Common-Mode Rejection Ratio Redefined

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Abstract—Redefining the common-mode rejection ratio (CMRR), a set of equations is derived for the common-mode gain and the CMRR of differential amplifiers. The results, which are consistent with practice and simulation, show that an amplifier’s CMR may exceed the CMR of the active device(s) by intentional unbalancing of the circuit.

I. INTRODUCTION

It is well known that the common-mode rejection of differential amplifiers is dependent on the degree of balance, or imbalance, between the amplifier’s half circuits. In op-amps, the common-mode rejection ratio (CMRR) is the result of the device’s CMRR, CMRR_d, and the CMRR due to the passive components external to the device, or CMRR_e, as defined by the following equation [1], [2]:

\[
CMRR = \frac{CMRR_d \cdot CMRR_e}{CMRR_d + CMRR_e}
\]

(1)

According to the classical definition of CMRR, CMRR_d, and CMRR_e are both positive quantities and the CMRR is at best equal to the lesser of the two rejection figures. In the ideal difference amplifier where the external resistors are perfectly balanced, i.e., \( R_2/R_1 = R_4/R_3 \) (see Fig. 1), CMRR_d = \( \infty \) and the amplifier’s CMRR is maximized at the device’s CMRR, CMRR = CMRR_d.

In actual practice it is known that the CMRR can exceed the device’s CMRR, however. Instead of tweaking the resistors for maximum CMRR_d, the resistance ratio is slightly unbalanced such that CMRR_e = -CMRR_d. The inconsistency is resolved by not restricting the definition of CMRR to magnitude only.

II. CMRR DERIVATION AND ANALYSIS

When a device such as an op-amp is strapped with feedback, the common-mode gain of the amplifier is altered by the circuitry restricting the definition of CMRR to magnitude only.

Substituting into (2) with \( r = R_2/R_1 \), \( R_2/R_1 \), and solving for \( V_{\text{cem}}/V_m \), the common-mode gain is

\[
A_{\text{cm}} = \frac{1}{1 + \frac{R_2/R_1}{A_v}} \left[ \frac{r - 1}{r + \frac{R_2}{R_1}} + \frac{1}{1 + \frac{R_1}{r R_2}} \right] \cdot \text{CMRR}_d
\]

(4)

if \( A_v \gg 1 \). For resistors that are balanced or slightly out of balance, \( r \approx 1, A_d \approx -R_2/R_1 \) and

\[
A_{\text{cm}} = \frac{1}{1 + \frac{R_2/R_1}{A_v}} \left( \frac{1}{1 + (R_2/R_1)} \right)
\]

(5)

where

\[
\text{CMRR}_e = \left( 1 + \frac{R_2/R_1}{A_v} \right) \left( \frac{1}{1 + (R_2/R_1)} \right) \cdot \text{CMRR}_d
\]

(6)

is the CMRR due to the components external to the device and

\[
\text{CMRR}_e = \frac{1}{1 + \frac{R_2/R_1}{A_v}} \cdot \text{CMRR}_d
\]

(7)

At low and moderate frequencies, \( 1 + R_2/R_1/A_v \approx 1 \).

Rewriting (5), the amplifier’s common-mode rejection ratio is

\[
\text{CMRR} = \frac{CMRR_d \cdot CMRR_e}{CMRR_d + CMRR_e}
\]

(8)

Compare (8) with (1). When CMRR_e ≈ CMRR_d the amplifier’s CMR exceeds the device’s common-mode rejection, the result of defining the CMRR as the ratio of the complex differential to common-mode gains (5)–(7) and not as a ratio of just the magnitudes. In contrast to Garrett [1] and Pallas-Arney and Webster [2], the denominator of the above equation takes the difference instead of the sum of the two CMRR components. The differential and common-mode gains used in the equation are consistent with those used by the latter, however.

