Intercept Point and Undesired Responses

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Abstract—A method for using the concept of intercept point to calculate the undesired-response rejection ratio of a single stage is presented. Single stages may be cascaded together to form a system and the undesired-response rejection ratio of the system may be found using a procedure similar to cascaded noise figure. When applied to receiver system design, this method allows easy calculation of such undesired receiver responses as intermodulation distortion and spurious responses.

I. INTRODUCTION

The transfer functions of electronic devices commonly used in amplifying and mixing circuits are seldom if ever identically linear in the case of an amplifier, square law in the case of a mixer. The nonideal characteristics inherent in these devices lead to undesired responses which are produced in addition to the desired response. Through a power series expansion representing the transfer function of a nonideal device, these undesired responses may be analyzed and the rejection ratio or the amount of rejection that a stage has to these undesired responses may be found. When several stages are cascaded together to form a system, the nonideal characteristics of each stage will contribute to the overall rejection ratio of the system. A system undesired response rejection ratio may therefore be defined to describe the overall system performance as it relates to undesired responses. The following sections outline a procedure for calculating the rejection ratio of a single stage as well as the rejection ratio of a system of cascaded stages.

II. EQUATIONS USED IN INTERCEPT POINT CALCULATIONS

Power Series Expansion

The transfer function f(x) of a nonideal stage can be expressed as a power series expansion:

\[ x = \frac{x_0}{a_0(x-1)^3 + a_1(x-1)^2 + a_2(x-1) + a_3(x-1)^3 + \cdots} \]

From this expansion it can be seen that undesired higher order responses are produced in addition to the fundamental response. These responses may be plotted as shown in Fig. 1, where the desired or fundamental response is shown with a slope of one, and an undesired higher order response is shown with a slope \( n \). The procedure for calculating numerical values for \( n \) will be covered in Section III.

Intercept Point

The two responses shown in Fig. 1 intersect at a point called the intercept point (IP) of the stage. From Fig. 1 it can be seen that the intercept point may be referenced to either the input or the output of the stage. If referenced to the input it is called the input intercept point \( IP_i \), and if referenced to the output it is called the output intercept point \( IP_o \). Note that \( IP_i \) and \( IP_o \) correspond to specific power levels at the input and output of the stage and may be related to each other by the gain of the stage. Let \( G \) be the gain of the stage in dB. Then

\[ P_{out} = P_{in} + G \]

where:

- \( P_{out} \) is the output power of stage in dB
- \( P_{in} \) is the input power to stage in dB.

For the IP coordinates shown in Fig. 1:

\[ IP_o = IP_i + G \]

or

\[ IP_i = IP_o - G \ (\text{dB}) \]  \hspace{1cm} (1)

As previously stated, the intercept point is the intersection of the fundamental and the undesired response. The IP of a
stage is determined by the device characteristics and the operating environment, and it remains constant as long as these two things remain the same. Therefore, the IP may be thought of as a fixed parameter associated with the stage and may be used to calculate the undesired-response rejection ratio of the stage. To derive such an expression, consider Fig. 2, where a signal with input power $P_{i1}$ is applied to a stage with gain $G$. From Fig. 2,

$$\text{IP}_o = P_{o_1} + d$$

and

$$\text{URR} = (m - 1)d.$$ 

Therefore

$$\text{URR}/(m-1) = d$$

and

$$\text{IP}_o = P_{o_1} + \text{URR}/(m-1)$$

or

$$\text{IP}_o = P_{o_1} + \beta \times \text{URR}$$

where

$$\beta = 1/(m-1).$$

In the general case for arbitrary $P_i$, this expression becomes

$$\text{IP}_o = P_o + \beta \times \text{URR} \quad (\text{dB})$$

(2)

$$\beta = 1/(m-1)$$

(3)

where $m$ is the slope of the undesired response (Section III). $P_o$ and URR can be measured at the output of the stage with a spectrum analyzer, as shown in Fig. 3.

In Fig. 3,

- $f_d$ = desired output response at frequency $f_d$ ($f_d$ is the unchannel signal for an amplifier stage and intermediate-frequency (IF) signal for a mixer stage)
- $f_u$ = undesired output response at frequency $f_u$
- $P_o$ = level of $f_d$ at output of stage (in dB)
- URR = difference in power level between $f_d$ and $f_u$ at output of stage (in dB).

$P_o$ and URR measurements should be made in the absence of output selectivity. See Appendix I.

Note that URR defines a rejection ratio between the fundamental and undesired response at the output of the stage. For receivers work the rejection ratio is defined as the difference between input signals necessary to produce the same output level. Therefore, an expression is needed for the rejection ratio referenced to the input of the stage URR. Consider the coordinates shown in Fig. 4.

Fig. 3. Spectrum analyzer display of desired and undesired responses at the output of a stage.

