Wireless Communication Protocols and Technologies
by Tatiana Madsen & Hans Peter Schwefel

- Mm1 Introduction into security aspects (hps)
- Mm2 ad-hoc networks I (TKM)
- Mm3 ad-hoc networks II (TKM)
- Mm4 Advanced Mobility Topics (HPS)
- Mm5 Simulation Techniques and Measurements (HPS)

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Content

1. Motivation & Background
   • Performance Analysis in Wireless Settings
   • Review of Basic Concepts: Random Variables, Exponential Distributions, Stochastic Processes

2. Simulation Models
   • Basics: Discrete Event Simulation
   • Random Number Generation
   • Output Analysis

3. Summary
**Intro: Packet-Based Transport**

- Advantages of Packet-Based Transport (as opposed to circuit switched)
  - Flexibility
  - Optimal Use of Link Capacities, Multiplex-Gain for bursty traffic
- Drawbacks
  - Buffering/Queueing at routers can be necessary
  - Delay / Jitter / Packet Loss can occur
  - Overhead from Headers (20 Byte IPv4, 20 Byte TCP)

... and it makes performance modeling harder!!

Main motivation for Performance Modeling:
- Network Planning
- Evaluation/optimization of protocols/architectures/etc.

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**Challenges in Packet Switched Setting**

Challenges in IP networks:
- Multiplexing of packets at nodes (L3)
- Burstiness of IP traffic (L3-7)
- Impact of Dynamic Routing (L3)
- Performance impact of transport layer, in particular TCP (L4)
- Wide range of applications → different traffic & QoS requirements (L5-7)
- Feedback: performance → traffic model, e.g. for TCP traffic, adaptive applications

Challenges in Wireless Networks:
- Wireless link models (channel models)
- MAC & LLC modeling
- RRM procedures
- Mobility models
- Cross layer optimization
  → Analysis frequently with ‘stochastic’ models
Basic concepts

• Probabilities
  – ‘Random experiment’ with set of possible results $\Omega$
  – Axiomatic definition on event set $V(\Omega)$
    • $0 \leq \Pr(A) \leq 1$; $\Pr(\emptyset) = 0$; $\Pr(A \cup B) = \Pr(A) + \Pr(B)$ if $A \cap B = \emptyset$ $[A, B \in \mathcal{P}(\Omega)]$
  – Conditional probabilities: $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

• Random Variables (RV)
  – Definition: $X: \Omega \rightarrow \mathbb{E}$; $\Pr(X=x) = \Pr(X^{-1}(x))$
  – Probability density function $f(x)$, cumulative distribution function $F(x) = \Pr(X \leq x)$,
    reliability function (complementary distr. Function) $R(x) = 1 - F(x) = \Pr(X > x)$
  – Expected value, moments: $E(X^n) = \int x^n f(x) \, dx$
  – Relevant Examples, e.g.:
    • number of packets that arrive at the access router in the next hour (discrete)
    • Buffer occupancy (#packets) in switch $x$ at time $y$ (discrete)
    • Number of downloads (‘mouse clicks’) in the next web session (discrete)
    • Time until arrival of the next IP packet at a base station (continuous)

Basic concepts: Exponential Distributions

Important Case: Exponentially distributed RV

• Single parameter: rate $\lambda$
• Density function $f(x) = \lambda \exp(-\lambda x)$, $x > 0$
• Cdf: $F(x) = 1 - \exp(-\lambda x)$, Reliability function: $R(x) = \exp(-\lambda x)$
• Moments: $E[X] = 1/\lambda$; $\text{Var}[X] = 1/\lambda^2$, $C^2 = \text{Var}[X] / [E[X]]^2 = 1$

Important properties:

• Memory-less: $\Pr(X>x+y \mid X>x) = \exp(-\lambda y)$
• Properties of two independent exponential RV: $X$ with rate $\lambda$, $Y$ with rate $\mu$
  – Distribution of $\min(X,Y)$: exponential with rate $(\lambda+\mu)$
  – $\Pr(X<Y) = \frac{\lambda}{\lambda+\mu}$
Basic concepts III: Stochastic Processes

- Definition of process \((X_i)\) (discrete) or \((N_t)\) (continuous)
  - Simplest type: \(X_i\) independent and identically distributed (iid)
- Relevant Examples:
  - Inter-arrival time process: \(X_i\)
  - Counting Process:
    \[ N(t) = \max \{ n \mid \sum_{i=1}^{n} X_i \leq t \} \]
    alternatively \( N_i(\Delta) = N(i\Delta) - N((i-1)\Delta) \)