The common-mode rejection is

\[
\text{CMRR} = 20 \log |\text{CMRR}| \text{ dB}
\]

(9)

If the resistors are perfectly balanced or \( r = 1 \), CMRR_e = \( \infty \), CMRR_e = CMRR_d and CMRR = -CMRR_d, which is not the maximum CMRR attainable for the amplifier. If the device is ideal, on the other hand, CMRR_d = \( \infty \), CMRR = CMRR_d and the CMRR is maximized when the resistors are perfectly balanced.
To maximize the CMRR for real devices, let $A_{cm} = 0$. From (4),

$$\frac{R_1}{R_3} = \frac{R_2}{R_1} - \frac{(1 + R_2/R_1)(R_4/R_3)}{CMRR_d}$$

(10)

or

$$r = \frac{1}{1 + \frac{R_2}{R_4}}$$

(11)

A. Example

For the difference amplifier with a differential gain of $R_2/R_1 = 99$ and $CMRR_d = 31500$, $r = 0.9968$ and the required resistance ratio $R_4/R_3 = 0.9968(99) = 98.69$. If $R_3 = 1$ kΩ, the circuit is tweaked for $R_4 = 98.69$ kΩ instead of 99 kΩ for maximum common-mode rejection. The actual differential gain is $A_d = -99.32$.

If $R_4 = 100$ kΩ instead of 98.69 kΩ and the resistor imbalance is 1% or $r = 1.010$, the common-mode gain is, from (4), $A_{cm} = 0.01304$ or -37.7 dB. The differential gain is -99.00 and the CMRR drops to -99.00 and a CMR of 77.6 dB.

For $R_2/R_1 = 99.7$, $R_1 = R_3 = 1$ kΩ and $CMRR_d = -31500$, the common-mode gain is

$$A_{cm} = \frac{100 - 99.7}{100} + \frac{1 + 99.7}{100} = -1.95 \times 10^{-1} = -74.2 \text{ dB}$$

Since $r = 100/99.7 = 1.00301$, or a resistance imbalance of 0.3%, $A_d \approx -R_2/R_1 = -99.7 \Rightarrow -40.0 \text{ dB}$ and $CMR = 40.0 - (-74.2) = 114 \text{ dB}$, which exceeds the device’s CMR by 24 dB.

B. Differential Gain $A_d$

The amplifier’s differential gain, assuming a device with large or infinite open-loop gain, is

$$A_d = -\frac{R_2}{R_1} \left( \frac{r + 1}{2} + \frac{R_3}{R_1} \right)$$

(12)

for a balanced-to-ground input signal. For floating inputs, the ideal differential gain is

$$A_d = -\frac{R_2}{R_1} \left( \frac{1 + R_3}{1 + R_3} \right)$$

(13)

In either case it can be seen that $A_d \approx -R_2/R_1$ for slight resistor imbalances where $r \approx 1$.

C. Differential-Input Differential-Output Amplifier

Unlike the single-ended-output amplifier, the differential-output amplifier has, in addition to the classically defined common-mode (or average) output voltage, a differential component of the input common-mode signal.

The classically-defined output CM voltage is

$$V_{ocm} = \frac{V_{o1} + V_{o2}}{2} = \frac{(A_{cm} + A_{cmh})V_{icm}}{2}$$

(14)

The differential output CM voltage is

$$V'_{ocm} = V_{o1} - V_{o2} = (A_{cm} - A_{cmh})V_{icm}.$$  

(15)

Fig. 2. A differential-input differential-output amplifier.

Fig. 3. A two-stage differential amplifier.

For an amplifier with ideal symmetry, $A_{cm} = A_{cmh}$ and $V_{ocm} = V_{ocm1} = 0$, and $V_{ocm2} = A_{cm}V_{ocm1}$.

If the differential-output amplifier is followed by a second differential amplifier as shown in Fig. 3, the final output CM voltage is

$$V_{o1} = A_{cm}V_{ocm1} + A_{cm}V_{ocm1}$$

(16)

where $V_{o1}$ is the output common-mode or average voltage from the first stage and $V_{ocm1}$ is the output differential voltage of the first stage due to the input common-mode voltage.

