# Multi Rate Orthogonal Frequency Division Multiplexing

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*Abstract*— **A novel multi rate orthogonal frequency division multiplexing (OFDM) system is proposed in this paper. It is expected to provide a flexible physical layer support to next generation wireless networks. It is mainly suited to inter carrier interference limited situations, when a large number of sub carriers are used within the available bandwidth constraint. The proposal is to alter the otherwise uniform performance of the sub carriers in an OFDM system by using different bandwidths for different subcarriers. This is advantageous in the situation where different users and data channels in a wireless network will have different data rates and quality of service requirements. Different sub carriers with different performance figures can be mapped to the data channels with matching requirements. This scheme can enhance the conventional OFDM systems to become flexible. It is shown that, using different sub carrier bandwidths can improve the throughput of some sub carriers at the cost of the others. Results show that overall system performance also improves with the proposed scheme even without any optimal mapping of data stream to the sub carriers for slightly higher frequency offsets.**

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is expected to be one of the enabling physical layer technologies in future generation wireless networks. It has strong capability to withstand multipath fading, can provide very high spectral efficiency and can be implemented with a simple equalization architecture [1]. Though OFDM has the above mentioned advantages it suffers from high sensitivity to synchronization errors such as carrier frequency offset (CFO), sampling frequency offset and symbol timing offset errors. Carrier frequency offset arises because of non exact synchronization between receiver and transmitter carrier frequencies. CFO and doppler shifts give rise to inter carrier interference (ICI). Closely placed sub carriers cause high amount of ICI. Due to increasing requirement to support larger user base within the available scarce and costly bandwidth, larger number of sub carriers are needed. This is also needed to reduce the overhead of guard interval in OFDM systems. Such systems often use a cyclic prefixed guard interval. This causes a reduction in the efficiency. In order to improve the situation, large number of sub carriers are usually used to decrease the overhead of this guard interval on each OFDM symbol. This often leads to interference limited rather than noise limited situation. Existing implementations and wireless physical layer standards using OFDM systems use equal bandwidth for all sub carriers. This makes all the sub carriers face almost equal ICI power. Therefore all sub carriers are expected to have the same performance on an average. On the other hand future wireless networks will have to support a wide variety of data/user channels simultaneously. They will vary in data rate, error tolerance, latency, etc. requirements. It is highly desired that the physical layer have an inherent capability to support different data channels with different quality of services.

To address the above situation it is proposed in this paper to use different bandwidths for different sub carriers (Figure 1). In this system, different sub carriers experience different amount of ICI and hence will have different performance figures in terms of bit error rate and latency. This brings in the flexibility to the system. The uniform performance of all sub carriers in conventional OFDM systems can now be altered and bit error performance can now be exchanged between sub carriers to support the network needs. An appropriate mapping of the data channel requirements to the available sub carriers, with their performance figures, will optimize the implementation of such a system. Sub carrier bandwidths can be computed and allocated dynamically as well, based on the requirement of the network.

The rest of the paper is organized as follows. Section II describes the scheme in details. Section III provides derivation of the performance of system. Section IV has a detailed discussion on the performance of the system and Section V concludes the paper.

## II. MULTI RATE FRAMEWORK: DESCRIPTION

The system proposal is based on OFDM. It is known that, in such systems, the sub carriers are orthogonal to each other. In normal OFDM systems, sub carriers are of the same duration, have same sub carrier spacing and bandwidth. In the proposed system, it is assumed that the symbol duration of "$\mathbf{x}$" number of sub carriers to be "$\mathbf{T_x}$", and that of "$\mathbf{y}$" number of sub carriers to be "$\mathbf{T_y}$" and so on. The total number of sub carriers is then equal to "$\mathbf{x+y+...}$". The proposed scheme requires that no two symbol durations from the set of symbol durations "$\mathbf{T_x}$,$\mathbf{T_y}$, etc." be the same. Accordingly the bandwidth of any two group of sub carriers will not be the same. This is depicted in Figure 1. There are three different sub carrier bandwidths shown in Figure 1. Accordingly three different pulse durations appear. The system must be designed such that the sub carriers orthogonal, i.e. the maximum amplitude of one sub carrier and the nulls of all other sub carriers coincide (if "$\mathbf{T_z}$" and "$\mathbf{T_y}$" are integer multiples of "$\mathbf{T_x}$', i.e of the smallest pulse duration). The amplitude difference is shown to include the

