PhD Course: Performance & Reliability Analysis of IP-based mobile Communication Networks
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• Day 1 Basics Modeling approaches & bursty traffic models (HPS) [NJ14 4-117]
• Day 2 Wireless Link Models & Network Models (HS) [??]
• Day 3 Network Models contd (HS), Long-Range Dependence (HPS) [A5-006]
• Day 4 Simulation Techniques, Rare Event Simulation (SA) [A6-308]
• Day 5 Dependability Modeling (AB) [NJ14, 3-117]

Organized by HP Schwefel & H Schiøler

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Notation (Summary)

- Matrices: underlined capitals: $\mathbf{B}, \mathbf{V}, \ldots$
  - Unit Matrix: $\mathbf{I} = \text{diag}(1, 1, \ldots, 1)$
- Row vectors: underlined, lower-case: $\mathbf{p}, \mathbf{u}, \ldots$
- Column vectors: primed
  - $\mathbf{\varepsilon}' = [1, 1, \ldots, 1]'$
- Random Variables: Capitals: $X, U, \ldots$
- Expected Values: $E\{X\}, E\{X^2\}, \ldots$
- Coefficient of correlation: $r(X, Y) = E\{(X - E\{X\})(Y - E\{Y\})\} / (\text{std}(X) \cdot \text{std}(Y))$
- Autocorrelation coefficient (process $(X_i)$): $r(k) = r(X_i, X_{i+k})$
- Queueing Systems:
  - Infinite buffer: $G/G/1$
  - Finite loss systems: $G/G/1/B$
  - Finite number of customers: $G/G/1//K$

Matrix Exponential Distributions

- Replace single state with open system $S$ containing $m$ states, described by
  - Entrance vector $\mathbf{p} = [p_1, \ldots, p_m]$
  - State leaving Rates: $\mathbf{M} = \text{diag}(\mu_1, \ldots, \mu_m)$
  - Transition Probabilities: $\mathbf{P} = (p_{ij})$
  - Note: $\mathbf{q} = \mathbf{\varepsilon}' - \mathbf{P}\varepsilon' \neq 0$ (in contrast to discrete time Markov chains)
- Interested in Random Variable $X := \text{time to leave } S$ entered according to $\mathbf{p}$
Matrix Exponential Distributions: Properties

- Mean Time to leave S: \( E\{X\} = pV' \)
  where \( B = M(I - D), Y := B^{-1} \)
  - Trivial Example: Exponential distribution, \( E(X) = \frac{1}{\mu} \)

- Distribution of X: \( R(x) := P(X > x) = p \exp(-xB)e' \)
  - Density: \( f(x) = -\frac{dR(x)}{dx} = p \exp(-xB)e' \)

- k-th Moments: \( E\{X^k\} = k! pV^k e' \)

- Matrix Exponential Distributions \( \supset \) Phase-Type Distributions

Examples I: Hyper-exponential Distributions

- Weighted Mixture of two (or m) exponentials:
  \( R(x) = p_1 e^{-\mu_1 x} + (1 - p_1) e^{-\mu_2 x} \)

- Moments:
  \( E\{X\} = \frac{p_1}{\mu_1} + \frac{1-p_1}{\mu_2} = \frac{1}{\mu_1} + \frac{1}{\mu_2} \)
  \( E\{X^2\} = 2 \left( \frac{p_1}{\mu_1^2} + \frac{1-p_1}{\mu_2^2} \right) \) \( \Rightarrow C^2 > 1 \)

  \( \Rightarrow \) frequently used to approximate distributions with high variance

- Matrix-Exponential Representation: \( \langle p, B \rangle \)
Examples II: Erlangian Distributions

- Sum of T exponentially distributed RVs
- Probability density function:
  \[ f(x) = \mu (\mu x)^{T-1} e^{-\mu x} / (T-1)! \]
- Moments:
  \[ E\{X\} = E\{X^2\} = \frac{T}{\mu} \]
  \[ \Rightarrow C^2 = \frac{1}{T} < 1 \]
- Limit (T→∞, μ→1/T): deterministic RV
- Matrix-Exponential Representation

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### M/G/1 and M/ME/1 Queueing Models

- Poisson Arrivals (rate $\lambda$)
- Service times general (ME) distributed (i.i.d. RVs $X_i$, described by $<p, B>$)
  - Utilization: $\rho = \lambda \cdot E(X)$ \hspace{1cm} \[ Pr(Q=0) = 1 - \rho \text{ for infinite buffer model, as we will see later} \]
- Solution Approaches:
  - Embedded Markov Chain (see e.g. Kleinrock)
  - Pollaczek-Khinchin Formula:
    \[ E(Q) = \frac{\rho}{1 - \rho} \left[ 1 + \rho \cdot \frac{C^2 - 1}{2} \right] \]
  - Quasi-Birth Death Processes \rightarrow existence of Matrix-geometric solution
  - Limit of finite-customer systems: M/ME/1//N models

