Proportional $\mathcal{H}_\infty$ Control

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Abstract

The static $\mathcal{H}_\infty$ output feedback problem will be considered in this paper for both the continuous time and the discrete time case. In this approach, there are no conditions on the direct feedthrough terms from the disturbance input signal and the control input signal to the output signal. It turns out that the $\mathcal{H}_\infty$ control problems are solvable by static output feedback controllers if and only if there exists positive semidefinite solutions to two certain quadratic matrix inequalities.

1 Introduction

The static output feedback problem has been considered in many papers, see [5] for a recent eigenstructure assignment approach. A static output feedback controller can assign at most $s$ closed-loop poles, where $s$ is the smallest of (1) no. of states, (2) no. of inputs + no. of outputs - 1, [5]. Further, an algorithm for assigning these poles, whenever it is possible is given in [5]. There is no guarantee, however, that the unstable poles are among the assignable ones, and henceforth pole assignment strategies do not provide sufficiently general systematic design techniques.

The static output control problem has recently been investigated in connection with covariance control [7, 8]. It has turned out that the static output feedback problem can be solved by considering simultaneous solvability of two Riccati-like inequalities. Moreover, the connection between covariance and $\mathcal{H}_\infty$ control theory has also been described in [8]. From these covariance results, the necessary and sufficient conditions for the solvability of the static $\mathcal{H}_\infty$ output feedback problem has been derived in [10, 13] for a certain class of continuous time systems.

The motivation for this paper is to solve the static $\mathcal{H}_\infty$ output feedback problem as directly as possible. The problem is formulated by considering a parameterization of a certain class of static $\mathcal{H}_\infty$ state feedback controllers along with a side constraint requiring that the state feedback gain can be described as a matrix multiplied by the system measurement matrix. Further, there will be no conditions on the two direct feedthrough terms $D_{11}$ and $D_{12}$ from disturbance and control input to the system output in the continuous time case. In the discrete time case, however, we will only consider systems where $D_{11}$ is zero.

The paper is organized as follows. In section 2, the continuous time case is considered followed by section 3, where the discrete time case is shortly considered. A conclusion is given in section 4.

2 The Continuous Time Case

The continuous time case will be considered in this section. First, some preliminary results are given. ($M^+$ will in the following denote the Moore-Penrose inverse of a matrix $M$).

2.1 Preliminaries

In the sequel we shall consider the following continuous time system:

\[ \dot{x} = Ax + B_1w + B_2u \]
\[ z = C_1x + D_{11}w + D_{12}u \]
\[ y = C_2z \]

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $z \in \mathbb{R}^q$, and $y \in \mathbb{R}^r$. Further, $(A, B_2)$ is assumed to be stabilizable and $(A, B_1, C_1, D_{12})$ is assumed to have no invariant zeros on the imaginary axis. Without loss of generality we assume that $C_2$ has full row rank.

An $\mathcal{H}_\infty$ state-feedback controller for the system described by (1) with $C_2 = I$ is given by $u(t) = Fz(t)$ where $F$ makes $A + B_2F$ stable and satisfies a suitably selected $\mathcal{H}_\infty$ norm bound $\gamma$:

\[ \| (C_1 + D_{12}F)(sI - A - B_2F)^{-1}B_1 + D_{11} \|_\infty < \gamma \]  

We need the following result which is a generalization of results from Stoorvogel [11] and Skelton et al [10]. The proof will be omitted due to space limitations.

Theorem 1 Consider the system (1). Let $\gamma > 0$. Assume that $D_{12}$ is injective. Then the following statements are equivalent:

(i) A static state feedback law exists such that after applying this compensator to the system (1) the resulting closed loop system is internally stable and the closed-loop transfer function from $w$ to $z$ has $\mathcal{H}_\infty$ norm less than $\gamma$. 