Fig. 4. Relationship between IP and output responses of a stage.
Then from the definition of slope,

\[ m = (m - 1) a_0 / U_R. \]

or

\[ U_R = (m - 1) x / m. \]

But

\[ d = P_0 - P_0 = [P_1 + G - P_0]. \]

Therefore

\[ U_R = [(m - 1) / m] [P_1 + G - P_0]. \]

But

\[ P_0 = P_1 + G. \]

Hence

\[ U_R = [(m - 1) / m] [P_1 + G - P_1 - G] \]

or

\[ U_R = \alpha [P_1 - P_1] \quad \text{(dB)} \]

where

\[ \alpha = (m - 1) / m. \]

In (4),

- \( U_R \) = Undesired response rejection ratio at input of stage (in dB)
- \( P_1 \) = Input intercept point of stage for Undesired response (in dBm)
- \( P_I \) = Input power level to stage (in dBm)

A summary of the previous derivations is shown in Fig. 5. These results form the basis for intercept point calculations.

III. DETERMINING THE SLOPE OF THE UNDESIRABLE RESPONSE

Power Series and Subexpansion

Consider a stage having a transfer function given by the power series expansion:

\[ x = [f(x)] - a_0 (x)^0 + a_1 (x)^1 + a_2 (x)^2 + \cdots a_n (x)^n \quad \text{or} \quad x = A + B + C + \cdots. \]

Let \( x \) be a sum of signals, \( A, B, C, \cdots \), appearing at the input:

\[ x = A + B + C + \cdots. \]

Then the \( n \)th order term of the expansion, (6), has the form

\[ a_n (x)^n = a_n [A + B + C + \cdots]^n. \]

Consider the case where \( x = A + B \); then the \( n \)th order term in (6) becomes

\[ a_n (x)^n = a_n [A + B]^n \]

\[ = a_n k_0 (A)^n + k_1 (A)^{n-1} B + k_2 (A)^{n-2} B^2 \]

\[ + \cdots + k_{n-1} A B^{n-1} + k_n B^n \] (7)

where

\[ k_i (i = 0, 1, 2, \cdots n) \] are the expansion coefficients.

Let the expansion (7) of the \( n \)th order term in (6) be called a subexpansion, and consider the \( i \)th term of this subexpansion. Let the powers of \( A \) and \( B \) in this term be designated by \( P_{IA} \) and \( P_{IB} \) so that the \( i \)th term of (7) becomes

\[ a_n k_i [P_{IA}]^{i-1} [P_{IB}]. \]

(8)

To determine the slope of the undesired response, the order \( n \) of the term in the power series expansion from which it comes (6) must first be identified. Once the order of this term is known, a subexpansion of the term is performed as in (7). From this subexpansion, the term which produces the undesired response must be identified. The term in the subexpansion which produces the undesired response will consist of a combination of the input signals, each raised to specific powers. Depending on the purpose of the stage, some of the input signals may be allowed to vary in input level while others may remain fixed. For example, a mixer may have two input signals applied to it, one being the desired radio frequency (RF) signal and the other being the local oscillator (LO) signal. Typically the RF signal will vary in input level while the LO signal will be held at a fixed input level. It is necessary to identify these input signals which will vary and those which will remain fixed.

Calculating the Slope

Once the term in (7) which produces the undesired response has been identified, the slope of the undesired response may be found by expressing this term in dB. An example is shown below for the \( i \)th term of the subexpansion (7). Let the \( i \)th term be the term which produces an undesired response:

\[ \text{Undesired response} = a_n k_i [P_{IA}]^{i-1} [P_{IB}]. \]
In dB this may be expressed as

$$\text{undesired response (dB)} = 20 \log \left[ a_x a_y A^2 B \right]$$

$$- 20 \log [a_x, a_y]$$

$$+ \left[ \Phi_x (4 \text{ in dB}) \right]$$

$$+ \left[ \Phi_y (2 \text{ in dB}) \right]$$

$$= \text{constant}$$

$$+ \left[ \Phi_x (A \text{ in dB}) \right]$$

$$+ \left[ \Phi_y (B \text{ in dB}) \right]$$

$$= (9)$$

The slope $m$ of the undesired response is found from (9) by summing the $\Phi$'s of the terms which are allowed to vary. For example, if signal $A$ can vary and increases by 1 dB, the undesired response would increase by $\Phi_x$ dB for a slope of $\Phi_x$. If signals $A$ and $B$ can vary and if both increase by 1 dB, the undesired response will increase by $\Phi_x + \Phi_y$ dB for a slope of $\Phi_x + \Phi_y$.

Examples

The calculation of the slopes for some common receiver-related undesired responses is shown in the following examples.

