Estimation of effective wind speed

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Abstract. The wind speed has a huge impact on the dynamic response of wind turbine. Because of this, many control algorithms use a measure of the wind speed to increase performance, e.g. by gain scheduling and feed forward.

Unfortunately, no accurate measurement of the effective wind speed is online available from direct measurements, which means that it must be estimated in order to make such control methods applicable in practice.

In this paper a new method is presented for the estimation of the effective wind speed. First, the rotor speed and aerodynamic torque are estimated by a combined state and input observer. These two variables combined with the measured pitch angle is then used to calculate the effective wind speed by an inversion of a static aerodynamic model.

1. Introduction

With the increasing competition in the wind energy market it is becoming very important to have control algorithms with which the structural fatigue is minimised without compromising the energy production. In contrast to many other control problems, the dynamics of wind turbines are driven by a disturbance, namely the wind speed. This means that the wind not only excites oscillations in various structural components but is also one of the main variables to select the operating condition of wind turbines – together with different control strategies like the rating of generator speed and power production.

One of the ways to handle the variations in operating conditions is the use of gain scheduling or adaptive control [1, 2, 3, 4, 5]. In these control methods, the controller variables are updated online on the basis of scheduling variables that are measured or constructed from measured variables. In these control methods it is very important that all operating conditions can be determined uniquely from the scheduling variables. This means that in the case of wind turbines, all variables determining the aerodynamics must be used, e.g. wind speed, rotor speed and pitch angle. In practice the number of variables is usually reduced by assuming a certain operating trajectory. Typically the pitch angle is chosen in full load operation and the generator speed is used in partial load operation as in [1, 6, 7]. Alternatively the controller is scheduled on wind speed [8, 9, 3], which has the advantage that the same scheduling variable can be used over the entire operating envelope. However, the wind speed is not directly available and must therefore be estimated. Further, if combined with pitch angle and generator speed it is also possible to schedule for operating conditions outside the nominal trajectory, which can be advantageous in the context of derating strategies, extreme weather conditions, fault situations, etc.

Another important reason for considering efficient estimation of the wind speed is the use of feed forward control. The variations in wind speed not only changes the dynamic response of the
wind turbine but also the steady state values of important signals like shaft torque, tower thrust etc. To compensate for this the references for the controller is generated to give appropriate steady state values and to give a fast response, a nonlinear feed forward term is usually included from wind speed or aerodynamic torque to the relevant control signals [10, 11, 7, 12].

Both examples above indicate that there is a need for precise estimates of the wind speed in order to get a good performance in the overall control loop. It is assumed that the estimated wind speed will be used to schedule a controller with the main purpose of tracking generator speed references, tower thrustwise movement, etc. In this context, the main drivers of the variations in set point and dynamics are the torque, \( Q_a \), on the main shaft and the thrust, \( F_t \), on the tower. These two variables can be described by static functions of rotor speed, \( \omega_r \), pitch position, \( \beta \), and effective wind speed, \( v \), as in (1) with the effective wind speed being defined as the spatial average of the wind field over the rotor plane with the wind stream being unaffected by the wind turbine, i.e. as if the wind turbines was not there [13].

\[
Q_a = \frac{1}{2} \rho \pi R^2 \frac{v^3}{\omega_r} c_p(v, \lambda) \tag{1a}
\]
\[
F_t = \frac{1}{2} \rho \pi R^2 v^2 c_T(v, \lambda) \tag{1b}
\]

The constants, \( \rho \), and \( R \), describe respectively the air density and rotor radius and \( \lambda \) is the tip speed ratio defined as \( \lambda = \frac{\omega_r R}{v} \). The purpose is then to estimate the effective wind speed, \( v \), and it has been chosen to use (1a) together with a dynamic model of the drive train in the observer design.