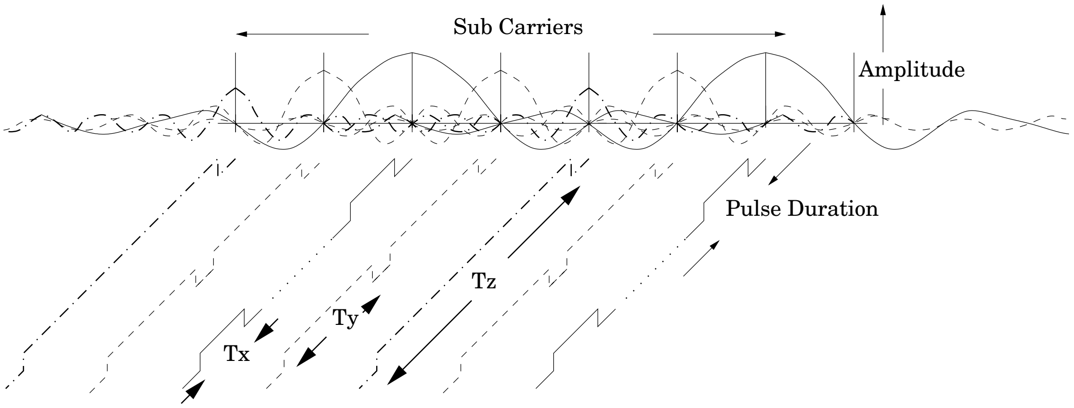

Fig. 1. Time Frequency diagram of Multi Rate OFDM

effect of equal energy pulses for each sub carrier. Compared to this, a normal OFDM based system will have all sub carriers using exactly the same bandwidth and amplitude (other than systems using adaptive modulation per sub carrier). The sub carrier bandwidths of the multi rate OFDM system need to be arranged such that ICI caused by frequency offset and doppler reach a minimum. The design or method of solving such a problem is not the primary goal of this paper. In such systems, careful allocation of bandwidths is very important to achieve the best result.

## III. ANALYTICAL MODEL

For simplicity, an analytical description of multi rate ODFM system with two different bandwidths is presented here. The goal of presenting the analysis is to show that it is feasible to use different sub carrier bandwidths to generate sub carriers with different bit error performances as compared to the uniform performance of sub carriers in standard OFDM systems. The analysis can be extended following similar systematic steps as explained in this paper to examine the effect of ICI for multi rate systems with more than two subcarrier types. In the current work one sub carrier set will be assigned a higher bandwidth than the other and will be called the high rate sub carriers, while the other set can be referred to as the low rate sub carriers. The high and the low rate sub carrier will be interleaved in an alternating fashion. The one with smaller inverse discrete fourier transform (IDFT) duration and hence larger sub carrier bandwidth will be indexed by 1 while the one with larger IDFT duration and smaller subcarrier bandwidth will be indexed by 2.

### A. Transmitter

The OFDM symbol of the $i^{th}$ type can therefore be written as

$$x_{i_{s_i}}(t) = \frac{1}{\sqrt{T_{i_u}}} \sum_{\substack{k_i=-\frac{N}{2},\\ k_i\to \text{even/odd}}}^{\frac{N}{2}-1} X_{i_{s_i}}\left[k_i\right] e^{j2\pi \frac{k_i}{T_{i_u}}(t-T_{i_g})} \Xi_{T_{i_s}}(t), \tag{1}$$

$$\Xi_T(t) = \begin{cases} 1 & \text{for} \quad 0 \le t < T \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