Example 1: Third-Order Intermodulation Response

a) Third-Order Intermodulation in an Amplifier: Power series expansion (6):$$f(x) = a_0(x)^0 + a_1(x)^1 + a_2(x)^2 + a_3(x)^3 + \cdots$$

Two input signals at frequencies $\Delta f$ and $2\Delta f$ from the on-channel input frequency are involved in producing a third-order intermodulation (IM) response in an amplifier [11, [2]. Let these two signals be designated by $A$ and $B$, where

$$A = f + \Delta f$$

$$B = f + 2\Delta f$$

and

$$x = A + B.$$  

The power series expansion becomes

$$f(x) = a_0(A + B)^0 + a_1(A + B)^1$$

$$+ a_2(A + B)^2$$

$$+ a_3(A + B)^3 + \cdots$$

(10)

The IM response is produced by the third-order term in (10). Evaluating the subexpression of this term gives

$$a_3[A^3 + 3A^2B + 3AB^2 + B^3]$$

or

$$a_3A^3 + 3a_2A^2B + 3a_1AB^2 + a_3B^3.$$  

(11)

The underlined term in the sub-expansion, (11), produces the IM response. To determine the slope $m$ of this response, express this term in dB.

$$\text{IM response (dB)} = 20 \log [3a_3A^2B]$$

$$= 20 \log [3a_3] + 20 \log [A^2]$$

$$+ 20 \log [B]$$

$$= \text{constant} + 2A(\text{dB})$$

$$+ [B (\text{dB})].$$

Since signals $A$ and $B$ are both allowed to vary, a 1-dB increase in $A$ and $B$ results in a 3-dB increase in undesired IM response. Hence for third-order IM in an amplifier stage,

$m = 3$.

Knowing the slope is useful when evaluating $\Phi_o$ (see Section II), because for every 1-dB change in the two input signals, the IM response seen on the spectrum analyzer should change by 3 dB. If it does not, something other than a third-order IM response is being observed. Note that $\Phi_o$ is observed at the on-channel frequency of the amplifier stage.

b) Third-Order Intermodulation in a Mixer: Power series expansion (6):$$f(x) = a_0(x)^0 + a_1(x)^1 + a_2(x)^2 + a_3(x)^3 + a_4(x)^4 + \cdots$$

Two input signals at frequencies $\Delta f$ and $2\Delta f$ from the on-channel input frequency as well as the LO signal are involved in producing a third-order IM response in a mixer [2], [3]. The two input signals can vary in amplitude, whereas the LO signal is usually held at a fixed amplitude. Let these signals be designated by $A$, $B$, and $C$, where

$$A = f + \Delta f$$

$$B = f + 2\Delta f$$

$$C = \text{LO signal}$$

and

$$x = A + B + C.$$  

The power series expansion becomes

$$f(x) = a_0(A + B + C)^0 + \cdots$$

$$+ a_4(A + B + C)^4 + \cdots.$$  

(12)

The IM response is produced by the fourth-order term in (12) and also by higher order even terms which are neglected.
The power series expansion becomes

\[ f(x) = a_0(x^0 + a_1(x) + a_2(x)^2 + a_3(x)^3 + a_4(x)^4 + \ldots) \]

where

\[ x = A + B. \]

The half-IF response is produced by the fourth-order term (also higher order even terms which are neglected here) in (14) [2], [3]. Evaluating the subexpansion of this term gives

\[ a_4[A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4] \]

or

\[ a_4A^4 + 4a_4A^3B + 6a_4A^2B^2 + 4a_4AB^3 + a_4B^4. \]

The underlined term in subexpansion (15) produces the half-IF response. To determine the slope \( m \) of this response, express this term in dB:

half-IF response (dB)

\[ = \text{constant} + 2[A \text{ (dB)}] + 2[B \text{ (dB)}]. \]

Since only signals \( A \) and \( B \) are allowed to vary, a 1-dB increase in \( A \) results in a 2-dB increase in undesired IM response. Hence, for third-order IM in a mixer stage,

\[ m = 3. \]

The slope of the third-order IM response is the same whether it is produced in an amplifier stage or in a mixer stage, even though the order of the term in the power series expansion is different for the two stages. The presence of the LO signal in the mixer stage merely converts the on-channel IM product produced by the \( A^2B \) term down to the intermediate frequency (output frequency) of the mixer. When \( IP_a \) is observed on a spectrum analyzer (note that \( IP_a \) will be observed at the intermediate frequency of the mixer), the IM response should change by 3 dB for every 1-dB change in the two input signals.

**Example 2: Half-IF Spur Response**

The half-IF response is produced in a mixer stage by an RF signal that lies halfway between the on-channel input frequency and the LO frequency (half an IF away from the desired input frequency).

From the power series expansion, (6):

\[ f(x) = a_0(x^0 + a_1(x) + a_2(x)^2 + a_3(x)^3 + a_4(x)^4 + \ldots). \]

Two signals are involved in producing the half-IF response: an RF signal and an LO signal. The RF signal is allowed to vary while the LO signal is held constant. Let these signals be designated by

\[ A = \text{RF signal} \]
\[ B = \text{LO signal}. \]

When \( IP_a \) is observed, the half-IF response should be seen to change by 2 dB for every 1-dB change in the RF signal. Note that \( IP_a \) is observed at the intermediate frequency of the mixer.