In the literature many different algorithms have been investigated. The most simple algorithm assume that there is a static relation between electrical power production and the effective wind speed [8, 14, 15]. This assumption means that for example the energy stored in the speed-up of the rotor is neglected – which is a very crude assumption. In [15] it is concluded that using dynamic models significantly improves the observer performance, and it is therefore estimated that the use of static relations does not give satisfactory performance.

As a solution to the above mentioned issues, most papers in the literature propose a method which utilises a simple drive train model as in (2) with \( Q_g \) as the generator reaction torque and \( Q_{loss} \) being a loss term describing for example friction. [10, 16, 7, 11]

\[
J \dot{\omega} = Q_a - Q_g - Q_{loss} \tag{2}
\]

This model assumes that the drive train is infinitely stiff which means that drive train oscillations are neglected and that the lag between rotor acceleration and generator acceleration in case of gusts is also neglected. The first issue can most likely be handled by a notch filter at the drive train eigenfrequency, whereas the second issue will need further investigations in order to understand its significance. The observer algorithm is simply to calculate \( Q_a \) from measurement \( Q_g \), differentiated measurement \( \omega_g \), and modelled loss term, \( Q_{loss} \). In practice this method is very sensitive to measurement noise as indicated in [11]. It is therefore necessary to low pass filter either \( \omega_g \) or the estimated output as in [16]. This approach imposes a very particular structure of the observer in order to reject measurement noise. It is well-known that a low pass filter will introduce a time delay in the estimated quantity and the particular observer structure can potentially lead to a poor trade-off between noise rejection and time delay. It is therefore as in [11] estimated that a better performance can be achieved by using dynamic observers.

Dynamic observers for estimating the effective wind speed has not been as intensively investigated as the above mentioned methods. The main trend in the design of dynamic estimators has been either to design a linear Kalman filter for estimating the aerodynamic torque and then calculate the effective wind speed using (1a) [11]. The alternative is to combine a linear
model of the drive train with the nonlinear aerodynamic model and use nonlinear algorithms to estimate the wind speed directly – either by online linearization (extended Kalman filter) [15] or by using more dedicated algorithms [17].

The main advantage of all three algorithms is that they are all dynamic observers, which means that the filtering is designed via the cost function for the design algorithm. However, they all have the disadvantage that none of them are suited directly for input estimation – only state estimation. To counter this, a model of the input is created with the unknown input as a state variable. In the most simple form, \( Q \), can be assumed to vary very slowly compared with the observer bandwidth. Then combined with the simple drive train model (2), the augmented model becomes

\[
\begin{bmatrix}
\dot{\hat{\omega}} \\
\dot{\hat{Q}_a}
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{J}{J} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\omega \\
Q_a
\end{bmatrix} +
\begin{bmatrix}
\frac{-g}{J} & 0 \\
0 & -\frac{\omega}{J}
\end{bmatrix}
\begin{bmatrix}
Q_g \\
Q_{loss}
\end{bmatrix}
\]

The dynamic observer is then constructed by combining the augmented model with an update term, \( L \cdot (\omega - \hat{\omega}) \), that updates the state vector based on the estimation error in measured output. This means that the observer is of the form (4) with the accent \( \hat{\cdot} \) denoting estimated variables

\[
\begin{bmatrix}
\dot{\hat{\omega}} \\
\dot{\hat{Q}_a}
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{J}{J} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{\omega} \\
\hat{Q}_a
\end{bmatrix} +
\begin{bmatrix}
\frac{-g}{J} & 0 \\
0 & -\frac{\omega}{J}
\end{bmatrix}
\begin{bmatrix}
Q_g \\
Q_{loss}
\end{bmatrix} +
\begin{bmatrix}
L_1 \\
L_2
\end{bmatrix} (\omega - \hat{\omega}) \tag{4}
\]

It is clear that there are two major issues in this way of estimating the unknown input. First of all it has to be chosen, which model to use for the aerodynamic torque. In the example, the most simple form, \( \hat{Q}_a = 0 \) was chosen, but it can be extended to models that reflect the expected spectrum of \( Q_a \). However, the difficulty in this part is to determine what spectrum to use, because wind turbines can encounter very different wind spectra depending on their respective location, e.g. plains, mountain areas, offshore, etc. A possible approach could be a self-tuning procedure, which would slowly identify location specific parameters and use these to adapt a wind spectrum model. This, however, would involve a comprehensive collection of representative data and is outside the scope of this paper.

