}{2} \sqrt{(\text{Re}\{Z_{b_1}\} + \text{Im}\{Z_{a_1}\})^2 + (\text{Re}\{Z_{a_1}\} - \text{Im}\{Z_{b_1}\})^2}$$
BACK TO THE EXAMPLE

Symmetric support

Asymmetric support
Modal amplitudes of rotor modes

- **Mode 3**: Symmetric mode
- **Mode 4**: Pure backward whirling mode
- **Mode 5**: Pure forward whirling mode

**Graphs:**
- **$A_0$**: Mode 3
- **$A_{BW}$**: Mode 4
- **$A_{FW}$**: Mode 5

- Mode 4: Backward whirling mode with forward whirling component
- Mode 5: Forward whirling mode with backward whirling component
Frequency response due to harmonic excitation of horizontal rotor support DOF
Frequency response due to harmonic excitation of horizontal rotor support DOF

\[ A_{0}(\omega) + 10^{-6} \]

\[ A_{BW}(\omega) \]

\[ A_{FW}(\omega) \]

\[ A_{BW}(\omega + \Omega) \]

\[ A_{FW}(\omega - \Omega) \]
Anisotropic rotors


Simple 5 DOF system considered:

Example: \( G_1 = 1.1G_b \) and \( G_2 = G_3 = 0.95G_b \)
Anisotropic rotor:
Tilt response to excitation of tilt DOF
OPEN- AND CLOSED-LOOP AERO-SERVO-ELASTICITY
Aeroelastic model

- Nonlinear kinematics based on co-rotational Timoshenko elements.

- Blade Element Momentum coupled with unsteady aerodynamics based on Leishman-Beddoes.

- Uniform inflow to give a stationary steady state that approximates the mean of the periodic steady state.

- Analytical linearization about the stationary steady state that include the linearized coupling terms from the geometrical nonlinearities.
Linear open-loop aeroelastic equations

\[
\begin{align*}
M\ddot{X}_s + (C + G + C_a) \dot{X}_s + (K + K_{sf} + K_a) X_s + A_f x_a &= F_s \\
\dot{x}_a + A_d x_a + C_{sa} \dot{x}_s + K_{sa} x_s &= F_a
\end{align*}
\]

\(X_s\) = elastic and bearing degrees of freedom
\(x_a\) = aerodynamic state variables
\(F_s, F_a\) = forces due to actuators and wind disturbance

Open-loop first order equations

\[
\dot{x} = Ax + B_{act} u + B_{wind} \begin{cases}
v_{\text{mean}} \\
v_{\text{ver}} \\
v_{\text{hor}}
\end{cases}
\]
Closed-loop aero-servo-elastic equations

Additional output matrices in HAWCStab2

\[ \begin{aligned}
\dot{x} &= Ax + B_{act}u + B_{wind} \begin{bmatrix} \nu_{\text{mean}} \\ \nu_{\text{ver}} \\ \nu_{\text{hor}} \end{bmatrix} \\
y &= Cx + Du
\end{aligned} \]

Present PI controller states

\[ \begin{aligned}
\dot{x}_c &= A_c x_c + B_c y \\
u &= K_g x_c
\end{aligned} \]

Present closed-loop equations

\[ \begin{aligned}
\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} &= \begin{bmatrix} A & B_{act}K_g \\ B_c C & A_c + B_c D K_g \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} + B_{wind} \begin{bmatrix} \nu_{\text{mean}} \\ \nu_{\text{ver}} \\ \nu_{\text{hor}} \end{bmatrix} \\
y &= Cx + DK_g x_c
\end{aligned} \]
Example: Collective and cyclic pitch controllers
Closed-loop aero-servo-elastic equations

\[
\dot{x} = Ax + B_{act} \begin{bmatrix} Q_{gen} \\ \theta_{col} \\ \theta_{cos} \\ \theta_{sin} \end{bmatrix} + B_{wind} \begin{bmatrix} v_{mean} \\ v_{ver} \\ v_{hor} \end{bmatrix}
\]

\[
\dot{x}_c = A_c x_c + B_c y
\]

Filters and integrators

\[
y = \begin{bmatrix} \Delta \Omega \\ m_{tilt} \\ m_{yaw} \end{bmatrix} = Cx
\]

Tuning parameters

- P-gain on speed: \( k_P \)
- I-gain on speed: \( k_I \)
- Lead angle: \( \psi_0 \)
- P-gain on cyclic: \( k_P^c \)
- I-gain on cyclic: \( k_I^c \)
- Filter frequencies
- Filter damping ratios

\[
x_c = \begin{bmatrix} \Delta \tilde{\Omega} \\ \Delta \dot{\Omega} \\ \phi \\ \dot{m}_{tilt} \\ \ddot{m}_{tilt} \\ M_{tilt} \\ \ddot{m}_{yaw} \\ \dddot{m}_{yaw} \\ M_{yaw} \end{bmatrix}^T
\]

\[
\begin{bmatrix} Q_{gen} \\ \theta_{col} \\ \theta_{cos} \\ \theta_{sin} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_P & 0 & k_I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_P^c \cos \psi_0 & 0 & k_I^c \cos \psi_0 & k_P^c \sin \psi_0 & 0 & k_I^c \sin \psi_0 \\ 0 & 0 & 0 & -k_P^c \sin \psi_0 & 0 & -k_I^c \sin \psi_0 & k_P^c \cos \psi_0 & 0 & k_I^c \cos \psi_0 \end{bmatrix} x_c
\]
Lead angle from open-loop analysis

\[ \theta_{\text{cos}} \rightarrow \begin{cases} m_{\text{tilt}} \\ m_{\text{yaw}} \end{cases} \]

Response to cosine pitch variations

NREL 5 MW turbine
Open and closed-loop wind shear response

\[ k_P = 0.9 \text{ rad/(rad/s)} \]
\[ k_I = 2.2 \text{ rad/rad} \]
\[ \psi_0 = 28 \text{ deg} \]
\[ k_P^c = 1.0 \text{ deg/MNm} \]
\[ k_I^c = 2.0 \text{ deg/MNm/s} \]
Aero-servo-elastic modes and damping

\[ k_P = 0.9 \text{ rad/(rad/s)} \]
\[ k_I = 2.2 \text{ rad/rad} \]
\[ \psi_0 = 28 \text{ deg} \]
\[ k_P^C = 1.0 \text{ deg/MNm} \]
\[ k_I^C = 2.0 \text{ deg/MNm/s} \]
HAWC2 simulations at 17 m/s with NTM
Validation of transfer functions with HAWC2
Generator torque to generator speed (LSS)
Validation of transfer functions with HAWC2
Collective pitch to generator speed (LSS)
Summary

- HAWCStab2 can be used for performing open-loop and closed-loop
eigenvalue and frequency-domain analysis of three-bladed turbines:
  - Controller equations are still hardcoded. A suitable interface is under
    consideration, for example based on DLLs as in HAWC2.
  - Full order analyses can be performed both inside or outside
    HAWCStab2 by writing out system matrices for each operation point.
  - Reduced order modelling capabilities are currently performed outside
    HAWCStab2. Automated procedures for obtaining models with desired
details will be implemented in HAWCStab2, or in Matlab scripts.

- HAWCStab2 is a common tool for both control engineers and mechanical
  engineers:
  - It can provide first-principle models for model-based controllers.
  - It can explain phenomena observed in load simulations.