**Example 3: Seventh-Order Ablle-Baker Response**

When harmonics of the RF and LO signals in a mixer stage differ by unity (\( |p - q| = 1 \); see [2]), spurious responses can be generated by signals which fall very close to the desired on-channel input frequency. These spurious responses are sometimes referred to as Able-Baker spurs. Consider the seventh-order Able-Baker spur; i.e., \( |p - q| = 1 \) and \( p + q = 7 \).

From the power series expansion, (6):

\[ f(x) = a_0(x^0 + a_1(x) + a_2(x)^2 + a_3(x)^3 + a_4(x)^4 + \ldots) \]

Two signals are involved in producing the seventh-order Able-Baker (7AB) response: an RF signal and an LO signal.

\[ a_4[A^4 + 4a_4A^3B + 6a_4A^2B^2 + 4a_4AB^3 + a_4B^4]. \]
The RF signal is allowed to vary while the LO signal is held constant. Let these signals be designated by

\[ A = \text{RF signal} \]
\[ B = \text{LO signal}. \]

The power series expansion becomes

\[ f(x) = a_0(A + B)^0 + a_1(A + B)^1 + \cdots + a_m(A + B)^m + \cdots \]

where

\[ x = A + B. \]

The seventh-order Able-Baker response is produced by the seventh-order term (also higher order odd terms which are neglected here) in (16). Evaluating the subexpression of this term gives

\[ a_7A^7 + 7a_7A^6B + 21a_7A^5B^2 + 35a_7A^4B^3 + 35a_7A^3B^4 + 21a_7A^2B^5 + 7a_7AB^6 + B^7 \]

The undelineed terms in this subexpression produce seventh-order Able-Baker responses. To determine the slopes of these responses, consider each term separately.

**Case 1:** \( A^6B^3 \) term (for high-side injection (HSI) [2]). Expressing this term in dB:

\[ 7AB \text{ response (dB)} \]
\[ = 20 \log [35a_7A^6B^3] \]
\[ = 20 \log [35a_7] + 20 \log [A^6] + 20 \log [B^3] \]
\[ = \text{constant} + 4[A \text{ (dB)}] + 3[B \text{ (dB)}]. \]

Since only signal \( A \) is allowed to vary, a 1-dB increase in \( A \) produces a 4-dB increase in the \( 7AB \) response. Hence for the \( A^6B^3 \) term,

\[ m = 4. \]

When \( IP_0 \) is observed (at the intermediate frequency of the mixer), the \( 7AB \) response should change by 4 dB for every 1-dB change in RF signal.

**Case 2:** \( A^5B^4 \) term (for low-side injection (LSI) [2]). Expressing this term in dB:

\[ 5AB \text{ response (dB)} \]
\[ = 20 \log [35a_7A^5B^4] \]
\[ = 20 \log [35a_7] + 20 \log [A^5] + 20 \log [B^4] \]
\[ = \text{constant} + 3[A \text{ (dB)}] + 4[B \text{ (dB)}]. \]

Since only signal \( A \) is allowed to vary, a 1-dB increase in \( A \) produces a 3-dB increase in the \( 7AB \) response. Hence for the \( A^5B^4 \) term,

\[ m = 3. \]

When \( IP_0 \) is observed (at the intermediate frequency of the mixer), the \( 7AB \) response should change by 3 dB for every 1-dB change in RF signal.

A summary of the above examples is contained in Table I.

### IV. CASCADED IP\(_1\)S AND SELECTIVITY

**Cascaded Intercept Formula**

It can be shown that \( IP_1 \)s of cascaded stages may be combined according to

\[ \frac{1}{IP_{tot}} = \left( \frac{1}{IP_{1}} \right)^q + \left( \frac{G_1}{IP_{2}} \right)^q + \left( \frac{G_2G_3}{IP_{3}} \right)^q + \cdots \right)^{1/q} \quad \text{(numeric)} \]

where

\[ q = (m - 1)/2 \]

and

\[ m \]

is the slope of the undesired response.

Note that \( IP_1 \) and \( G \) are expressed in numeric values rather than in dB. Intercept points of stages which do not contribute to the undesired response are considered to be infinite. Equation (18) allows the calculation of an overall system \( IP_1 \), which may then be used along with the system input sensitivity level, \( P_1 \), to find the system rejection ratio of the undesired response according to (4):

\[ \text{U} \times 10^{-\text{alpha} \times (P_1 - P_1)} \]

\[ \text{System input sensitivity level} \]

\( (20 \text{ dB quiting} \text{, etc.}) \text{ (dBm)} \)

\[ \text{System input intercept point (dBm)} \]

\[ \text{System response rejection ratio (dB)} \]

An example follows.