Further, there is a positive semidefinite solution $P_0$ to the algebraic Riccati equation

$$0 = A'P_0 + P_0A + C_1'C_1 - H'G^{-1}H$$

such that $A_{cl}$ is asymptotic stable where:

$$A_{cl} = A - (B_2 B_1) G^{-1}H$$

where $G$ and $H$ are given by

$$G = \begin{pmatrix} D_{12}'D_{12} & D_{12}'D_{11} \\ D_{11}'D_{12} & D_{11}'D_{11} - \gamma^2 I \end{pmatrix}$$

$$H = \begin{pmatrix} B_2'P_0 + D_{12}'C_1 \\ B_1'P_0 + D_{11}'C_1 \end{pmatrix}$$

Furthermore, if a $P_0 \geq 0$ exists satisfying the above conditions, one state feedback controller $F_0(P_0)$ is given by

$$F_0(P_0) = -R^{-1}(D_{12}'C_1 + B_2'P_0 + D_{11}'D_{11} - \gamma^2 I)$$

$$V_2 = 0$$

where $V_2 = (I - C_2^+C_2)$

and $V_2$ has full column rank.

However, instead of the condition (10) on the state feedback gain, we want to have an inequality in order to facilitate (a modified version of) the numerical solution method described in [4].

2.3 The regular case

The main result in the regular case is the following.

**Theorem 2** Consider the system (1). Let $\gamma > 0$. Assume that $D_{12}$ is injective. Then the following statements are equivalent:

(i) There exist a matrix $K \in \mathbb{R}^{n \times r'}$ such that when applying the static output feedback law $u = Ky$, the resulting closed-loop system is internally stable, and the $\mathcal{H}_\infty$ norm from $w$ to $z$ is smaller than $\gamma$.

(ii) $D_{11}$ satisfies

$$\|D_{11}\| < \gamma$$

Further, there is a positive semidefinite solution $P$ to the algebraic Riccati inequality

$$A'P + PA + C_1'C_1 - H'G^{-1}H < 0$$

such that $A_{cl}$ is asymptotic stable, and for some $W > 0$

$$F = FC_2^+C_2$$

where $F$ is given by the expression (7).

Proof. Obviously, all static $\mathcal{H}_\infty$ output feedback controllers are given as all $\mathcal{H}_\infty$ state feedback controllers $F$ which satisfies the additional constraint $FKer C_2 = 0$ or equivalently,

$$FV_2 = 0$$

where

$$V_2 = (I - C_2^+C_2)$$

Based on the above parameterization of all static state feedback controllers satisfying the $\mathcal{H}_\infty$ norm conditions in (2), we are available to formulate the static $\mathcal{H}_\infty$ output feedback problem.

2.2 Problem formulation

Let us again consider the system in (1). If there exist static output feedback controllers satisfying the stability conditions and the $\mathcal{H}_\infty$ norm condition, they are given by

Theorem 2 Consider the system (1). Let $\gamma > 0$. Assume that $D_{12}$ is injective. Then the following statements are equivalent:

(i) There exist a matrix $K \in \mathbb{R}^{n \times r'}$ such that when applying the static output feedback law $u = Ky$, the resulting closed-loop system is internally stable, and the $\mathcal{H}_\infty$ norm from $w$ to $z$ is smaller than $\gamma$.

(ii) $D_{11}$ satisfies

$$\|D_{11}\| < \gamma$$

Further, there is a positive semidefinite solution $P$ to the algebraic Riccati inequality

$$A'P + PA + C_1'C_1 - H'G^{-1}H < 0$$

such that $A_{cl}$ is asymptotic stable, and for some $W > 0$

$$F = FC_2^+C_2$$

where $F$ is given by the expression (7).
Moreover, all static $\mathcal{H}_\infty$ output feedback gains $K$ are then given by

$$K = (F_0(P) + R^{-1/2}MW^{1/2})C_2^T$$

where a parametrization of $M$ can be found by the algorithm given in [10].

Note that this result is a generalization of the result derived in [10].

Proof. First, let us prove the necessity for obtaining static $\mathcal{H}_\infty$ output feedback controllers. The first inequality in (ii) gives the condition for obtaining all equivalence classes of $\mathcal{H}_\infty$ state feedback controllers. The Riccati inequality (13) can be rewritten into the Riccati inequality (8) by rewriting $H'G'H$ and introducing $W > 0$ and the state feedback gain $F$ given by (7). The additional condition in Theorem 2, (10) or (11), can be rewritten into

$$V_2'(F'D_2(I - \gamma^{-2}D_1D_1')^{-1}D_2F)V_2 = 0$$

which prove the necessity of (13) in Theorem 3.