The other issue is the tradeoff between state estimation and input estimation. When there is an estimation error in the measured variable, \( \omega - \hat{\omega} \), it must be identified how much this error shall affect the update of the state vector, \( \hat{\omega} \), and how much the unknown input shall be updated. This is essentially the tradeoff between the sizes of \( L_1 \) and \( L_2 \). If \( L_1 \) becomes too large compared to \( L_2 \), \( Q_a \) is not updated sufficiently leading to small estimation error in the state vector, but high estimation error in the unknown input, \( Q_a \). On the other hand, if \( L_1 \) becomes too small compared to \( L_2 \), the state vector is not updated correctly. Then the estimation error, \( \omega - \hat{\omega} \) will increase (not necessarily to instability) and give poor estimates in both state vector and \( Q_a \). Besides the balance between \( L_1 \) and \( L_2 \), their size must also be balanced between time propagation and time update. In the Kalman filter approach this is done in an optimisation function which minimises the mean square error of the state (and unknown input) estimate weighted by constant scalings.

To summarise: The simplified method of using steady state equations is very easy to design, but does not give sufficient estimation quality. The method that uses differentiation of generator speed is in its direct form also very simple to design, but amplifies to a large extend the measurement error and drive train oscillations. This problem can be handled by proper filtering which, however, introduces a time delay in the process and complexity in the design. Finally, the observer based estimator has the major advantage that filtering is included in the algorithm. The disadvantage is on the other hand that the complexity in the algorithm is increased – especially regarding the choice of model for the unknown input and the weighting between state estimation and input estimation.
This paper presents a method that is quite similar to the observer based method presented above. The major difference is that instead of augmenting the state model as in (3), the state and input estimation problem is split into two separate problems. A dynamic observer based on the Kalman filtering approach is designed for the state estimation and an input observer based on ideas from tracking controllers is designed for estimation the aerodynamic torque. In this setting it is expected that the tracking performance will be significantly better than the steady state estimation method. Further it is expected that the tradeoff between noise rejection and time delay is improved when compared with the method using differentiation of measured generator speed. Finally it is expected to have similar performance to other dynamic observers in the literature. However by splitting the observer problem into a state estimator and an input estimator, the design problem is simplified as it will be illustrated and the choice of wind model is transformed into the choice of observer structure for the input estimator – from experience in tracking controllers, this problem is efficiently solved by a PID structure.

In Section 2 we present a method for the design of an observer to estimate the angular velocity of the rotor and the aerodynamic torque acting on the low speed shaft. Then in Section 3, these two variables together with measured pitch position are used to calculate the effective wind speed by inversion of the aerodynamic model. Finally in Section 4 the conclusions are given.

2. Estimation of Rotor Speed and Aerodynamic Torque

In this section we take advantage of methods from the field of state estimation and combine them with ideas from tracking controllers to obtain what is known as disturbance estimators [18]. In the following it is assumed that the drive train to a sufficient level of accuracy can be described by two inertias interconnected by a spring and damper and with viscous friction on each inertia. The external forces to this 2-DOF system is then the aerodynamic torque, \( Q_a \), on the slow speed shaft and generator reaction torque, \( Q_g \), on the high speed shaft. This results in the system of equations in (5) which for simplicity in the notation will be referred to via the general state space form (6) with \( x = \begin{bmatrix} \omega_r & \omega_g & \theta_\Delta \end{bmatrix}^T \) as the state vector.