**Example 4:** Find the system IM intercept point and calculate the system IM rejection ratio (IMr), given the following stages:

\[ \begin{align*}
P_1 &= \text{IM intercept point} \\
G_1 &= 10 \text{ dB} \\
G_2 &= 7 \text{ dB} \\
G_3 &= -6 \text{ dB} \\
P_{tot} &= 7 \text{ dBm} \\
P_1 &= 10 \text{ dBm} \end{align*} \]

\[ \text{IP}_{\text{IM}} = ? \]

\( \text{IMr} = ? \)

See Appendix II for derivation.
Solution:
Converting gains and intercept points to numeric values gives

<table>
<thead>
<tr>
<th>dB</th>
<th>Numeric</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 dB</td>
<td>10</td>
</tr>
<tr>
<td>4 dB</td>
<td>2.51</td>
</tr>
<tr>
<td>-6 dB</td>
<td>0.251</td>
</tr>
<tr>
<td>12 dBm</td>
<td>0.0138</td>
</tr>
<tr>
<td>7 dBm</td>
<td>0.00501</td>
</tr>
<tr>
<td>17 dBm</td>
<td>0.00901</td>
</tr>
</tbody>
</table>

For third-order IM,

\( m = 3 \)

\( q = 1 \)

and (18) becomes:

\[
\frac{1}{IP_{3,01}} = \frac{1}{IP_{3,sys}} = \frac{1}{IP_{1,4} + G_1/IP_{3,2} + G_2/G_3/IP_{3,3}} \quad \text{(numeric)}
\]

\[
= \frac{1}{0.0138 + 10/0.00501 + (10)(2.51)/0.00501}
\]

\[
= 2.56 \times 10^3
\]

\[
IP_{3,sys} = \frac{1}{(2.56 \times 10^3)} = 0.391 \times 10^{-3} \quad \text{(numeric)}
\]

\[
= -4.08 \text{ dBm}
\]

From (4)

System IMR

\[
= (2/3)(IP_{sys,1} - P_i)
\]

\[
= (2/3)[-4.08 \text{ dBm} - (-110 \text{ dBm})]
\]

\[
= (2/3)[-4.08 + 110]
\]

\[
= 70.6 \text{ dB}
\]

If the stages in the above example comprised the blocks of a receiver, this would be the IM performance level of the receiver.

Effect of Input Selectivity on Intercept Point

Consider an input signal to a stage at a spurious frequency \( f_1 \) which generates an undesired response at frequency \( f_2 \) at the output of the stage (see Fig. 6).

Adding \( \Delta dB \) of input selectivity at \( f_1 \) reduces its input level to the stage by \( \Delta dB \) from \( P_{t1} \) to \( P_{t1}' \). Hence the undesired output response is also reduced. To bring the undesired output response back up to its previous level (to produce 20 dB quieting in an IM receiver, for example), the level of \( f_2 \) at the output of the selectivity must be increased by \( \Delta dB \). Since the input level required to produce the same output has now increased by \( \Delta dB \), the rejection ratio has been increased by \( \Delta dB \). Hence a dB-per-dB relationship exists between rejection ratio and input selectivity when the stage plus selectivity are considered as one equivalent stage, and an intercept point \( IP_{1}' \) may be defined for the equivalent stage. This \( IP_{1}' \) reflects the improvement in the undesired response rejection ratio, \( UR_{t1} \), due to the addition of selectivity. Fig. 7 shows a stage with intercept point \( IP_{1,2}' \) preceded by a stage of selectivity.

An expression for \( IP_{1}' \) can be found from (4):

\[
UR_{t1} = \alpha [IP_{1}' - P_i]
\]

Assuming a loadless passband response \( (G_1 = 0 \text{ dB in Fig. 7)} \), the input sensitivity level, \( P_i \), does not change with the addition of selectivity. Therefore holding \( P_i \) constant in (4) while allowing \( IP_{1}' \) to vary gives

\[
\Delta UR_{t1} = \alpha \Delta IP_{1}'
\]
Stage preceded by lossy selective stage:
\[ IP'_1 = IP_{t_2} + \left(\frac{1}{a}\right)[\Delta \text{dB selectivity}] - G_1 \quad \text{(dB).} \quad (19c) \]

Effect of Loss on URR:

When a stage (or the input to a system of cascaded stages) is preceded by a lossy nonselective stage, \( IP'_{\text{tot}} \) improves \( \Delta \text{dB for} \Delta \text{dB with added loss (19b), but in order to maintain the same signal power at the output, } P_t \text{ must also increase } \Delta \text{dB for} \Delta \text{dB. Hence the increases in } IP'_{\text{tot}} \text{ and } P_t \text{ offset each other in (4), and } \text{URR does not change.} \]

When working with intercept points and cascaded stages, it is important to remember the following:
\( P_t \) equals on-channel input sensitivity or reference level.
Lossless selectivity at \( f_2 \) does not change \( P_t \).
\( G \) equals on-channel stage gain; where \( \Delta \text{dB of selectivity at } f_2 \) is added to an existing stage, the on-channel gain of the equivalent stage \( G' \) may be found as
\[ G' = G_1 + G_2 \quad \text{(dB).} \quad (19d) \]

Stage contributing to an undesired response which occur in a system prior to a stage of selectivity will degrade the effectiveness of the selectivity on the system URR.