### M/ME/1//N Queueing Systems: steady state queue-length distribution

- Define
  - $r(n; N) := Pr(n \text{ customers at } S_1)$
  - $(p(n; N))_i := Pr(n \text{ customers at } S_1 \text{ & server } S_1 \text{ in state } i)$
- Balance Equations
  \begin{align*}
  (1) \quad & n = 0 \quad \lambda p(0; N) = p(1; N)B^p \\
  (2) \quad & n = N \quad \lambda p(N; N) = \lambda p(N - 1; N)Y \\
  (3) \quad & n = 1, \ldots, N - 1 \quad \lambda p(n; N)(B + Y) = p(n + 1; N)B^p + p(n - 1; N)Y
  \end{align*}

with
\[ A := I + \frac{1}{\lambda}B - \epsilon p \hspace{1cm} U := A^{-1} \]
- Matrix-Geometric Solution
  \begin{align*}
  n = 0, \ldots, N - 1 \quad & \pi(n; N) = r(0; N)pU^n \\
  n = N \quad & \pi(N; N) = r(0; N)\lambda pU^{N-1}Y
  \end{align*}
M/ME/1//N Queueing Systems (cntd.)

• Normalization ($\Sigma r(n)=1$)

$$r(0; N) = \frac{1}{pK(N)} \quad K(N) := \sum_{n=0}^{N-1} L^n + \lambda L^{N-1} \nu$$

$$= (1 - L)^{-1} (I - L)^{-1} + \lambda L^{N-1} \nu$$

• Probabilities at
  – Arrival instances
    $$n = 0, ..., N-1 \quad a(n; N) = \frac{1}{1 - r(N; N)} \pi(n; N)$$
  – Departure instances
    $$n = 0, ..., N-1 \quad d(n; N) = \frac{r(n; N)}{1 - r(N; N)} P$$

M/ME/1 Queue

Stability Criterion: $\rho := \lambda \nu (\mathbb{I} - L)^{-1} \pi < 1$

$$\pi(n) = (1 - \rho) \pi L^n$$

$$a(n) = \pi L^n$$

$$d(n) = r(n) \pi = (1 - \rho) \pi L^n \nu$$

Performance Parameters:

mean QL:
$$\bar{q} = \rho + \frac{\lambda}{1 - \rho} \nu (\mathbb{I} - L)^{-1} \nu$$

mean system time:
$$\bar{s} = \bar{q}$$

system time distribution:
$$\pi(k) = (1 - \rho) \pi L^{-1} \nu$$

overflow probability:
$$P_r(Q(0) \geq B) = \sum_{k=B}^{\infty} a(k) = (1 - \rho) \nu \sum_{k=B}^{\infty} \nu L^k$$

$$= (1 - \rho) \nu (\mathbb{I} - L)^{-1} \nu$$
**ME/M/1 Queueing Systems**

- Definition:
  - Service Times exponential (rate $\lambda$).
  - Inter-Arrival Times $X_i$ iid, described by $<\mu, B>$.

- In principle same approach as for M/ME/1:
  - Existence of Matrix-geometric solution known from QBD theory.
  - Consider solution of finite ME/M/1//N system.
  - Take limit $N \to \infty$.

- Finite ME/M/1//N system identical to M/ME/1//N system (but consider $S_2$ instead of $S_1$).
  - $\zeta = 1 / [E(X) \lambda]$.

**Obstacles**

- $\lim U^N$ does not exist ($U$ has eigenvalues $> 1$).

- Consider $\lim s^N U^N$ where $s$ is the smallest eigenvalue of $A$.

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**ME/M/1 Queueing Systems: Solution**

- Solution (Matrix-geometric solution reduces to geometric!)

  $$
  r_2(i) = 1 - \xi i
  
  r_2(k) = (1 - s) \xi s^{k-1}, \quad k > 0
  
  a_2(k) = d_2(k) = (1 - s) s^k
  $$

- Performance Parameters

  $$
  \text{mean QL:} \quad \bar{Q}_2 = \frac{\xi}{1 - s}
  
  \text{mean system time:} \quad \bar{S} = \frac{\xi}{1 - s}
  
  \text{system time distribution:} \quad b_2(x) = \frac{1 - s}{x^2} e^{-\xi x^2}
  
  \text{overflow probability:} \quad P_{\text{r}}(Q^{(a)} \geq B) = \sum_{k=B}^{\infty} a_2(k) = s^B
  $$
ME/M/1 Queue: Geometric parameter s

