Carrier Tracking Loop; Loop Filter

GPS Signals And Receiver Technology MM12
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Today’s Subjects

• Demodulation of the GPS signal
• Tracking loop introduction
• Carrier tracking
  – Phase Lock Loop (PLL)
  – Frequency Lock Loop (FLL)
• Loop filters
Demodulation

Or how to turn the radio waves back into the data message that we are interested in
Demodulation

• The transmitted signal from satellite $k$ is:

$$S^k(t) = \sqrt{2P_c} (C^k(t)D^k(t)) \cos(2\pi f_{L1}t)$$
$$+ \sqrt{2P_{PL1}} (P^k(t)D^k(t)) \sin(2\pi f_{L1}t)$$
$$+ \sqrt{2P_{PL2}} (P^k(t)D^k(t)) \cos(2\pi f_{L2}t)$$

$L1$ C/A signal
$L1$ P(Y) signal
$L2$ P(Y) signal

$D$ – data
$C$ – C/A code
$P$ – P(Y) code
Demodulation

• The transmitted signal from satellite $k$ is:

\[
S^k(t) = \sqrt{2P_c} (C^k(t)D^k(t)) \cos(2\pi f_{L1} t) + \sqrt{2P_{PL1}} (P^k(t)D^k(t)) \sin(2\pi f_{L1} t) \\
+ \sqrt{2P_{PL2}} (P^k(t)D^k(t)) \cos(2\pi f_{L2} t)
\]

• After an RF front-end (L1 only):

\[
S^k(t) = \sqrt{2P_c} (C^k(t)D^k(t)) \cos(\omega_{IF} t) + \sqrt{2P_{PL1}} (P^k(t)D^k(t)) \sin(\omega_{IF} t)
\]

• The signal from one satellite after the ADC (narrow filters and low sampling frequency):

\[
S^k(n) = C^k(n)D^k(n) \cos(\omega_{IF} n) + e(n)
\]
Demodulation

- The signal from two satellites after the ADC:
  \[ S(n) = C^1(n)D^1(n) \cos(\omega_{if1}n) + C^2(n)D^2(n) \cos(\omega_{if2}n) + e(n) \]

- The code phase and the intermediate frequency of the carrier wave should be known parameters to demodulate the navigation data from e.g. satellite 1.
Demodulation

- Convert the signal down to baseband:

\[
S(n) \cos(\omega_{if1} n) = C^1(n)D^1(n) \cos(\omega_{if1} n) \cos(\omega_{if1} n) + \\
C^2(n)D^2(n) \cos(\omega_{if2} n) \cos(\omega_{if1} n) + e(n)
\]

\[
\cos(a) \cdot \cos(b) = \frac{1}{2} \cos(a+b) + \frac{1}{2} \cos(a-b)
\]

\[
S(n) \cos(\omega_{if1} n) = \frac{1}{2} C^1(n)D^1(n) + \frac{1}{2} C^1(n)D^1(n) \cos(2\omega_{if}) + \\
C^2(n)D^2(n) \cos(\omega_{if2} n) \cos(\omega_{if1} n) + e(n)
\]

\[
S(n) \cos(\omega_{if1} n) = \frac{1}{2} C^1(n)D^1(n) + \frac{1}{2} C^1(n)D^1(n) \cos(2\omega_{if}) + \\
+ \frac{1}{2} C^2(n)D^2(n) \cos(\omega_{if2} - \omega_{if1} n) + \\
+ \frac{1}{2} C^2(n)D^2(n) \cos(\omega_{if2} + \omega_{if1} n) + e(n)
\]
Demodulation Visualized

![Diagram of demodulation visualization](image)
Demodulation

• **Code wipe off:**

\[ S(n) \cos(\omega_{if1} n) C^1(n) = \frac{1}{2} D^1(n) + \frac{1}{2} D^1(n) \cos(2\omega_{if} ) + e(n) \]

• **After low-pass filtering (integration):**

\[ S(n) \cos(\omega_{if1} n) C^1(n) = I = \frac{1}{2} D^1(n) + e(n) \]

• **Received signal amplitude vs. tracking errors**

\[ I_i = \frac{\sin(\pi \Delta f_i T)}{(\pi \Delta f_i T)} \sqrt{2 \frac{S}{N_0}} R(\tau_i) D_i \cos(\Delta \phi_i ) + e_i \]

• **Conclusion:** perfectly aligned code and carrier replicas are required to do demodulation. These replicas can be tracked using two tracking loops.
Tracking Loop

A way to generate exact copy of the received signal
What does tracking do and why?