To prove the sufficiency, we need the following lemma.

**Lemma 1** Let $T \in \mathbb{R}^{n\times n}$ and $Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{pmatrix} \in \mathbb{R}^{n\times n}$ be given matrices with $Q < 0$, and assume that $Q_{22} + TT' < 0$. Then there exists a matrix $\Lambda \in \mathbb{R}^{(n-r)\times n}$ such that

$$Q + LL' < 0, \quad L := \begin{pmatrix} A \\ T \end{pmatrix}$$

Proof. See [10].

To prove sufficiency in Theorem 3, assume that $P > 0$ exists satisfying (14) and (13). Since

$$D_2'(I - \gamma^{-2}D_1D_1')D_2 > 0$$

it is possible to choose $T \in \mathbb{R}^{n\times n}$ as any matrix such that

$$TT' = V_2'(F'D_2'(I - \gamma^{-2}D_1D_1')^{-1}D_2F)V_2$$

Then with

$$W := A'P + PA + C'_1C_1 + (PB_1 + C'_1D_1)(\gamma^2I - D_1D_1')^{-1}(B'_1P + D_1C_1)$$

$$-F'(D_2'(I - \gamma^{-2}D_1D_1')^{-1}D_2F) < 0$$

we have that

$$V_2'WV_2 + TT' = V_2'(A'P + PA + C'_1C_1 + (PB_1 + C'_1D_1)\times(\gamma^2I - D_1D_1')^{-1}(B'_1P + D_1C_1))V_2 < 0$$

Introducing the singular value decomposition of $C_2$:

$$C_2 = (U_1 \quad U_2) \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix} (V_1 \quad V_2),$$

and defining

$$Q := V'WV, \quad (W = VQV')$$

or

$$\begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{pmatrix} := \begin{pmatrix} V_1'WV_1 & V_1'WV_2 \\ V_2'WV_1 & V_2'WV_2 \end{pmatrix}$$

we get from (21) and (22) that

$$Q < 0 \quad \text{and} \quad Q_{22} + TT' < 0$$

Applying Lemma 1 to (23) we obtain the existence of $\Lambda \in \mathbb{R}^{(n-r)\times n}$ such that

$$Q + \begin{pmatrix} A \\ T \end{pmatrix}' \begin{pmatrix} A' & T' \end{pmatrix} < 0$$

Finally, defining $L := V \begin{pmatrix} A \\ T \end{pmatrix}$ we have

$$W + LL' < 0$$

which is the inequality (13). The equality (17) follows directly from (20). This concludes the proof of Theorem 3.

\[ \square \]

2.4 The singular case

When $D_{12}$ is not injective, we get a singular $\mathcal{H}_\infty$ problem. The derived results from section 2.3 can therefore not be applied directly in this case. For deriving equivalent results in the singular case, we transform the system described in (1) into [11]:

$$\begin{pmatrix} \dot{z} \\ \dot{w} \\ \dot{u} \end{pmatrix} = \begin{pmatrix} A_D & B_{D1} & B_{D2} \\ C_{D1} & C_{D2} & 0 \end{pmatrix} \begin{pmatrix} z \\ w \\ u \end{pmatrix}$$

where

$$\begin{pmatrix} A_D \\ B_{D1} \\ B_{D2} \\ C_{D1} \\ D_{D12} \end{pmatrix} = \begin{pmatrix} A + B_1(\gamma^2I - D_1D_1')^{-1}D_1C_1 \\ -B_1(\gamma^2I - D_1D_1')^{-1}D_1C_1 \\ B_2 + B_1(\gamma^2I - D_1D_1')^{-1}D_1D_1D_2 \\ (\gamma^2I - D_1D_1')^{-1}C_1 \\ (\gamma^2I - D_1D_1')^{-1}D_1D_2 \end{pmatrix}$$

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still with the condition that the $H_\infty$ norm of $D_{11}$ is less than $\gamma$.

The connection between the two systems described by \((1)\) and \((24)\) is [11]:

**Lemma 2** Consider the systems described by \((1)\) and \((24)\). Then the following statements are equivalent:

(i) The system in \((1)\) with the controller $u = C(s)y$ is internally stable and the closed-loop transfer function from $w$ to $z$ has $H_\infty$ norm less than $\gamma$.