When input selectivity is added to a stage, the intercept point of the equivalent stage is found according to (19a) or (19c), and the gain of the equivalent stage is found according to (19d). The cascaded intercept formula, (18), may then be used to find the system intercept point.

Example 5: Consider the following system:

The mixer (Stage 5) generates an undesired response from a signal \( f_2 \), which can enter the front-end via the antenna. To improve the rejection of the system to \( f_2 \), a stage of lossless selectivity is added in front of the mixer as shown. Assume that the first three stages do not contribute to the generation of the undesired response, hence their \( IP'_t \)'s are considered infinite. Find the intercept point of the system \( IP'_{\text{tot}} \) and
From (18), with \( m = 2 \) and \( \alpha = 1/2 \),
\[
\frac{1}{\text{IP}_{\text{tot}}} = \frac{1}{\text{IP}_{\text{sys}}}
\]
\[
\frac{1}{\text{IP}_{\text{sys}}} = \left( \frac{1}{\text{IP}_{11}} \right)^{1/2} + \left( \frac{G_1}{\text{IP}_{12}} \right)^{1/2} + \left( \frac{G_1 G_2}{\text{IP}_{13}} \right)^{1/2}
\]
\[
+ \left( \frac{G_1 G_2 G_3}{\text{IP}_{15}} \right)^{1/2} \right)^2
\]
\[
\frac{1}{\text{IP}_{15}} = \left[ \left( \frac{1}{\infty} \right)^{1/2} + \left( \frac{G_1}{\infty} \right)^{1/2} + \left( \frac{G_1 G_2}{\infty} \right)^{1/2}
\]
\[
+ \left( \frac{G_1 G_2 G_3}{\text{IP}_{15}} \right)^{1/2} \right)^2
\]

\[
\text{IP}_{\text{sys}} = \frac{\text{IP}_{15}'}{G_1 G_2 G_3} \quad \text{(numeric)}.
\]

Since the right side consists of only one term, it may be conveniently expressed in dB as
\[
\text{IP}_{\text{sys}} = \text{IP}_{15}' - G_1 - G_2 - G_3 \quad \text{(dB)}
\]
\[
= 50 \text{ dBm} - (-2 \text{ dB}) - (-12 \text{ dB}) - (-4 \text{ dB}) = 36 \text{ dBm}
\]

The system half-IF rejection ratio, \( \text{IP}_1 \), is now calculated by means of (4):
\[
\text{IP}_1 = \frac{1}{2} (\text{IP}_{\text{sys}} - \text{IP}_2)
\]
\[
= \frac{1}{2} \left[ 36 \text{ dBm} - (-110 \text{ dBm}) \right]
\]
\[
= \frac{1}{2} (36 + 110)
\]
\[
= 73 \text{ dB}.
\]

If the stages shown in this example comprised the blocks of a receiver, then the system \( \text{IP}_1 \) calculated above would be the half-IF spur rejection ratio of the receiver.

Example 6: Suppose Stage 2 now contributes to the half-IF response of the system in the previous example. Find \( \text{IP}_{\text{sys}} \) and calculate the system \( \text{IP}_1 \).

---

Fig. 7. Equivalent intercept point of a stage with selectivity.

calculate the rejection ratio UIRr of the system to this undesired response. For this example, let the undesired response be the half-IF spur.

**Solution:** From Table 1 the half-IF response has the following properties:

\( m = 2 \)

\( \alpha = 1/2 \).

Equation (4) becomes

\[
\text{UIRr} = \text{IP}_1 = (1/2)(\text{IP}_1 - \text{IP}_2)
\]

This expression gives the half-IF spur rejection ratio (\( \text{IP}_1 \)) for a single stage. After the selectivity is accounted for, stage \( \text{IP}_2 \)'s may be combined according to (18) to calculate the overall half-IF rejection ratio for the system.