• The main goal is to receive the GNSS signal “as clear and loud” as possible – the local carrier and spreading code must be well aligned with the ones in the signal
• GNSS adds one more requirement: to track signal arrival (time) as precise as possible
• Advanced receivers can detect multipath to some extent
• Additional task can be signal quality monitoring
The Tracking Loop Idea

- Generate a local signal
- Correlate it with the received signal
- Measure (time/code-phase, frequency, phase) error between the local and the received signals
- Steer local signal generators to minimize the error
- Pass demodulated data bit value stream to the data processing task
- Repeat procedure
Main Parts Of A Tracking Loop

The error measurement

Tracked signal

Error measurement unit (called error detector or discriminator)

Generated signal

Filter

Scaling of error – used to control sensitivity to the tracking error

Filter has two main tasks:
• To filter noise in the error measurements (due to noise in the tracked signal)
• To shape the tracking loop’s response to the tracking error

$\Theta_1(s)$

$\Theta_2(s)$

$\Theta_c(s)$

$K_d$

$F(s)$

$VCO$

$K_0/s$
Types Of Tracking Loops

- There are 3 main types of the tracking loops depending on the tracked property of the tracked signal:
  - Phase lock loop (PLL)
  - Frequency lock loop (FLL)
  - Delay lock loop (DLL)

- There are few error detectors for each type of the tracking loop with different properties

- Variations of filter parameters will shape the filter response and amount of noise filtering
Plan For The Tracking Topic

- PLL (FLL) and DLL are explained in sections on carrier and code tracking
- Each section will also cover a set of error detectors applicable in a given tracking loop
- Tracking loop filter is explained in a separate section as the same theory is used for all types of tracking loops
The Carrier Tracking Loop

The Phase Locked Loop (PLL)
The goal of the Carrier Tracking Loop is to produce a perfectly aligned carrier replica. The most common way is to use a PLL:

\[
D(n) \cos(\omega_f n) \cos(\omega_f n + \phi)
\]

\[
\sin(\omega_f n + \phi)
\]
Carrier Tracking Loop

- The demodulation in the In-phase (I) branch:
  \[ D(n) \cos(\omega_{if} n) \cos(\omega_{if} n + \phi) = \frac{1}{2} D(n) \cos(\phi) + \frac{1}{2} D(n) \cos(2\omega_{if} n + \phi) \]

- The demodulation in the Quadrature-phase (Q) branch:
  \[ D(n) \cos(\omega_{if} n) \sin(\omega_{if} n + \phi) = \frac{1}{2} D(n) \sin(\phi) + \frac{1}{2} D(n) \sin(2\omega_{if} n + \phi) \]

- The I signal:
  \[ I = \frac{1}{2} D(n) \cos(\phi) \]

- The Q signal:
  \[ Q = \frac{1}{2} D(n) \sin(\phi) \]
Carrier Tracking Loop

- **Costas loop:**

\[
I = \frac{1}{2} D(n) \cos(\phi)
\]

\[
Q = \frac{1}{2} D(n) \sin(\phi)
\]

**Diagram:**
- **Incoming signal:** \(D(n) \cos(\omega_{if} n)\)
- **NCO Carrier Generator**
- **Carrier Loop Filter**
- **Carrier Loop Discriminator**
- **Integrate and Dump**
- **\(\cos(\omega_{if} n + \phi)\)**
- **\(\sin(\omega_{if} n + \phi)\)**

2009 Danish GPS Center
Carrier Tracking Loop

- To find the phase error:

\[ \frac{Q}{I} = \frac{\frac{1}{2} D(n) \cos(\phi)}{\frac{1}{2} D(n) \sin(\phi)} = \tan(\phi) \]

\[ \phi = \tan^{-1} \frac{\frac{1}{2} D(n) \sin(\phi)}{\frac{1}{2} D(n) \cos(\phi)} \]

\[ \phi = \tan^{-1} \frac{Q}{I} \]
Carrier Tracking Loop

- The Costas loop is independent on the phase shifts caused by the data bits

- Phasor diagram:

An output example:
Carrier Tracking Loop

• Different kinds of phase lock loop discriminators:
  – Arctan

\[ D = \tan^{-1} \frac{Q}{I} \]

• Much time consuming (not a big problem today)
• The output is the real phase error

  – Sign product

\[ D = Q \cdot \text{sign}(I) \]

• Fast method
• The discriminator output is proportional to \( \sin(\varphi) \)
Carrier Tracking Loop

- Different kinds of phase lock loop discriminators:
Carrier Tracking Loop

- **Costas loop:**

\[ I = \frac{1}{2} D(n) \cos(\phi) \]

\[ Q = \frac{1}{2} D(n) \sin(\phi) \]

\[ \phi = \tan^{-1}\left(\frac{I}{Q}\right) \]
Carrier Tracking Loop

- The signal energy in I and Q when PLL has locked on the signal:

![Graphs showing signal energy in I and Q over time.](image)
The Carrier Tracking Loop

The Frequency Locked Loop (FLL)
The discriminators are a bit different than in PLL. They measure change in carrier phase over an interval of time.

Less noise sensitive than PLL – it can track at lower SNR

The tracking loop has more noise than PLL

Can be used for the re-acquisition or pull-in states due to bigger frequency lock range
Loop Filters
Why The Filter Is Needed Anyway?

- The error measurement is there, so just correct the generator frequency and job is done, right?
- The answer is NO:
  - There is an error measurement noise (even at good SNR)
  - There is a steady state error caused by Doppler
The Typical Tracking Loop

- Phase error detector has gain $K_d$. The transfer function is showed in figure a)
- The VCO has a center frequency $\omega_0$ and gain $K_0$. The transfer function is showed in figure b)
- The filter coefficients depend on $K_d$ and $K_0$
There is an initial frequency between tracked and generated signals (27Hz here)

Figures show:
- The filter “accumulates” offset over time and keeps it
- The result of damping – different convergence times
A Simple Digital Loop Filter

- The $C_1$ and $C_2$ depend on loop noise bandwidth $B_L$, VCO and PD gains and loop damping factor $\zeta$.
- Damping factor controls how fast the filter reaches its settle point.
- Noise bandwidth controls the amount of allowed noise in the filter.

\[ C_1 = \frac{1}{K_0 K_D} \frac{8\zeta \omega_n T}{4 + 4\zeta \omega_n T + (\omega_n T)^2} \]

\[ C_2 = \frac{1}{K_0 K_D} \frac{4(\zeta \omega_n T)^2}{4 + 4\zeta \omega_n T + (\omega_n T)^2} \]

\[ \omega_n = \frac{8\zeta B_L}{4\zeta^2 + 1} \]
Different loop responses depending on the damping factor (first 20ms are due to loop filter initialization)

- Determines how much the loop filter "resists" to the control signal:
  - On one hand – how fast the loop will "fix" the tracking error
  - On other hand – how much the loop will overshoot 0 error point

- A compromise value is used or few values are used for different receiver modes
Noise Bandwidth

- Narrow noise bandwidth decreases noise in the tracking loop, AND – response speed
- At 100ms the loop noise bandwidth is switched from about 100Hz to 15Hz
Noise Bandwidth

• Loop noise bandwidth also determines maximum Doppler offset and rates tolerated by the loop

• Figures show a case of too big initial frequency error in acquisition
Loop Order

- The filter is a first order filter.
- The tracking loop (excluding filter) is a first order system, therefore the tracking loop is second order.
- Higher order filters approximate the error dynamics better (e.g. to be used for ships etc.)
An Example Of A PLL

- \( \omega = \frac{20}{0.7845}; \ T = 0.02; \)
  - \( k_1 = 2.4 \times \omega \)
  - \( k_2 = \frac{1.1 \times \omega^2}{20} \)
  - \( k_3 = \frac{\omega^3}{20} \)

- \( \omega = \frac{20}{0.53}; \ T = 0.02; \)
  - \( k_1 = 1.414 \times \omega \)
  - \( k_2 = \frac{\omega^2}{20} \)
  - \( k_3 = 0 \)

This slide contents is only available to the listeners of our courses.
2-nd Order Loop Response
3-rd Order Loop Response
Questions and Exercises