FAST AND ROBUST MEASUREMENT OF OPTICAL CHANNEL GAIN

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ABSTRACT

We present a numerically stable and computational simple method for fast and robust measurement of optical channel gain. By transmitting adaptively designed signals through the channel, good accuracy is possible even in severe noise conditions.

1. INTRODUCTION

The measuring of optical channel gains is a key element in many applications. Measuring channel gains means determining the change in intensity when a signal is transmitted from an emitter to a receiver. A well-known and simple application is an automatic door, which responds whenever a person is reflecting the emitted signal, and thereby significantly increasing the channel gain. Another example is measuring the thickness of paper. A more subtle example is determination of spatial position by comparing the intensities of a multitude of reflections from a single object. A typical way of making this type of measurements is emitting a simple signal, such as a harmonic or square wave signal, since they are both easily constructed and measured with analog electronics. Such solutions have two major disadvantages: The signals are sensitive to frequency located disturbances, and it is difficult to detect and avoid/neutralize such disturbances.

We propose a measuring method which is highly accurate in moderate noise conditions, and less accurate, but very robust, in severe noise conditions. This is achieved by using two closely related digital signal design algorithms; a "best case" and a "worst case" algorithm. The former is based on the wavelet transform (WT), while the latter is based on the Rudin-Shapiro transform (RST). They are both simple, numerically stable, and post-processing friendly making them ideal for implementation e.g. in a fixed point DSP. By introducing a signal processor it becomes possible to continuously redesign the signals for improved SNR, and thereby maintaining the accuracy in changing and/or severe noise conditions.

2. DESIGNING THE DIGITAL SIGNALS

The two design algorithms are based on the wavelet packet transform scheme; it is fast, numerically stable, works well in fixed point arithmetics, and has low program complexity.

The best case algorithm uses the classical WT to create signals which are near-orthogonal to expected noise occurrences, while the other algorithm uses the RST to create an all-spectrum signal, which by nature has low sensitivity with respect to time and frequency located noise occurrences. The difference is essentially that the WT algorithm "searches for holes" in the current noise, while the RST algorithm spreads information in time and frequency to reduce the impact of localized disturbances. The preferred method depends on the noise conditions. If there are easy-to-find holes in the noise, the former can provide very accurate measurements. If, however, the noise is difficult to define or is changing rapidly, the latter method provides less accurate, but more robust measurements.

A good introduction to the wavelet theory is Wickerhauser [6]. A mathematically rigorous treatment of the subject is given in Daubechies [3]. For more material on Rudin-Shapiro polynomials see Brillhart [1].
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of the polynomials
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The RST is delincd through a slightly
remarkable Rudin-Shapiro polynomials, introduced in
1959 by H. S. Shapiro in his master's thesis, and published in 1959 by
Rudin [3]. Define the polynomials
\[ P_{m+1}(z) = P_m(z) + (-1)^m z^m Q_m(z), \quad P_0 = 1 \]
\[ Q_{m+1}(z) = P_m(z) - (-1)^m z^m Q_m(z), \quad Q_0 = 1 \]
with \( m \in \{0, 1\}. \) It immediate follows that for all \( |z| = 1 \)
\[ |P_{m+1}|^2 + |Q_{m+1}|^2 = 2|P_m|^2 + 2|Q_m|^2 = 2^{m+2}. \]
Consequently,
\[ \max_{\xi} |P_m(e^{i\xi})| \leq \sqrt{2} |P_m(e^{i\xi})|, \quad \text{(2)} \]
guaranteeing a certain flatness of the polynomials. A construction similar to (1) is found in Byrnes [2]. The coefficients of the polynomials can also be constructed with the Rudin-Shapiro transform, which is really a modified wavelet packet Haar transform. Define the unitary transform \( H_n : \mathbb{R}^n \rightarrow \mathbb{R}^n, \quad n \geq 1, \) as
\[
\begin{bmatrix}
y
y_k
\end{bmatrix}
= \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & (-1)^k & 2^k
\end{bmatrix}
\begin{bmatrix}
y_{k+2^{n-1}}
\end{bmatrix}^{-1}
\]
for \( k = 0, \ldots, 2^n - 1 \) when mapping \( x \) to \( y. \) Then, with \( \tilde{H}_n \) being the inverse of \( H_n, \)
\[
\tilde{H} \equiv \prod_{n=1}^{N} \tilde{H}_n
\begin{bmatrix}
0
\end{bmatrix}
\]
transforms the canonical basis for \( \mathbb{R}^N \) into the coefficients of the \( 2^N \) possible \( P_m(z). \) Hence these coefficients constitutes an orthonormal basis, which not only consists of only \( \pm 1, \) but
also, due to (2), has a remarkable frequency response. In figure 2 is an example of such a basis element and its frequency response. The RST based algorithm is applied in much the same way as the WT method. A simple signal is inversely transformed prior to transmission. This will produce an all-spectrum signal. Upon transmission the signal is forwardly transformed yielding the original, single signal with noise. The post-processing is equivalent to that of the WT method.

3. CONCLUSION
Two computationally simple and numerically robust algorithms for measuring optical channel gains were presented. One of the algorithms provides excellent accuracy in moderate noise conditions, while the other has reduced, but very robust, accuracy even in severe noise conditions. Combining the two algorithms, either by applying the most suitable one, or jointly in two parallel systems, is easy due to their similar program and computational structure, and the result is a versatile optical channel gain method. The low complexity and numerical stability of the wavelet packet transform scheme and of the post-processing (mainly inner products) also makes this approach fast and suitable for low cost hardware implementation. For the commercial aspects of these methods, please refer to www.beamcontrol.com.

4. REFERENCES