(ii) The system in \((24)\) with the controller $u = C(s)y$ is internally stable and the closed-loop transfer function from $w$ to $z$ has $H_\infty$ norm less than $\gamma$.

Further, we need the following result for the existing of a state feedback controller in the singular case. From [11] we have:

Consider the system in \((24)\) with $C_2 = I$.

The existence of state feedback laws for is characterized by the following result.

**Theorem 4** The following two are equivalent.

1. There exists a state feedback gain $F$ such that $A + B_2F$ is stable and such that

$$\|(C_{D1} + D_{D1}F)(sI - A_D - B_{D2F})^{-1}B_D1\|_\infty < \gamma$$

2. There exists $P \geq 0$ such that the following three hold

\[
\begin{pmatrix}
F_{11} & F_{12} \\
F_{12} & F_{22}
\end{pmatrix}
=:
\begin{pmatrix}
C_P & D_P \\
D_P & D_{P'}
\end{pmatrix}
\begin{pmatrix}
C_P & D_P
\end{pmatrix}
\geq 0
\]

where

\[
F_{11} = A_P'P + P A_D + C_D' C_{D1} + \gamma^{-2}P B_{D1} B_{D1} P
\]

\[
F_{12} = P B_{D2} + C_{D1} D_{D12}
\]

\[
F_{22} = D_{D12} D_{D12}
\]

(b) \(\text{rank} \left( \begin{pmatrix}
C_P & D_P
\end{pmatrix}
\right) = m_{nq} := \max_{s \in \mathbb{C}^+} \text{rank} \left( (C_{1}(sI - A_D)^{-1}B_D + D_{D12}) \right)\)

(c) \(\text{rank} \left( \begin{pmatrix}
sI - A_D - \gamma^{-2}B_{D1} B_{D1} P & -B_{D2} \\
C_P & D_P
\end{pmatrix}
\right) = n + m_{nq}, \ \forall s \in \mathbb{C}^+\)

Whenever $P \geq 0$ exists satisfying the three conditions \((2a-2c)\) of Theorem 4 such $P$ can be found by solving a reduced order Riccati equation. Moreover, it can be shown that $P$ is unique (see [11]). $2a+b+c$ is also known as the Quadratic Matrix Inequality [11].

Based on this result, we can now give the equivalent result of Theorem 3 in the singular case.

**Theorem 5** The following two are equivalent.

1. There exists a static output feedback gain $K$ such that $A_K = A + B_2 KC_2$ is stable and such that

$$\|(C_1 + D_{12} KC_2)(sI - A_K)^{-1}B_1 + D_{11}\|_\infty < \gamma$$

2. There exists $P \geq 0$ and $W > 0$ such that the following four hold

\[
\begin{pmatrix}
F_{11W} & F_{12W} \\
F_{12W} & F_{22W}
\end{pmatrix}
=:
\begin{pmatrix}
C_P & D_P \\
D_P & D_{P'}
\end{pmatrix}
\begin{pmatrix}
C_P & D_P
\end{pmatrix}
\geq 0
\]

where

\[
F_{11W} = A'_P P + P A_D + C_{1W} C_{1W} + \gamma^{-2}P B_{D1} B_{D1} P
\]

\[
F_{12W} = P B_{D2} + C_{1W} D_{12W}
\]

\[
F_{22W} = D_{12W} D_{D12W}
\]

\[
\text{rank} \left( \begin{pmatrix}
sI - A_D - \gamma^{-2}B_{D1} B_{D1} P & -B_{D2} \\
C_P & D_P
\end{pmatrix}
\right) = n + m_{nq}, \ \forall s \in \mathbb{C}^+
\]

\[
V_{2}(A_P'P + P A_D + C_{D1} D_{D12} + \gamma^{-2}P B_{D1} B_{D1} P) V_2 < 0
\]

where \(\begin{pmatrix}
C_{1W}' \\
D_{12W}'
\end{pmatrix}
( C_{1W} D_{12W} ) := \left( \begin{pmatrix}
C_{D1} C_{D1} + W & C_{D1} D_{D12} \\
D_{D12} C_{D1} & D_{D12} D_{D12}
\end{pmatrix}\right)\)

A proof of Theorem 5 can be found in [13]. The proof is based on cheap control and a description of the solutions of the Quadratic Matrix Inequality from [9].