\[
\begin{align*}
J_r \dot{\omega}_r &= Q_a - B_r \omega_r - \mu (\omega_r - \omega_g) - K \theta_\Delta \quad \text{(5a)} \\
J_g \dot{\omega}_g &= -Q_g - B_g \omega_g + \mu (\omega_r - \omega_g) + K \theta_\Delta \quad \text{(5b)} \\
\dot{\theta}_\Delta &= \omega_r - \omega_g \quad \text{(5c)}
\end{align*}
\]

\[
\dot{x} = A x + B_r Q_a + B_g Q_g \quad \text{(6a)}
\]
\[
\omega_r = C_r x \quad \text{(6b)}
\]
\[
\omega_g = C_g x \quad \text{(6c)}
\]

2.1. Dynamic observer design.

For the observer design we assume that the generator speed, \( \omega_g \), and generator torque, \( Q_g \), is available through measurements. Let us for a moment assume also that the aerodynamic torque is available through measurements. Then we are left with only the state estimation problem. The state estimator is designed by propagating the input signals, \( Q_a \) and \( Q_g \), through (5). The state vector is furthermore updated by a scaling, \( L \), of the error in estimated output as described in (7)

\[
\begin{align*}
\dot{x} &= A \dot{x} + B_r Q_a + B_g Q_g + L (\omega_g - \dot{\omega}_g) \quad \text{(7a)} \\
\begin{bmatrix} \dot{\omega}_r \\ \dot{\omega}_g \end{bmatrix} &= \begin{bmatrix} C_r \\ C_g \end{bmatrix} \dot{x} \quad \text{(7b)}
\end{align*}
\]
The observer gain, $L$, can be designed using a number of different methods. It has been chosen to use the Kalman filtering approach, which is a method that minimises the expected value of the square of the estimation error: $E[(x - \hat{x})^2]$.

In practice, the aerodynamic torque, $Q_a$, is not measurable, which means that we need to extend the observer described by (7) with a term to estimate $Q_a$. In the literature this issue is handled by augmenting the dynamic model by a model of the unknown input to estimate as in (3) – typically with the 2-DOF drive train model instead of the 1-DOF model in the example.

In contrast to the methods in the literature, it has been chosen to split the observer design into two observers operating in a cascaded coupled setup. The inner part is a Kalman filter designed along standard lines on the basis of (7), i.e. under the assumption that $Q_a$ is available. The outer loop is then setup as a tracking configuration with $\omega_g$ as the tracking variable and $\hat{Q}_a$ as the “control signal”. The “controller” has been chosen to be of the PI structure in order to have an integral term taking care of the asymptotic tracking and a direct gain handling the faster variations. The complete observer structure is then as shown in Figure 1.

![Figure 1. Block diagram of observer structure.](image)

This approach has some resemblance to the observer with the augmented wind model because the PI controller can be considered a known wind model with which the model is augmented. The integral term corresponds in this context to the typical wind model, $\dot{Q}_a = 0$, but with the proportional term a quicker response is ensured. In this context it should be noted that by increasing the proportional gain it corresponds to an increase in $L$ in the input direction. This corresponds to increasing the bandwidth of the outer loop to something close to the bandwidth of the inner loop which can potentially lead to instability. Because of this issue, the stability of the interconnection must always be checked a posteriori to the observer design.

The major advantage of this method for observer design is that the split into two interconnected observers lead to a two-step design method. First a state estimator is designed to have a sufficient bandwidth and noise rejection as if the aerodynamic torque is available. Afterwards the input observer can be designed by investigating different structures – with the constraint that there is a sufficient gap in bandwidth between the inner and outer loop.