Combining Stages 4 and 5 to account for the effect of selectivity on \( \text{IP}_{15} \),

\[
\text{IP}_{15}' = \text{IP}_{15} + (1/e)(\Delta \text{ dB added selectivity})
\]

\[
= 30 \text{ dBm} + 2(10 \text{ dB})
\]

\[
= 50 \text{ dBm}
\]
Solution: Stages 4 and 5 are combined to give an equivalent intercept point as in the previous example:

\[ IP_{i4}' = 50 \text{ dBm}. \]

Converting gains and intercept points to numeric values:

<table>
<thead>
<tr>
<th>( i )</th>
<th>( 1/dB )</th>
<th>Numeric</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_1 )</td>
<td>2.43</td>
<td>0.631</td>
</tr>
<tr>
<td>( G_2 )</td>
<td>12 dB</td>
<td>15.849</td>
</tr>
<tr>
<td>( G_3 )</td>
<td>4 dB</td>
<td>2.512</td>
</tr>
<tr>
<td>( IP_{i2} )</td>
<td>17 dBm</td>
<td>0.0501</td>
</tr>
<tr>
<td>( IP_{i5}' )</td>
<td>30 dBm</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\[
\frac{1}{IP_{i4}} = \frac{1}{IP_{i5}'}
\]

\[
\frac{1}{IP_{i5}'} = \left[ \left( \frac{1}{IP_{i1}} \right)^{1/2} + \left( \frac{G_1}{IP_{i2}} \right)^{1/2} \right] \left[ \left( \frac{G_1 G_2}{IP_{i3}} \right)^{1/2} + \left( \frac{G_1 G_2 G_3}{IP_{i5}'} \right)^{1/2} \right]^2
\]

\[
= \left[ \left( \frac{1}{11} \right)^{1/2} + \left( \frac{0.631}{15.849} \right)^{1/2} \right] \left[ \left( \frac{0.631}{2.512} \right)^{1/2} + \left( \frac{0.631 \cdot 15.849 \cdot 2.512}{1.00} \right)^{1/2} \right]^2
\]

\[
= 1/73.3
\]

\[
= 0.0136 \quad \text{(numeric)}
\]

\[
IP_{i5}' = 11.3 \text{ dBm}
\]

System IF = \((1/2)(IP_{i5}' - P_i) = (1/2)[11.3 - (-110)] = (1/2)(11.3 + 110) = 60.7 \text{ dB}.

The effects of selectivity at Stage 4 are reduced due to the half-IF contribution of Stage 2. Selectivity could be added ahead of Stage 2 to bring the system IF back up to its previous level.

CONCLUSION

Undesired responses generated by the non-linear properties of electronic devices may be analyzed by means of power series expansions and intercept points. In a system consisting of cascaded stages, IP's of the same order may be combined to give an overall system intercept point. This system IP may then be used to calculate the system rejection to an undesired response of a given order.

When applied to receiver design, the concept of intercept point allows calculation of the receiver rejection to various distortion and spurious responses, such as IM and mixer spurs. By considering stage gains, selectivity, and device intercept points, a receiver system design may be analyzed on paper and the distortion and spurious response performance predicted.

Adjustments may then be made to individual stages in the system so that the receiver will meet its desired performance goals.

APPENDIX I

\( P_o \) and URR measurements must be made at the output of a stage in the absence of output selectivity. A mixer, however, will often have a resonant circuit at the output which will be tuned to \( f_d = f_{IF} \). The selectivity at \( f_d \) presented by this resonant circuit will distort the URR ratio by making URR appear larger than it really is. A method of measuring URR which overcomes this problem is shown below. The desired output at \( f_{IF} = f_d \) is first generated and \( P_o \) recorded.

The desired output at \( f_d \) should be centered in the passband of the output selectivity. This will cause \( f_d \) to be unattenuated so that its output level can be properly measured. After recording \( P_o \) at \( f_d \), the frequencies of the RF and LO signals are adjusted to put \( f_d \) in the center of the passband while maintaining the same frequency difference between \( f_d \) and \( f_{IF} \).
From (3):
\[ \beta = \frac{1}{m - 1} \]

where

\( m \) is the slope of the undesired response.

Substituting this expression for \( \beta \) into (21) gives:
\[
P_u = P_d \left[ 1 + m - 1 - IP_o (m - 1) \right] = mP_d - (m - 1)IP_o \tag{3B}.
\]

In numeric form, (22) becomes:
\[
P_u = P_d^m IP_o (m - 1). \tag{23}
\]

Equation (23) shows the relationship between output intercept point and the undesired power output from the \( m \)th order response.

Now consider the following system of two cascaded stages:

From (23)
\[
P_u = \frac{P_d^m}{IP_o (m - 1)}
\]

where

\( P_d \) = desired output power from fundamental response in Stage 1
\( P_{u1} \) = undesired output from \( m \)th order response in Stage 1
\( P_{d2} \) = desired output from fundamental response in Stage 2
\( P_{u2} \) = undesired output from \( m \)th order response in Stage 2.