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**Distributional Tails**

- A particularly important part of a distribution is the (upper) tail
- \( P[X > x] \)
- Large values dominate statistics and performance
- “Shape” of tail critically important

![Graph showing distributional tails](image)

Source: M. Crovella

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**Light Tails, Heavy Tails**

- Light – Exponential or faster decline
- Heavy – Slower than any exponential

![Graph showing light and heavy tails](image)

\[
f_1(x) = 2 \exp(-2(x-1))
\]

\[
f_2(x) = x^{-2}
\]

Source: M. Crovella
Examining Tails

- Best done using log-log complementary CDFs
- Plot \( \log(1-F(x)) \) vs \( \log(x) \)

![Graph showing log-log complementary CDFs](source: M. Crovella)

Heavy Tails Arrive

pre-1985: Scattered measurements note high variability in computer systems workloads

  - File sizes
  - Process lifetimes

1993 – 1998: Attention focuses specifically on (approximately) polynomial tail shape:
  “heavy tails”

post-1998: Heavy tails used in standard models

Source: M. Crovella
**Power Tails, Mathematically**

We say that a random variable $X$ is power tailed if:

$$P[ X > x ] \sim x^{-\alpha} \quad 0 < \alpha \leq 2$$

where $a(x) \sim b(x)$ means

$$\lim_{x \to \infty} \frac{a(x)}{b(x)} = 1.$$

Focusing on polynomial shape allows

- Parsimonious description
- Capture of variability in $\alpha$ parameter

Source: M. Crovella

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**A Fundamental Shift in Viewpoint**

- Traditional modeling methods have focused on distributions with “light” tails
  - Tails that decline exponentially fast (or faster)
  - Arbitrarily large observations are vanishingly rare
- Heavy tailed models behave quite differently
  - Arbitrarily large observations have non-negligible probability
  - Large observations, although rare, can dominate a system’s performance characteristics

Source: M. Crovella
**Heavy Tails are Surprisingly Common**

- Sizes of data objects in computer systems
  - Files stored on Web servers
  - Data objects/flow lengths traveling through the Internet
  - Files stored in general-purpose Unix filesystems
  - I/O traces of filesystem, disk, and tape activity
- Process/Job lifetimes
- Node degree in certain graphs
  - Inter-domain and router structure of the Internet
  - Connectivity of WWW pages
- Zipf's Law

Source: M. Crovella

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**Evidence: Web File Sizes**

Barford et al., *World Wide Web, 1999*

Source: M. Crovella
The Bad News

- Workload metrics following heavy tailed distributions are extremely variable
- For example, for power tails:
  - When $\alpha \leq 2$, distribution has infinite variance
  - When $\alpha \leq 1$, distribution has infinite mean
- In practice, empirical moments are slow to converge – or nonconvergent
- To characterize system performance, either:
  - Attention must shift to distribution itself, or
  - Attention must be paid to timescale of analysis

Source: M. Crovella
**Heavy Tails in Practice**

Power tails with $\alpha=0.8$

Large observations dominate statistics (e.g., sample mean)

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(Truncated) Power-Tail Distributions

- **Definitions:**
  - Sub-exponential distributions
  - Power-Tail distributions
  - Example: Pareto Distribution

- **Properties:**
  - Moments
  - Residual Times

- **Truncated Power-Tail Distributions**
  - Representation as hyper-exponentials
    \[ p_T = \frac{1 - \theta}{1 - \theta^T} \left[ 1, \theta, \theta^2, \ldots, \theta^{T-1} \right]. \]
    \[ b_T = \mu \text{ diag } \left( 1, \gamma^{-1}, \ldots, \gamma^{-(T-1)} \right). \]
  - Systematic control of Power-tail Range
  - [easy to generate in simulation models]
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Self-Similar Properties:

- $\Delta = 10$ s
- $\Delta = 1$ s
- $\Delta = 0.1$ s
- $\Delta = 0.01$ s

Measurement of intervall-based Counting Process

Long-Range Dependence in Measurements

- Correlation Plot (Inter-cell times)
- Aggregated Variance Plot

$r(k) \sim k^{-\alpha} \text{ with } \alpha \approx 1.4$

$\text{Var } X^m \sim m^{-\alpha} \text{ Var } X$

→ Long-Range Dependence found in ATM measurements
Self-Similarity/Long-Range Dependence

• Self-Similarity with Hurst Parameter \( H \): \[ \mathbb{E}[N_i(\Delta)]_i \stackrel{d}{=} s^{-H} \mathbb{E}[N_j(s \Delta)]_j \]