3 The Discrete Time Case

The discrete time case will be considered in this section. The discrete time static $H_\infty$ output feedback controller is equivalent to the continuous time case. Therefore, the results will only be given shortly without proofs. The main difference between the continuous time and the discrete time case is that the singular case does not exist in discrete time. The reason is the structure of the Riccati equation in the discrete time case. Instead of requiring injectivity of $D_{12}$ in the continuous time case, a condition based on the Riccati solution is obtained in the discrete time case [12].

3.1 Preliminaries

In the sequel we shall consider the following discrete time system:

\[
\begin{align*}
x(t + 1) &= A x(t) + B_1 w(t) + B_2 u(t) \\
x(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\
y(t) &= C_2 x(t)
\end{align*}
\]

where \(x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^p, z(t) \in \mathbb{R}^s \) and \(y(t) \in \mathbb{R}^s\). Further, \((A, B_2, C_1, D_{12})\) has no invariant zeros on the unit circle.
An $\mathcal{H}_\infty$ state-feedback controller for the system described by (25) with $C_2 = I$ is given by $u(t) = Fx(t)$ where $F$ makes $A + B_2 F$ stable and satisfies a suitably selected $\mathcal{H}_\infty$ norm bound $\gamma$:

$$\| (C_1 + D_{12} F)(zI - A - B_2 F)^{-1} B_1 + D_{11} \|_\infty < \gamma$$  \hspace{1cm} (26)

From Stoorvogel et.al. [12], we have the following result:

**Theorem 6** Consider the system (25). Let $\gamma > 0$. Then the following statements are equivalent:

(i) A static state feedback law exist such that after applying this compensator to the system (25) the resulting closed-loop system is internally stable and the closed-loop transfer function from $w$ to $z$ has $\mathcal{H}_\infty$ norm less than $\gamma$.

(ii) $D_{11}$ satisfies

$$\| D_{11} \| < \gamma$$

Further, there is a positive semidefinite solution $P$ such that:

$$\gamma^2 I - D_{11} D_{11}' - B_1' P B_1 > 0$$  \hspace{1cm} (27)

$$P = A' PA + C_1' C_1 - H' G^* H$$  \hspace{1cm} (28)

$$\text{rank}_R \left( \begin{pmatrix} zI - \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} \right) = n + q + \text{rank}_R \left( C_1 (zI - A)^{-1} B_2 + D_{12} \right)$$  \hspace{1cm} (29)

where

$$\hat{A} = A, \quad \hat{B} = (B_2 \quad B_1)$$

$$\hat{C} = \begin{pmatrix} B_2' P A_2 + D_{12}' C_1 \\ B_2' P A_1 + D_{11}' C_1 \end{pmatrix}, \quad \hat{D} = \begin{pmatrix} B_2' P B_2 + D_2 D_{12} \\ B_2' P B_1 + D_2 D_{11} \\ B_1' P B_2 + D_1 D_{12} \\ B_1' P B_1 + D_1 D_{11} - \gamma^2 I \end{pmatrix}$$

$G$ and $H$ are given by

$$G = \begin{pmatrix} D_{12}' D_{12} & D_{12}' D_{11} \\ D_{11}' D_{12} & D_{11}' D_{11} - \gamma^2 I \end{pmatrix}$$

$$+ \begin{pmatrix} B_2' \\ B_1' \end{pmatrix} P \begin{pmatrix} B_2 & B_1 \end{pmatrix}$$

$$H = \begin{pmatrix} B_2' P A_2 + D_{12}' C_1 \\ B_2' P A_1 + D_{11}' C_1 \end{pmatrix}$$

Further, if a $P \geq 0$ satisfying the above conditions, a state feedback controller $F$ is then given by

$$F = -V^* (B_2' P A_2 + D_{12}' C_1 + [B_2' P B_1 + D_{11}' D_{12}])$$

$$\times R^{-1} [B_1' P A_2 + D_{11}' C_2] + (I - V^* V) F_0$$  \hspace{1cm} (30)

and $F_0$ is required to stabilize $A + B_2 F_0$.