Important Example: Poisson Process
- Assume i.i.d. exponential packet inter-arrival times (rate \(\lambda\)): \(X_i := T_i - T_{i-1}\)
- Counting Process: Number of packets \(N_t\) until time \(t\)
  - \( \Pr(N_t = n) = (\lambda t)^n \exp(-\lambda t) / n! \)
- Properties:
  - Merging: arrivals from two independent Poisson processes with rate \(\lambda_1\) and \(\lambda_2\) \(\Rightarrow\) Poisson process with rate \((\lambda_1 + \lambda_2)\)
  - Thinning: arrivals from a Poisson process of rate \(\lambda\) are discarded independently with probability \(p\) \(\Rightarrow\) Poisson process with rate \((1-p)\lambda\)
  - Central Limit Theorem: superposition of \(n\) independent processes results in the limit \(n \to \infty\) in a Poisson process (under some conditions on the processes)

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3. Summary
Simulation Models (I)

- Basic principles of discrete event simulation
  - Virtual simulation time $t$
  - System state $S(t)$
  - Events occur at certain times $t_i$
    - Instantaneous changes of system state $S(t_{i-1}) \rightarrow S(t_i)$
    - Possibly scheduling of follow-up events
  - Events stored in ordered event list
  - System description:
    - Entities, attributes, and activities
    - Frequently object oriented implementation

- Important aspects
  - Initial state $S(0)$
  - Termination Criterion
    - Fixed simulation time $T$
    - Fixed number of packets/connections
    - Occurrence of certain events (e.g. Loss of connectivity)

Simulation Models (II)

- Application to wireless networks: Main components
  - Topology definition: nodes and connectivity
  - Link properties: e.g., Propagation models
  - Node functionalities: e.g., schedulers, buffer management, L2/L3 protocol implementation
  - Traffic models (and transport protocol implementation)
  - Mobility Models

- probabilistic elements in several of these components $\Rightarrow$ stochastic simulation
  - 'alternative': trace-driven simulations

- Output parameters, statistics collection, e.g.
  - Packet based
    - End-to-end packet delay
    - Packet loss rate
    - Energy per packet
  - Connection based
    - File Transfer times
    - Fraction of blocked calls
    - Throughput
  - Node/Link Properties
    - Buffer occupancy
    - Link utilizations
    - Throughput
Types and Examples of Simulation Tools

• ‘Libraries’ and programming languages with basic functionalities and data types:
  – Simula [e.g. R. Pooley: An Introduction to Programming in SIMULA, 1987]

• General Purpose Simulation Environments, e.g.
  – DEMOS/MAOS [Birtwistle, A system for discrete event modelling on SIMULA (DEMOS), 1979]
  – GPSS [http://www.minutemansoftware.com]

• Network Simulation Tools, e.g.
  – NS2 [http://www.isi.edu/nsnam/ns/]
  – OPNET
  – WIPSIM [http://www.wipsim.net/]
  – Glomosim [http://pcl.cs.ucla.edu/projects/glomosim]

Random Number Generation

• Uniform Random Number Generator (RNG)
  – Sequence $U_1, U_2, \ldots$ of i.i.d. ‘random’ numbers, uniformly distributed in $[0,1[$
  – Pseudo-random: same seed $X_0 \rightarrow$ same sequence
    – example: linear congruential generator
      • $X_{i+1}=(a X_i + b) \mod c$, $U_{i+1}=X_{i+1}/c$
      • E.g. $a=7^5=16807$, $b=0$, $c=2^{31}-1$ (prime)

• Random Variables from general distributions
  – $Y_1, Y_2, \ldots$ with cumulative distribution function $F(x)$
    derived from uniform stream $U_1, U_2, \ldots$ by
      • Inversion: $Y_i=F^{-1}(U_i)$
      • Other Techniques: Rejection, Convolution/Composition, etc.
Exercises I:

1. Random Number Generation: Write a MATLAB function to generate exponentially distributed random numbers based on the uniform RNG rand() from MATLAB.
   a. Check mean and coefficient of variation of the created number stream.
   b. Plot a histogram and compare with the exponential density function.
   c. Write a program to compute the empiric cumulative distribution function and compare with the exponential distribution.
   d. Check independence properties.

2. Write a simulation program for an M/M/1 queueing model.
   a. Plot the queue-length process for different simulation runs with lambda=0.5 and mu=1.
   b. Plot the average queue length over time t.
   c. Investigate the time to reach queue length B=5.
   d. Investigate correlation properties of the busy period duration.