In the above two equations $x_{i_{s_i}}(t)$ denotes the OFDM symbol of the $i^{th}$ type where $i$ can take values $1, 2$. $T_{i_u}$ is the IDFT duration and $T_{i_s}$ is the symbol duration (IDFT duration + guard interval) of the $i^{th}$ type of sub carriers. $X_{i_{s_i}}\left[k_i\right]$ denotes the data symbol in the $k_i^{th}$ sub carrier index of the $i^{th}$ sub carrier type. $T_{i_g}$ is the guard interval duration of the $i^{th}$ sub carrier type, while $\Xi_T(t)$ is the gate pulse. $N$ is the total number of sub carriers in the system. Sub carrier bandwidth for type 1 is $\Delta f_1 = \frac{1}{T_{1_u}}$, while that for type 2, is $\Delta f_2 = \frac{1}{T_{2_u}}$. It is assumed that $T_{2_u} = MT_{1_u}$, where $M$ is an integer. *It is important to note the frequencies as indicated in the exponentials, type* $1$ *and type* $2$ *are orthogonal.* Therefore the composite signal transmitted can be expressed as

$$x\left(t\right) = \sum_{s_1=-\infty}^{\infty} x_{1_{s_1}}(t - s_1 T_{1_s}) + \sum_{s_2=-\infty}^{\infty} x_{2_{s_2}}(t - s_2 T_{2_s}) \tag{3}$$

### B. Channel

If $h_\tau(t)$ is the channel impulse response, and $\tau$ is the delay, then the signal after passing through the wireless link can be expressed from [2] as $r(t) = \int_0^{\tau_{\max}} h_\tau(t) s\left(t-\tau\right) d\tau$ where $\tau_{\max}$ is the maximum channel delay spread. An assumption that is made throughout the analysis is that $h_\tau(t)$ is quasi–static over a period $\max(T_{1_s}, T_{2_s})$.

### C. Receiver

The $X^{'}$ is the receiver counterpart parameter of the $X$ parameter of the transmitter. The down converted received baseband signal which, after low pass filtering can be evaluated to

$$\begin{aligned}\tilde{r}(t) = \int_0^{\tau_{\max}} h_\tau(t) \Bigg[ \sum_{s_1=-\infty}^{\infty} x_{1_{s_1}}(t - s_1 T_{1_s} - \tau) \\ + \sum_{s_2=-\infty}^{\infty} x_{2_{s_2}}(t - s_2 T_{2_s} - \tau) \Bigg] d\tau \, e^{j2\pi\delta f t} + \nu(t),\end{aligned} \tag{4}$$

where $\delta f$ is the difference in the receiver and transmitter local oscillator carrier frequencies and $\nu(t)$ is the noise component. It will be simpler to present the analysis for the two rates separately. First the analysis for the high rate (smaller $T_u$) will be done, and then the analysis for the low rate (larger $T_u$) shall follow.

*1) High Rate:* The received signal has to be appropriately windowed for DFT operation. The windowed received signal for high rate data can be written as

$$\tilde{r1}_{s_1}(t) = \tilde{r}(t)\Xi_{T'_{1_u}}(t - s_1T^{'}_{1_s} - T^{'}_{1_g} - \delta T_1) \tag{5}$$

For both the high rate and the low rate cases, the assumption made is that there is appropriate DFT window alignment, such that ISI is avoided. Therefore a received sub carrier for the high data rate system can be written, for $k^{'}_1 \rightarrow$ even as,

$$R_{1_{s1}}\left[k^{'}_1\right] = \frac{1}{\sqrt{T^{'}_{1_u}}}\int_0^{T^{'}_{1_u}} \tilde{r1}_{s_1}(t)e^{-j2\pi\frac{k^{'}_1}{T^{'}_{1_u}}(t-s_1T^{'}_{1_s}-T^{'}_{1_g})}\,dt$$

$$=\frac{1}{\sqrt{T^{'}_{1_u}}}\int_{\Xi_{T^{'}_{1_u}}(\ldots)}\int_0^{\tau_{\max}} h_\tau(t)\left[\left[\underbrace{\sum_{s_1=-\infty}^{\infty} x_{1_{s_1}}(t - s_1T_{1_s} - \tau)}_{\textsc{i},\text{high rate sub carriers}}\right.\right.$$

$$\left. + \underbrace{\sum_{s_2=-\infty}^{\infty} x_{2_{s_2}}(t - s_2T_{2_s} - \tau)}_{\textsc{ii},\text{low rate sub carriers}}\right]\,d\tau\, e^{j2\pi\delta ft} +$$

$$\nu(t)]\; e^{-j2\pi\frac{k^{'}_1}{T^{'}_{1_u}}(t-s_1T^{'}_{1_s}-T^{'}_{1_g})}\,dt \tag{6}$$