By applying a Riccati inequality instead of a Riccati equation in Theorem 6 we directly obtain a parameterization of all equivalence classes of static state feedback controllers satisfying the $\mathcal{H}_\infty$ norm conditions in (26).

Based on this result, we are able to formulate the static $\mathcal{H}_\infty$ output feedback problem.

### 3.2 Main results

Due to the complexity of the discrete time Riccati equations for the state feedback control, we will only give the static $\mathcal{H}_\infty$ output feedback results for $D_{11} = 0$. Using the $\mathcal{H}_\infty$ state feedback results from [14], we can then give the following formulation of the discrete-time static $\mathcal{H}_\infty$ output feedback problem.

**Theorem 7** Consider the system (25). Let $\gamma > 0$. Then the following statements are equivalent:

(i) There exist a matrix $K \in \mathbb{R}^{p \times r}$ such that when applying the static output feedback law $u(t) = K y(t)$, the resulting closed-loop system is internally stable, and the $\mathcal{H}_\infty$ norm from $w$ to $z$ is smaller than $\gamma$.

(ii) There exists an $F$ and a positive semidefinite solution $P$ to the algebraic Riccati inequality

$$\begin{pmatrix} A + B_2 F & (A + B_2 F)' P (A + B_2 F) - P \\ + P (A + B_2 F)' (A + B_2 F)' P B_1 + (C_1 + D_{12} F)' C_1 + D_{12} F \end{pmatrix} < 0$$

such that

$$F V_2 = 0$$

**Proof.** The result is the bounded real lemma for discrete time systems combined with the output constraint, see e.g. [15].

Based on Theorem 7, we can give the main result for the discrete-time static $\mathcal{H}_\infty$ output feedback problem.

**Theorem 8** Consider the system (25). Let $\gamma > 0$. Then the following statements are equivalent:

(i) There exist a matrix $K \in \mathbb{R}^{p \times r}$ such that when applying the static output feedback law $u(t) = K y(t)$, the resulting closed-loop system is internally stable, and the $\mathcal{H}_\infty$ norm from $w$ to $z$ is smaller than $\gamma$. 

where

$$A_\infty = A - B_2 V^* [B_2' P A_2 + D_{12}' C_1]$$

$$C_\infty = C_1 - D_{12} V^* [B_2' P A_2 + D_{12}' C_1]$$

$$V = B_2' P B_2 + D_{12}' D_{12}$$

$$R = \gamma^2 I - D_{11} D_{11}' - B_1' P B_1 + (B_1' P B_2 + D_{11} D_{12}) V^* (B_2' P B_1 + D_{12} D_{11})$$

and $F_0$ is required to stabilize $A + B_2 F_0$. 


(ii) There is a positive semidefinite solution $P$ to the algebraic Riccati inequalities

\begin{align}
A'PA - P + C_1'C_1 \\
+ PB_1(\gamma^2I - B_1'PB_1)^{-1}B_1'P \\
-(B_2'PA)'(D_{12}'D_{12} + B_2'PB_2)^{-1}(B_2'PA) < 0
\end{align}

(31)

\begin{align}
V_2'(A'PA - P + C_1'C_1 \\
+ PB_1(\gamma^2I - B_1'PB_1)^{-1}B_1'P)V_2 < 0
\end{align}

(32)

and

\begin{align}
D_{12}'D_{12} + B_2'PB_2 > 0
\end{align}

(33)
\begin{align}
\gamma^2I - B_1'PB_1 > 0
\end{align}

(34)

Proof. The proof of Theorem 8 follows the same line as in the continuous-time case.

In analogy to the continuous time case, given $P$ satisfying (31-34) it is possible to put forward an algorithm in which all discrete time static output feedbacks are parameterized. However, due to space limitations we shall not give details here.

4 Conclusion

Necessary and sufficient conditions have been given for the existence of static output feedback controllers satisfying an $H_\infty$ norm bound for both continuous time as well as for discrete time systems. The design problem has been formulated directly by considering all equivalence classes of state feedback controllers together with the condition that the controllers must be output controllers. It has been shown that the derived results are generalizations of the results in [10, 13] in the continuous time case.

References