• Second Order Self-Similarity: \( r^{(m)}(k) = r(k) \) for all \( m,k=1,2,... \)
  where \( r^{(m)}(k) \) is autocorrelation of averaged, \( m \)-aggregated process

• Asymptotic Second Order Self-Similarity: \( \lim_{k \to \infty} \frac{r^{(m)}(k)}{r(k)} = 1 \)
  for all \( m=1,2,... \)

• Long-Range Dependence: \( \sum r(k) \to \infty \) (assuming \( r(k) \geq 0 \))
  with special case: \( r(k) \sim \frac{1}{k^{(\alpha-1)}} \), \( 1<\alpha<2 \)

The latter processes are asymptotically second order self-similar!

Self-Similar/LRD Traffic Models

• Fractional Brownian Motion / Fractional Gaussian Noise
  \[ N(t) = m t + \sqrt{a m} Z(t) \]
  where \( Z(t) \) continuous, normally distributed, \( E(Z^2) = t^{2H} \), \( 0.5<H<1 \)

  ... is self-similar with parameter \( H \)
  and long-range dependent with \( \alpha=3-2H \)

• ON/OFF Traffic with Power-Tailed ON or OFF periods (\( 1<\alpha<2 \))
  ... is long-range dependent

• ... [others, e.g. chaotic maps, F-ARIMA]
Generalized ON/OFF Models

Parameters:
- N sources, each average rate $\kappa$
- During ON periods: peak-rate $\lambda_p$

MMPP representation (exponential case):

$$P(\text{ON}>t) = e^{-\beta t}$$

MMPP Representation: Multiplexed ON/OFF models with ME ON times

$$Q_N = \begin{bmatrix} Z_0 & X_0 \\ Y_1 & Z_1 & X_1 \\ \vdots & \vdots & \vdots \\ Y_{N-1} & Z_{N-1} & X_{N-1} \\ Y_N & Z_N \end{bmatrix}$$

$$L_N = \begin{bmatrix} 0 \\ \beta_1 \lambda_p I \\ \beta_2 2 \lambda_p I^{\otimes 2} \\ \vdots \\ \beta_N N \lambda_p I^{\otimes N} \end{bmatrix}$$

where:
- $X_i = \frac{N-i}{Z} I^{\otimes i} \otimes p, \quad i = 0, \ldots, N-1$
- $Y_i = -\beta_i (B \otimes i)^{\otimes i}, \quad i = 1, \ldots, N$
- $Z_i = -\beta_i B^{\otimes i} - \frac{N-i}{Z}, \quad i = 0, \ldots, N$

$\beta_i = 1$ (needed for TCP models, not done here)
ON/OFF models with Heavy-Tails

- Power-Tail distribution → Very long ON periods can occur
- Exponent $\alpha$: Heavier Tails for lower $\alpha$; Impact on exponent in AKF
- Truncated Tail: Power-Tail Range, Maximum Burst Size (MBS)

Heavy tails in ON or OFF periods?

- Coefficient of Correlation of Inter-packet times
  - LRD resulting from TPT ON or OFF periods
  - Here: 1-Burst, $\lambda_p=10$ with Poisson background traffic of $\lambda_{BG}=0.1$

- Average Queue-lengths: 1-Burst/M/1
  - TPT distribution in ON periods dominating
  - Only for $\rho\to1$, TPT in OFF period strong impact
  - Avg. QL shown relative to M/M/1 queue
**Performance relevance: variance & correlation**

- Coefficient of Variation
  - 1=exp,
  - 2=TPT ON,
  - 3=TPT OFF,
  - 4=TPT ON&OFF

- Coefficient of Correlation

- Experiment with Poisson background rate very small
  \((10^{-3})\)

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**Performance Impact: Multiplexed Case**

- `Blow-up Regions` \(i_0=1,...,N\) for LRD traffic
- Radical delay increase at transitions
- Power-Tailed queue-length-distr.
- ...with changing exponents \(\beta(i_0)\)
**Cause of Blow-up Regions**

- Oversaturation periods caused by a minimum of \( i_0 \) long-term active sources.
- Duration of oversaturation periods: PT with exponent \( \beta = i_0(\alpha - 1) + 1 \)

\[ \beta \text{ matters for performance, not } \alpha \text{ or } H \]

**Consequences: Power-Laws for BOP & CLP**

- Buffer Inefficiency: BOP ~ \( B^{1-\beta} \)
  CLP ~ \( B^{1-\beta} \)
- Drop-off for \( B > d_N \)
- Power-Law growth of \( mCD(MBS) \) when \( \beta < 2 \)