$R_{1_{s1}}\left[k^{'}_1\right]_{\mathbf{I}}$, is the component which has the useful signal to be demultiplexed and interference from high rate sub carriers, while $R_{1_{s1}}\left[k^{'}_1\right]_{\mathbf{II}}$, will contain interference from low rate sub carriers. The $\tau$ in the exponential can be taken out and used to integrate the $\int_0^{\tau_{\max}} h_\tau(t)$ to yield, $H_s[k]$, using the previous assumption that the channel is quasi static over a period $\max(T_{1_s}, T_{2_s})$. It is also assumed that the thermal noise contribution is much less as compared to the ICI and hence can be removed from the computation of ICI noise contributions. It is understood that $T_{2_u} > T_{1_u}$. Upon substituting expression as in previous equations and solving the integral, the following is arrived at,

$$R_{1_{s1}}\left[k^{'}_1\right]_{\mathbf{II}} = \frac{T^{'}_{1_u}}{\sqrt{T^{'}_{1_u}T_{2_u}}}\sum_{\substack{k_2=-\frac{N}{2},\\ k_2\rightarrow\text{odd}}}^{\frac{N}{2}-1} H_{s_2}[k_2]X_{2_{s_2}}[k_2]$$

$$.e^{j2\pi\{\frac{s_1T^{'}_{1_s}+T^{'}_{1_g}}{T^{'}_{1_u}}k^{'}_1-\frac{T_{2_g}+s_2T_{2_s}}{T_{1_u}}k_2M\}}$$

$$.e^{j\frac{2\pi}{T^{'}_{1_u}M}\phi_{k_2M,Mk'_1,MT'_{1_u}}(s_1T^{'}_{1_s}+T^{'}_{1_g})}\; e^{j\frac{\pi}{M}\phi_{k_2M,Mk'_1,MT'_{1_u}}}$$

$$.\text{sinc}\left(j\frac{\pi}{M}\phi_{k_2M,Mk'_1,MT'_{1_u}}\right) \tag{7}$$

where $\phi_{k_2M,Mk'_1,MT'_{1_u}} = (1+\zeta)k_2M - Mk^{'}_1 + M\delta fT^{'}_{1_u}$ and $\zeta$ is the sampling frequency error, defined as $\frac{T^{'}-T}{T^{'}}$, where $T$ denotes the sampling duration at the transmitter and $T^{'}$ denotes the sampling duration the receiver. The variance of $R_{1_{s1}}\left[k^{'}_1\right]_{\mathbf{II}}$ would give the power of the interference term from the low rate sub carriers to a high rate sub carrier. The following assumptions are made.

- $H_s[k]$, are iid, and $E_{H_s[k_2].H_s[k_1]^*} = E_{|H_s[k]|^2}\delta_{k_2,k_1}$
- $E_{X_s[k_2].X_s[k_1]^*} = E_{|X_s[k]|^2}\delta_{k_2,k_1}$
- $E_{X_s[k]} = 0$, $E_{|X_s[k]|^2} = 1$, and $E_{|H_s[k]|^2} = 1$

where $E$ denotes averaging. Therefore the variance can be approximated for small angles, following the works [3], [4], to

$$\begin{aligned}\sigma^2_{R_{1_{s1}}[k'_1]_{\mathbf{II}}} &\approx E_{|X_{2_{s_2}}[k_2]|^2}E_{|H_{s_2}[k_2]|^2}\tfrac{\pi^2}{4M}(\delta fT^{'}_{1_u})^2\\ &= \tfrac{\pi^2}{4M}(\delta fT^{'}_{1_u})^2\end{aligned} \tag{8}$$