But
\[
P_{d2} = P_d G_2.
\]

Hence
\[
P_{d1} = \frac{P_{d2}}{G_2}.
\]

and
\[
P_{d1}^m = P_{d2}^m G_2^m.
\]
The undesired output at $R_L$ from Stage 1 is

$$P_{u1}G_2 = \frac{P_{d1}mG_2}{I_{c1}^{(m-1)}} - \frac{P_{d2}mG_2}{G_2mIP_{c1}^{(m-1)}}$$

$$= \frac{P_{d2}m}{G_2^{(m-1)}IP_{c1}^{(m-1)}}.$$  

The undesired output at $R_L$ from Stage 2 is

$$P_{u2} = \frac{P_{d2}m}{IP_{c2}^{(m-1)}}.$$  

The undesired voltage at $R_L$ from Stage 1 is

$$V_{u1} = \left[\frac{P_{d2}mR_L}{G_2^{(m-1)}IP_{c1}^{(m-1)}}\right]^{1/2}.$$  

The undesired voltage at $R_L$ from Stage 2 is

$$V_{u2} = \left[\frac{P_{d2}mR_L}{IP_{c2}^{(m-1)}}\right]^{1/2}.$$  

Total undesired voltage at $R_L$ is

$$V_U = V_{u1} + V_{u2} = \sqrt{P_{d2}mR_L}$$

$$= \left[\left(\frac{1}{G_2 IP_{c1}}\right)^{(m-1)/2} + \left(\frac{1}{IP_{c2}}\right)^{(m-1)/2}\right].$$  

Total undesired power across $R_L$ is

$$P_{uT} = \frac{V_U^2}{R_L} = \frac{P_{d2}mR_L}{R_L} \left[\left(\frac{1}{G_2 IP_{c1}}\right)^{(m-1)/2} + \left(\frac{1}{IP_{c2}}\right)^{(m-1)/2}\right].$$  

The definition of URR is

$$URR = \frac{P_{d2}}{P_{uT}} = \frac{1}{P_{d2}^{(m-1)}} \left[\left(\frac{1}{G_2 IP_{c1}}\right)^{(m-1)/2} + \left(\frac{1}{IP_{c2}}\right)^{(m-1)/2}\right].$$  

At the intercept point, $URR = 1$ and $P_{d2} = IP_{c1}\text{tot}$ (for cascaded system). Hence

$$1 = \frac{1}{IP_{c1}\text{tot}^{(m-1)}} \left[\left(\frac{1}{G_2 IP_{c1}}\right)^{(m-1)/2} + \left(\frac{1}{IP_{c2}}\right)^{(m-1)/2}\right].$$  

Taking the square root of both sides:

$$\frac{1}{IP_{c1}\text{tot}^{(m-1)/2}} = \left[\left(\frac{1}{G_2 IP_{c1}}\right)^{(m-1)/2} + \left(\frac{1}{IP_{c2}}\right)^{(m-1)/2}\right].$$  

Let

$$q = (m-1)/2.$$  

Then

$$\frac{1}{IP_{c1}\text{tot}^{q}} = \left[\left(\frac{1}{G_2 IP_{c1}}\right)^q + \left(\frac{1}{IP_{c2}}\right)^q\right].$$  

Raising both sides to the 1/q power:

$$\frac{1}{IP_{c1}\text{tot}} = \left[\left(\frac{1}{G_2 IP_{c1}}\right) + \left(\frac{1}{IP_{c2}}\right)\right]^{1/q}.$$  

Now

$$IP_{c1}\text{tot} = IP_{c1}\text{tot} / G_{1}G_{2}.$$  

Multiplying both sides by $G_{1}G_{2}$:

$$G_{1}G_{2}IP_{c1}\text{tot} = (1/G_{1} IP_{c1}^*) + (1/IP_{c2})^{1/q}G_{1}G_{2}.$$  

But

$$IP_{c1} = IP_{c1}/G_{1}$$

and

$$IP_{c2} = IP_{c2}/G_{2}.$$  

Hence

$$1 = \left[\left(\frac{1}{G_{1} IP_{c1}}\right)^q + \left(\frac{1}{G_{2} IP_{c2}}\right)^q\right]^{1/q}.$$
where
\[ q = (m - 1)/2. \]

Extending the above procedure to more than two stages gives
\[
\frac{1}{P_{\text{alt}}} = \left[ \left( \frac{1}{P_{I_1}} \right)^q + \left( \frac{G_1}{P_{I_2}} \right)^q + \left( \frac{G_1 G_2}{P_{I_3}} \right)^q + \cdots \right]^{1/q},
\]

where
\[ q = (m - 1)/2. \]

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Correction to “Co-Channel Interference and its Avoidance in Close-Spaced Systems”

ARThUR C. STOCKER

The equation on page 146 of the above paper\(^1\) should have read
\[
P_I = \frac{K^2 m^a}{(1 - m^a + K^2 m^a)^2}
\]

where \( a \) is the distance attenuation constant shown in the first paragraph under the section heading Propagation on page 145.

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