The output CM voltage of stage 1 is

$$V_{ocm1} = A_{cm1}V_{icm1}$$

where

$$A_{cm1} = \frac{A_{cm} + A_{cmh}}{2}.$$  

The differential output common-mode voltage of stage 1 is

$$V'_{ocm1} = V_{o1} - V_{o2} = \Delta A_{cm1}V_{cm1}$$

where

$$\Delta A_{cm1} = A_{cm} - A_{cmh}.$$  

Equation (16) becomes

$$V_{ocm2} = (A_{cm1}A_{cm2} + \Delta A_{cm1}A_{o2})V_{ocm1}$$

and the common-mode gain is

$$A_{cm} = \frac{V_{ocm2}}{V_{ocm1}} = A_{cm1}A_{cm2} + \Delta A_{cm1}A_{o2}.$$  

(17)

The differential gain is approximately $A_d = A_{d1}A_{o2}$ and

$$\frac{1}{CMRR} = \frac{A_{cm1}A_{cm2} + \Delta A_{cm1}}{A_{cm2}}$$

(18)

where

$$CMRR' = A_{d1} = \frac{1}{\Delta A_{cm1}}.$$  

It should be noted that $CMRR = CMRR_1 CMRR_2$ only if $CMRR_1 \gg CMRR_1 CMRR_2$, or $A_{cm} \approx A_{cmh}$.

If $A_{cm} = A_{cmh}$, $CMRR_1 = CMRR_2 = CMRR_1 CMRR_2$. If $A_{cm} \neq A_{cmh}$, $CMRR_1 \approx CMRR_2$. If $CMRR_1 \ll CMRR_2$, $CMRR_1 \approx CMRR_2$.
D. Maximum CMRR

The amplifier’s CMRR is maximized by equating the right side of (18) to zero. Therefore, let

\[
\text{CMRR}_2 = - \frac{\Delta A_{cm2}}{\Delta A_{cm1}}.
\]

For the 3-op-amp instrumentation amplifier, CMRR = \( A_{d1} \) and

\[
\text{CMRR}_2 = - \frac{\Delta A_{cm2}}{\Delta A_{cm1}} = - \frac{1}{\Delta A_{cm1}}.
\]

For an ideal amplifier where \( \Delta A_{cm1} \) is zero, CMRR = \( \frac{49}{50} \)

\[
\text{CMRR}_2 = - \frac{\Delta A_{cm2}}{\Delta A_{cm1}} = - \frac{1}{\Delta A_{cm1}}.
\]

For the 3-op-amp instrumentation amplifier, CMRR = \( \frac{49}{50} \)

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\]

III. EXPERIMENTAL RESULTS

Using an LM747CN strapped with balanced resistors \((r = 1)\) and a differential gain of 500, the measured common-mode gain is \(7.00 \times 10^{-3} \) V/V at low frequencies (10 Hz), \( R_1 = R_3 = 1.0018 \) kΩ and \( R_2 = R_4 = 500.5 \) kΩ. The common-mode rejection ratio for the device is, therefore, CMRR = \( 500 / 7.00 \times 10^{-3} = 71430 \), and \( CMRR = 97.07 \) dB, which agrees with the published data.

Tweaking the circuit for minimum common-mode gain by adjusting resistor \( R_1 \) to unbalance the resistive network, the common-mode gain decreases to \( 8.75 \times 10^{-4} \) at \( R_1 = 497.3 \) kΩ. The CMRR increases to \( 5.714 \times 10^5 \), or a CMR of 115.1 dB, an improvement of 18.0 dB over the device’s common-mode rejection.

The measured \( R_1 \) is equal to the expected theoretical value. For maximum CMRR, the resistance ratio is, from (11), \( r = 1/(1 + 500.6/71430) = 0.993 \), and \( R_4 = r R_3 (R_2/R_1) = 0.993(500.5) = 497 \) kΩ.

IV. CONCLUSIONS

The study shows that, by redefining the common-mode rejection ratio to include the phase angle, the theoretically derived CMRR is consistent with practice and simulation; that an amplifier’s CMR may exceed the CMR of the device(s) or the external components. The improved CMR is achieved by the unbalancing of the components external to the device.

REFERENCES


Correction to “Comments on ‘The Modulo Time Plot: A Useful Data Acquisition Diagnostic Tool’”

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Fig. 1 Mathcad simulation: spurious components due to quantization.

In the above paper\(^1\), an error occurred in Fig. 1. An important part of the figure (the reordering of raw samples) was omitted. Shown here is the correct figure in its entirety.