The ICI power in a high rate sub carrier from the high rate sub carriers can be derived similarly,

$$\sigma^2_{R_{1_{s1}}[k'_1]_{\mathbf{I}}} \approx \tfrac{\pi^2}{12}(\delta fT^{'}_{1_u})^2 \tag{9}$$

The total ICI power experience by a high rate sub carrier is then

$$\begin{aligned}\sigma^2_{R_{1_{s1}}[k'_1]} &= \sigma^2_{R_{1_{s1}}[k'_1]_{\mathbf{I}}} + \sigma^2_{R_{1_{s1}}[k'_1]_{\mathbf{II}}}\\ &= \frac{\pi^2}{12}(\delta fT^{'}_{1_u})^2 + \frac{\pi^2}{4M}(\delta fT^{'}_{1_u})^2\end{aligned} \tag{10}$$

The ICI power encountered by a sub carrier in a normal OFDM system is as given by [5]

$$\sigma^2_{ICI} \approx \frac{\pi^2}{3}(\delta fT^{'}_{1_u})^2 \tag{11}$$

*2) Low Rate :* The windowed received signal for Low rate data can be written as

$$\tilde{r2}_{s_2}(t) = \tilde{r}(t)\Xi_{T'_{2_u}}(t - s_2T^{'}_{2_s} - T^{'}_{2_g} + \delta T_2) \tag{12}$$

A received sub carrier for the low data rate system can be written as $R_{2_{s2}}\left[k^{'}_2\right]_{k'_2\rightarrow\text{odd}} =$

$$\frac{1}{\sqrt{T^{'}_{2_u}}}\int_{\Xi_{T^{'}_{2_u}}(\ldots)}\int_0^{\tau_{\max}} h_\tau(t)\left[\left[\underbrace{\sum_{s_1=-\infty}^{\infty} x_{1_{s_1}}(t - s_1T_{1_s} - \tau)}_{\textsc{ii},\text{high rate sub carriers}}\right.\right.$$

$$\left. + \underbrace{\sum_{s_2=-\infty}^{\infty} x_{2_{s_2}}(t - s_2T_{2_s} - \tau)}_{\textsc{i},\text{low rate sub carriers}}\right]\,d\tau\, e^{j2\pi\delta ft} +$$

$$\nu(t)]\; e^{-j2\pi\frac{k^{'}_2}{T^{'}_{2_u}}(t-s_2T^{'}_{2_s}-T^{'}_{2_g})}\,dt \tag{13}$$

$$\tag{14}$$

$R_{2_{s2}}\left[k_2^{'}\right]_{\mathbf{I}}$, is the component which has the useful signal to be de–multiplexed and interference from low rate sub carriers, while $R_{2_{s2}}\left[k_2^{'}\right]_{\mathbf{II}}$, will contain interference from high rate sub carriers only. The ICI power as experienced by a low rate sub carrier can be derived, following the analysis of high rate sub carriers as,

$$\sigma^2_{R_{2_{s2}}[k_2^{'}]_{\mathbf{II}}} = \frac{\pi^2}{4}(\delta f T_{1_u}^{'})^2, \sigma^2_{R_{2_{s2}}[k_2^{'}]_{\mathbf{I}}} = \frac{\pi^2}{12}(\delta f T_{1_u}^{'})^2. \tag{15}$$

Therefore total ICI power is

$$\sigma^2_{R_{2_{s2}}[k_2^{'}]} = \sigma^2_{R_{2_{s2}}[k_2^{'}]_{\mathbf{I}}} + \sigma^2_{R_{2_{s2}}[k_2^{'}]_{\mathbf{II}}} = \frac{\pi^2}{3}(\delta f T_{1_u}^{'})^2 \tag{16}$$

while the ICI power experienced by a sub carrier in a normal OFDM system using DFT duration $T_{2_u} = MT_{1_u}$ is [5]

$$\sigma^2_{ICI} \approx \frac{\pi^2}{3}(M\delta f T_{1_u}^{'})^2 \tag{17}$$

Therefore it can be seen from the above that the interference power for the high rate sub carriers is less than that of the normal OFDM system using the same bandwidth, while the interference power of the low rate sub carriers is the same as the normal OFDM system using sub carrier bandwidth as that of the low rate system. The SINR can be computed as