**BUT:** Steady-State reached in 4-8 daily Busy Hours?

BOP = Buffer Overflow Probability (N-Burst/M/1), CLP = Cell Loss Probability (N-Burst/M/1/B)
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**Background/Motivation**

- Bursty, non-elastic traffic $\rightarrow$ ON/OFF models ('1-Burst')

  - Average packet-rate $\lambda$, < peak-rate $\lambda_p$ $\rightarrow$ burstiness $b := \frac{\lambda}{\lambda_p}$ (0 $\leq b \leq 1$)
  - OFF time exponential, mean $Z$
  - ON period matrix-exponential, mean ON $\rightarrow$ MMPP representation
  - (truncated) Power-tail distributions $\rightarrow$ long-range dependent properties

- Single N-Burst/M/1 Queue (bottleneck analysis)
  Known performance results
  - Steady-state queue-length distribution (itc16)
  - Transient parameters (itc17)
  - Extensions for elastic traffic (Infocom01)

  **Behavior in queueing networks?**
**First Step: Tandem queue with ON/OFF input**

- Interesting Scenarios: Q2 as bottleneck, \( v_2 < v_1 \)
- Exact calculation via truncation \( \rightarrow \) QBD with level-dependent blocks
- Traffic decoupling approximation:
  - First queue: 1-Burst/M/1 queue (exact, known results: steady-state, transient)
  - Second queue: 1-Burst ON/OFF approximation
    \[
    \lambda^{(2)} = \lambda, \quad b^{(2)} = 1 - \frac{\lambda}{\nu_1} = 1 - \rho_1 \quad ON^{(2)} = BF_1.
    \]
  - Busy-Period Analysis of Q1 \( \rightarrow \) ON period at Q2 (at least) HYP-2 distributed

**Output process: more details**

- Each busy period has at least one departure
- But: MMPP models allow for ‘empty bursts’
- Markovian Arrival Process (MAP) modification
  - associate packet with end of ON period
  - Appropriate modifications to keep mean rate \( \lambda \)
Quality of output process approximation (1)

Candidates
- MMPP 1-Burst (allowing for ‘empty bursts’) [dashed]
- MAP 1-Burst [solid]
- Different ON period distributions
  - Exact (not feasible)
  - Exponential
  - HYP-2

Parameters to judge ‘quality’
- Coefficient of variation
- Auto-correlation (inter-packet times)
- Queueing behavior at Q2

Quality of output process approximation (2)

Candidates (again)
- MMPP 1-Burst (allowing for ‘empty bursts’) [dashed]
- MAP 1-Burst [solid]
- Different ON period distributions
  - Exact (not feasible)
  - Exponential
  - HYP-2

Observations:
- MMPP versions over-estimate for low b (additional burstiness due to ‘empty bursts’)
- MAP 1-Burst with HYP-2 fit provides close approximation for whole range of b
- Other scenarios (different utilization, etc.) → same conclusions
Summary

First Steps for network calculus

- Traffic type (after 1st stage at the latest):
  - 1-Burst MAP with HYP-2 ON-time distribution
    → representation with 3 states at each server
  - MAP representation was shown to be important

- Rules for traffic composition along N-stage tandem queue

\[ \lambda^{(2)} = \lambda, \quad \bar{\nu}^{(2)} = 1 - \frac{\lambda}{\bar{\nu}_1} = 1 - \rho_1 \quad \text{ON}^{(2)} = \text{BP}_1. \]

- Only one ‘complex’ operation: BP analysis MAP/M/1 queue

- Other ingress traffic → only impacts first tandem stage
  - But: approximation quality needs to be investigated

Future Work

- ‘Full Network Calculus’
  - Flow Aggregation/Multiplexing → N-Burst/M/1
  - Splitting of traffic
  - Feedback Loops (in network, elastic ingress traffic)
    → fix-point iteration (?)

- Comparison to stochastic network calculus

- Improved approximation
  - Truncated Power-Tailed ON periods
  - Correlated busy periods

- Other performance parameters
  - Correlation between queue-length distributions
  - Transient analysis
### Appendix: Exact computation of joint qld

For truncation Nmax of overall filling:

\[
\hat{Q} = \begin{bmatrix}
Q - D_0 & \mathcal{L} & 0 \\
0 & Q - D_1 & \nu_1 I \\
\nu_2 I & 0 & Q - D_2 \\
& \vdots & \ddots & \ddots
\end{bmatrix}
\]

### Appendix: Busy Period Analysis (Q1)

[Graphs showing busy period analysis for different scenarios]
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References

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