Output Analysis (I): General

• Goal: Obtain Estimator Ž of desired performance parameter μ
  – Note: Ž often multi-dimensional
  – Ž is a random variable with some distribution fŽ(x)
  – Considered case here: Ž is estimator of μ=E(Z)

• Properties of estimator: Žt is called
  • Unbiased when E(Žt)=μ
  • Consistent when lim Žt=μ for t→∞ (stochastic convergence)

• Types of Simulations
  – Terminating simulations ~ ‘transient parameters’
  – Non-terminating simulations ~ Steady-state parameters
**Output Analysis II: Terminating Simulations**

- Terminating Simulations
  - Explicite stopping criterion, e.g.
    - Fixed simulation time
    - Fixed number of arrivals/connections
    - Specific event (e.g., buffer overflow, component failure)
  - Approach: Independent Replications
    - Repeat Experiment R times, each time with different seeds
    - independent outcomes \( Z_1, Z_2, \ldots, Z_R \)
    - Estimator \( \hat{Z} = 1/R \sum Z_i \)
      - Unbiased
      - Asymptotically normal distributed

- Relevant examples:
  - Determine average buffer-occupancy during busy hours 9-17hrs (starting empty at 9hrs)
  - Determine probability that call will be dropped before its desired end (given initial conditions)
  - Determine probability of buffer-overflow within \( n \) packet arrivals (given empty buffer in beginning)

**Output Analysis III: Confidence Intervals**

- Example: Estimate Probability \( \gamma = \Pr(\text{Overflow before simulation time } t) \)
  - \( R \) replications with indep. outcomes \( Z_i \)
  - Estimator \( \hat{Z} = 1/R \sum Z_i \)
    - \( E(\hat{Z}) = \gamma \) (unbiased!),
    - Estimates \( \hat{S}^2 \) of \( \text{Var}(Z_i) \)
      - \( \hat{S}^2 = 1/(R-1) \sum (Z_i - \hat{Z})^2 \)
      - \( \hat{S}^2 / R \) is estimate of \( \sigma^2 = \text{Var}(\hat{Z}) \)

Approaches for Confidence Intervals, confidence level \( 1-\alpha \) (often \( 1-\alpha = 95\% \)):

- Convergence to normal distribution
  - \( (\hat{Z} - \gamma) / \sigma \) in the limit standard normal distributed
  - Hence for \( n_{\text{conv}} \), quantile of normal distribution at level \( (1-\alpha)/2 \)
    - \( \Pr(\hat{Z} - \sigma n_{\text{conv}} < \gamma < \hat{Z} + \sigma n_{\text{conv}}) = 1-\alpha \)
  - Using the variance estimate \( \hat{S}^2 / R \):
    - \( \Pr(\hat{Z} - \sqrt{\hat{S}^2} / \sqrt{R} n_{\text{conv}} < \gamma < \hat{Z} + \sqrt{\hat{S}^2} / \sqrt{R} n_{\text{conv}}) = 1-\alpha \)

- General variance estimate \( \sqrt{\hat{R}} (\hat{Z} - \gamma) \) approx. Student-t distributed with \( (R-1) \) degrees of freedom
- Other approaches, e.g., variance stabilization for probability estimates [see Heyman/Sobel]
Special case: Binary Outcome

- Example: Estimate Probability $\gamma=\Pr(\text{Overflow before simulation time } t)$
  - $R$ replications with indep. outcomes $Z_i=\{1 \text{ when overflow occurred}, \ 0 \text{ otherwise}\}$
  - Estimator $\hat{Z}=1/R \sum Z_i$
    - $R*\hat{Z}$ Binomially distributed: expected value $\gamma R$, variance $R(1-\gamma)$
    - $E(\hat{Z})=\gamma$ (unbiased!), $Var(\hat{Z})=\gamma(1-\gamma)/R$
    - Estimates $\hat{\sigma}^2$ of $Var(Z)=\sigma^2$
      - $\hat{\sigma}^2 = \hat{Z}(1-\hat{Z})$ [for probabilities]

Output Analysis IV: non-terminating case

- Steady state parameters $\implies$ in theory infinite simulation needed
- Finite simulation length causes biased estimator
- Approaches:
  - Independent replications, impact of transient phase
    $\implies$ 'avoid' transient phase
  - Single, long simulation run
    - Problem: correlated samples require adjustment of variance estimate
    - Alternatives
      - Batching
      - Regenerative Simulation
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Exercises II:

3. Simulations: Use your M/M/1 simulation program from MM1 to obtain the probability that the queue reaches length 10 within simulation time t=100.
   a) Use independent replications and plot the distribution of the MEAN ESTIMATOR for N=10, N=20, N=50, N=100 replications. Compare with a normal distribution.
   b) Plot the behavior of the transient overflow probability for lambda in [0.05:0.05:0.5]. What happens with the confidence intervals for small lambda?
   c) Use the approach of batch means to develop an estimate (and 95% confidence intervals) of the average queue-length. Compare with independent replications.
   d) Transient phase: Use a queue with lambda=0.95 and mu=1. Check how the estimate for average queue-length (using independent replications) is influenced by dropping the first K samples out of N samples in the individual simulation run.
      i. K=0, N=100
      ii. K=50, N=100
      iii. K=0 N=1000
      iv. K=50 N=1000
      v. K=500 N=1000
      vi. K=1000, N=5000
References

Simulation models


* more details in lecture 'Discrete Event simulation' (9th Sem DIRS/NPM)