In the case of estimation of the aerodynamic torque, a $PI$ controller is a natural choice for estimation of the effective wind speed. Process knowledge can also be used to improve the estimation process. It is well-known that the generator speed might suffer from oscillations at the drive train eigenfrequency without having noticeable oscillations in the aerodynamic torque. Because of this it might be advantageous to filter the signal to the $PI$ observer at the drive train eigenfrequency without filtering the signal for the state estimator. In this way the state estimator still estimates the rotor speed correctly, and the noise on the estimate of aerodynamic torque is reduced. Also gain-scheduling of the $PI$ observer can be introduced in order to take into account that there might be different requirements to bandwidth in high wind speeds as opposed to low wind speeds.

2.2. Simulation results
The performance of the observer designed in Section 2.1 is tested against an estimator designed on the basis of solving the differential equation (2) by differentiating the generator speed
measurement. In this estimator, the loss term is described by viscous friction, i.e. \( Q_{\text{loss}} = B_r \cdot \omega \). Further the rotor rotational speed, \( \omega \) is assumed equivalent to the generator speed with the drive train eigenfrequency filtered out. The differentiation will amplify the measurement noise and a first order low pass filter will be used to smoothen out the estimation. This leads to the estimator structure in Figure 2 with \( T \) as the tuning parameter for the tradeoff between time delay in the estimation and noise rejection.

\[ \omega_g \rightarrow \frac{1}{N} \dot{\omega}_r \rightarrow B_r \rightarrow \frac{1}{T_s \cdot (s+1)} \rightarrow \dot{Q}_a \]

**Figure 2.** Block diagram of differentiation based estimator.

For the verification, both estimators have been simulated with identical controller parameters and wind conditions (according to IEC 1A), and in Figure 3 a comparison of the performance of the two estimators is given. It can be observed that the estimation of aerodynamic torque is improved slightly and when comparing standard deviations it can be concluded that there is an improvement of approximately 18% (standard deviation is respectively 72 kNm and 88 kNm for the two algorithms). For the case of estimation of rotor speed, the dynamic observer shows significantly improved performance.

![Illustration of operating region](image1)

**Figure 3.** Left: Simulation results of variables to estimate. Right: estimation error for selected signals (Light blue: observer based estimate, and black: differentiation based estimate).

If we zoom in on a time interval with a large change in aerodynamic torque as in Figure 4, it can be observed that the differentiation based method suffers from a larger time delay in the estimation which is caused by the low pass filtering of the estimate. To counter this, the time constant in the filter can be decreased which has the side-effect that the high frequency noise will be increased and result in an even worse overall performance.
3. Calculation of wind speed

In the previous section a dynamic observer was presented for estimating the rotor speed and aerodynamic torque. These two variables together with measured pitch angle will be used in this section to calculate the effective wind speed by using (1a). First (1a) is rewritten as in (8) and under the assumption that the air density is known, all variables on the left hand side of (8b) will be online available. The right hand side will be a function of $\lambda$ alone, because $\beta$ is online available.

\[
Q_a \omega_r = \frac{1}{2} \rho \pi R^2 \frac{R^3 \omega_r^3}{\lambda^3} c_P(\beta, \lambda) \implies (8a)
\]

\[
\frac{2Q_a}{\rho \pi R^5 \omega_r^2} = \frac{c_P(\beta, \lambda)}{\lambda^3} \quad (8b)
\]

In the following, $c_P(\beta, \lambda)$, for a particular choice of $\beta$ will be denoted $c_{P, \beta}(\lambda)$. The effective wind speed is then calculated by first solving (8b) for $\lambda$ and then calculating the effective wind speed as $v = \frac{\omega_r}{\lambda}$.

In order to be able to solve (8b) for $\lambda$ we first need to understand the monotonicity properties of $\lambda^{-3} \cdot c_{P, \beta}(\lambda)$. $\lambda^{-3}$ is clearly a monotonously decreasing function, but $c_{P, \beta}(\lambda)$ is concave which means that two different tip speed ratio will lead to the same power coefficient, $c_P$: one for the stall region and one for the pitch region. When multiplying these two factors the result is monotonous for some values of $\beta$, whereas it is non-monotonous for other values – determined by the region where the positive slope of $c_{P, \beta}(\lambda)$ is steeper than the negative slope of $\lambda^{-3}$. This issue is illustrated in Figure 5 from which it can be seen that the function is invertible for large pitch angles whereas it is not invertible for small pitch angles.