$$\text{SINR} = \frac{E_{|X_s[k]|^2} E_{|H_s[k]|^2} \text{sinc}(\delta_f T_u)^2}{\sigma^2_{ICI} + \sigma^2_n} \tag{18}$$

where $\sigma^2_n$ is the noise variance of the sub carrier. The symbol error rate for QAM modulation can be computed using [6] as

$$\text{SER} \leq 4\mathbf{Q}\left(\sqrt{\frac{3\text{SINR}}{2(M_o - 1)}}\right) \tag{19}$$

where is $\mathbf{Q}(x) = \frac{1}{2}\text{erfc}(\frac{x}{\sqrt{2}})$, where erfc is the complementary error function, and $\mathrm{M_o}$ is the modulation order. For 64-QAM, $\mathrm{M_o} = 64$, for 16-QAM $\mathrm{M_o} = 16$, etc. For computing the throughput for each sub carrier

$$N_b(1 - \text{SER})\frac{T_u}{T_s}$$

can be used, where $N_b$ is the number of bits in the modulation used for the system.

## IV. PERFORMANCE AND DISCUSSION

### A. Environment

For numerical evaluation of the performance, a bandwidth of 5 MHz is taken at a carrier frequency of 5 GHz. Number of sub carriers considered were 1024 and 2048. Different amount of carrier frequency offsets {represented in parts per million of carrier frequency (ppm) of the carrier frequency} are considered. Carrier frequency offset is caused not only by the mismatch between the local oscillators of the transmitter and receiver but also due to doppler shifts due to movement of transmitter, receiver and reflectors. Although plots are obtained for several values of M, the one for M=2 is shown here. Increasing the value of M does not provide significant improvement for smaller frequency offsets. Modulation scheme for a sub carrier type is chosen such that the signal to interference noise ratio (SINR) is greater than the minimum required SNR for the modulation order. $\mathrm{MR1}$ indicates high rate sub carrier in the multi rate system , while $\mathrm{MR2}$ indicates low rate sub carrier. $\mathrm{MR}$ indicates the combined performance the high and low rate sub carriers of the multi rate system without using any resource allocation scheme. $\mathrm{T2}$ indicates performance of a normal OFDM system using sub carrier bandwidth of the low rate sub carriers of the multi rate system. $T2$ has 2048 sub carriers, while, $MR$ has 1024 sub carriers, in the same bandwidth, with $MR1$ having the larger sub carrier bandwidth while $MR2$ having the smaller sub carrier bandwidth.

Figure 2 shows the performance of each type of sub carrier for M=2. For very small CFO (in the range of 0.01 ppm), the performance of $T2$ is the best. For slightly higher frequency offsets (more than 0.1 ppm), the performance of $T2$ drops significantly. The gives multi rate system an advantage over the normal OFDM system under such situation. At offsets little above 0.1 ppm CFO the high rate sub carriers $MR1$ have more than 70% better throughput. The low rate sub carriers $MR2$ have about 15% worse throughput as compared to $T2$. Thus, it can be said that by using the multi rate system of the type discussed in this paper, two different kind of subcarriers have been generated with different performances. These can be used to support multi media traffic (characterized by traffic generation rate, tolerable delay, jitter, bit error rates, etc.). For example voice may be supported by $MR2$ (lower bandwidth, larger delay, worse performance), while video may be supported by $MR1$ (larger bandwidth, smaller delay, better performance) in an optimal way than using a normal OFDM system with sub carriers separated by spacing as much as the bandwidth of the low rate sub carriers. The best utilization of the differential physical layer depends on the resource management unit of the communication network. It is very difficult to compare the combined performance of the sub carriers of the proposed multi rate scheme with a normal OFDM system, since an appropriate resource allocator is needed for the purpose. A very simple comparison though unfair with respect to the multi rate system, is to assume only one type of traffic being present in the network. Figure 3 shows such a comparison. In this case as well, the performance of the normal OFDM system $T2$ is the highest for very small offsets. When the carrier offset is slightly above 0.1 ppm, the performance of the combined sub carriers of the multi rate system is better than the normal OFDM system using sub carrier bandwidth same as that of the low rate sub carriers. At around 0.12 ppm, the over all performance of the multi rate system is 25% better than T2. For comparison, T1 is also plotted. It indicates performance of a normal OFDM system using sub carrier bandwidth same as that of the high rate sub carriers of the multi rate system. It has 1024 uniformly spaced sub carriers in the same bandwidth as the other two systems. The combined performance of the sub carriers of the multi rate system is better than T1 system for slightly higher offsets. For lower offsets, both T1 and T2 are better than multi rate system.