Because the right hand side of (8b) is not invertible for specific choices of the pitch angle, knowledge about operation of the wind turbine is used to calculate the most likely $\lambda$ that would solve the equation. From the above discussion, the monotonicity can only occur in the stall region because the slope of $c_{P, \beta}(\lambda)$ must be positive. This means that the issue is unlikely to occur during nominal operation because the algorithm is designed for pitch controlled wind turbines not operating in stalled operation. Gusts, or fault situations might on the other hand lead to short periods of time operating in the stalled region. In this case the largest tip speed ratio satisfying the equation is used, which is based on the assumption that it is more likely that the wind turbine is operating in slight stall than in deep stall.

If we investigate a bit further in which operating conditions the problem of monotonicity might happen, a plot of the nominal tip speed ratio and pitch angle is given in Figure 6. From
Figure 5. Illustration of non-monotonicity of $\lambda^{-3} \cdot c_{P,\beta}(\lambda)$.

the illustration it can be seen that the region 10-14 m/s is the most critical operating region for having numerically stable inversion of (8b), because both tip speed ratio and pitch angle are relatively small. Below 10 m/s the issue is still relevant, but the nominal tip speed ratio will be larger relative to the local maximum of $\lambda^{-3} \cdot c_{P,\beta}(\lambda)$, and in order to reduce $\lambda$ to a critical size the gust must therefore be large. For larger mean wind speeds the nominal pitch angle will be larger which means that the function will be monotonously decreasing during nominal operation.

For the signals presented in Figure 3, the procedure described above has been applied to calculate the wind speed estimate shown in Figure 7. From the plot on the right hand side, it can be observed that wind speed estimate is slightly improved by using the dynamic observer as base when comparing with the differentiation based method. And when comparing the standard deviation of the estimation errors it can be seen that the estimate is improved by approximately 15% (standard deviation is respectively 0.20 m/s and 0.23 m/s for the two methods). This improvement in standard deviation between the two methods is similar to that of the estimation of $Q_a$, which indicates that the significantly improved estimation of $\omega_r$ does not increase the performance much in terms of estimation of the effective wind speed.

Figure 6. Nominal operating range.

Figure 7. Left: simulated wind speed. Right: estimation error (Light blue: observer based estimate, and black: differentiation based estimate).

4. Conclusions

This paper has presented a method for estimation of the effective wind speed. The observer consists of two components: A state and input observer for the estimation of rotor speed and aerodynamic torque and a calculation of the effective wind speed by inversion of the monotonous part of a static model of the aerodynamics.
The state and input observer showed a significant improvement in performance, when comparing with methods that solve the estimation problem by solving the differential equation using differentiation.

The calculation of the effective wind speed has shown to be numerically stable during nominal operation. Further investigations are necessary, though to understand how the algorithm will perform in the case of large and fast increases in the mean wind speed – especially in the region around rated generator speed.

It is expected from this improvement in quality of the estimate of effective wind speed that control algorithms that at present time use this variable will benefit from using this algorithm. Those algorithms will typically be controller that are gain-scheduled on wind speed or which use wind speed in a feed forward setting.

In order to achieve an even higher precision in the estimation it might be required to design the observer as one single component, because it is not clear how the estimation error in $Q_a$ and $\omega_r$ transforms into estimation error in effective wind speed. In that case it is necessary to take the aerodynamic model into account, which means that the presented method needs to be extended to nonlinear methods, e.g. by using the unscented Kalman filter for the state estimation. Also the $PI$ observer might need to be modified to a nonlinear observer. This will make the design problem harder in practice but will potentially give a better performance.