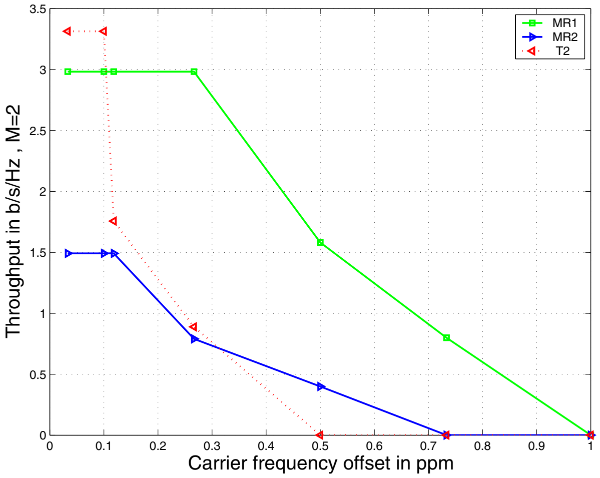

Fig. 2. Throughput comparison of each sub carrier type, for M=2

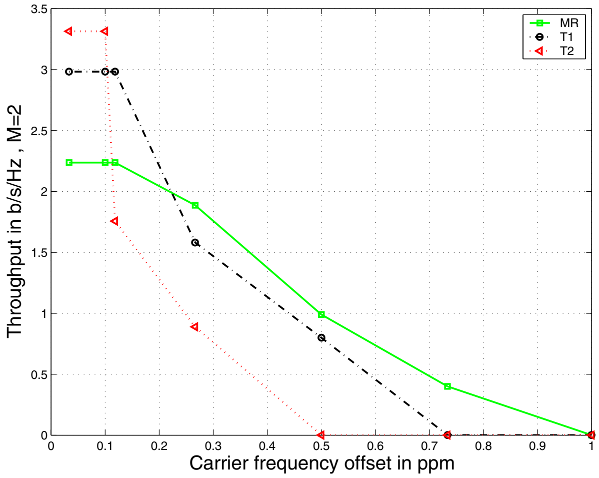

Fig. 3. Throughput comparison of Multi Rate and Normal OFDM, for M=2

These comparison are unfair in the sense that the flexibility of the multi rate system has not been harnessed in these cases. With the use of an appropriate resource management mechanism suited to the multi rate scheme, that maps the requirement of the multi media traffic to the sub carriers, the advantage offered by multi rate system can be leveraged.

## V. CONCLUSION

A novel multi rate, orthogonal frequency division multiplexing has been presented in this article. It has been shown that sub carriers can be generated with different performance figures thus creating a flexible physical layer support for heterogenous multi media traffic (voice, video, text, etc). This is much needed for future networks where different traffic items have different requirements such as bit error rates, latency, etc. For the case of two types of sub carriers it has been found that, in the environment under discussion, a set of sub carriers with $70\%$ better performance and another with $15\%$ worse performance can be created for frequency offsets little more than $0.1$ ppm. It has also been shown that for higher frequency offsets (due to both local oscillator mismatch and doppler shift) the presented multi rate scheme provides improved performance as compared to normal OFDM systems even without leveraging the benefit of the flexible physical layer that the multi rate scheme provides. The presentation in this article is shown for a particular set of parameters. With different environmental parameters, the performance is expected to change but will behave in a similar way as shown here.

## ACKNOWLEDGEMENT

The authors are grateful to TCS India for funding the project and to the WING lab members of Aalborg University for delightful discussions.

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