High Level Model Predictive Control for Plug-and-Play Process Control with Stability Guaranty

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Abstract—In this paper a method for designing a stabilizing high level model predictive controller for a hierarchical plug-and-play process is presented. This is achieved by abstracting the lower layers of the controller structure as low order models with uncertainty and by using a robust model predictive controller for generating the references for these. A simulation example, in which the actuators in a process control system are changed, is reported to show the potential of this approach for plug and play process control.

I. INTRODUCTION

A complex process, such as a power plant or a water distribution system, might comprise hundreds or thousands of sensors and actuators. Adding or removing just one sensor or actuator, however, might in extreme cases require a complete re-design of the entire control system, with a tremendous cost involved. Currently, such changes are primarily implemented during a scheduled re-commissioning of the process control system, even though changes to the process control system, at the same time the sensor or actuator configuration is changed would have yielded a more optimal performance. The lack of flexibility in such a system and the expenses involved with reconfiguration make plant operators reluctant to implement advanced control technology in the first place or even upgrade the subsystems, for instance by adding sensors or actuators, in order to achieve optimal performance.

Traditionally, the high cost of controller design has been lowered by using PID controllers, and tuning these using heuristic tuning rules. See e.g. [1], [2]. This makes PID control the most commonly used controllers in industrial process control, because of the simple structure and ease of understanding it.

The reluctance towards using model based control technology might in part be ascribed to the expenses involved with re-commissioning, even though, once the model based control system is operational it would yield a better performance.

It would be desired that new hardware, e.g. a new actuator or sensor, could be integrated in a process in a plug and play fashion, i.e., the controller automatically recognises that new hardware has been added to the process, and, using reliable numerical methods, as a result reconfigures itself to accommodate these changes, thus reducing or even removing the load on the designer.

A problem here is that the majority of the existing design methodologies are monolithic, i.e., given an open loop model of a process to be controlled they output a single multi-variable controller.

Drastic changes to a control system, such as implementing a new, single controller when a new piece of hardware has been introduced, are not desirable, since it might be difficult to merge the new controller with the existing software, and the new behaviour of the controlled process might differ significantly from the previous behaviour.

Plug and Play Process Control aims at lowering the cost associated with reconfiguring a process by automatically synthesising new controllers after a process has been reconfigured. See e.g. [3], [4], [5]. This should not be confused with flexible manufacturing systems, where the purpose is to have a single manufacturing system that can manufacture many different types of goods, see e.g. [6] for a survey.

The purpose in plug and play process control is to have one controller that is flexible regarding the individual subsystems, sensors and actuators of the process to be controlled. Figure 1 shows a system, where a new actuator has been plugged in. The controller must then utilise the new hardware.

![Diagram of a plug and play process control system](image)

Fig. 1. A plug and play process control system; a new actuator is added to an existing system.

Plug and play algorithms that reconfigure a model based control system in a localised manner were investigated in [7], where the process to be controlled was divided into a hierarchical system, such that localised changes to the controller could be made. This approach had previously been used for solving difficult problems, see e.g. [8], [9] and [10].

Synthesising a controller for a network of identical subsystems has been reported in [11], where the controllers are synthesised using all available information. Stability of a network of nonlinear systems has been investigated in [12] and [13].

This paper aims at designing a high level model predictive...
controller for a hierarchical system, that remains stable after changes in the plant, in the form of faster actuators.

II. PROBLEM FORMULATION

In the following a plant, actuator and controller setup as shown in Figure 2 is considered.

![Diagram](image-url)

Fig. 2. The structure of the considered problem.

The goal is to construct a model predictive controller that is flexible with respect to the actual actuators plugged in to the system, while minimizing the reference tracking error $e(t) = r(t) - y(t)$.

Only discrete time systems are considered in the following, and, where there is no risk of confusion, the time dependence of signals will be omitted, e.g. $x = x(t)$. Furthermore $+$ will be used as a shorthand for $(t+1)$, i.e., $x^+ = x(t+1)$.

A. Plant Model

The plant model is given as a state space model:

$$x^+ = Ax + \sum_{n=1}^{N_a} B_n (u_n + w_n),$$

where $u_n$ is the reference to actuator $n$, $w_n$ is the tracking error of actuator $n$, and $N_a$ is the number of actuators. Letting $B = [B_1, \ldots, B_{N_a}]$, $u = [u_1, \ldots, u_{N_a}]^T$, and $w = [w_1, \ldots, w_{N_a}]^T$ gives

$$x^+ = Ax + Bu + Bw.$$  \hspace{1cm} (2)

The states of the plant must be constrained to the set of the form

$$X = \{x \in \mathbb{R}^{n_x} | C_x x \leq k_x \},$$

where $\leq$ is taken as an element wise less than or equal, and $C_x$ is a matrix, so that $X$ is the intersection of a set of half planes. Similarly the inputs to the plant must be in the set

$$U = \{u \in \mathbb{R}^{n_u} | C_u u \leq k_u \}.$$  \hspace{1cm} (4)

The origin must be interior points in $X$ and $U$.

B. Actuator

The actuators are modelled as stable first order systems with a steady state gain of one, i.e.,

$$x_n^+ = a_n x_n + b_n u_n$$

$$y_n = x_n,$$

where $|a_n| < 1$ and $b_n = 1 - a_n$ for all $n \in \{1, \ldots, N_a\}$, $u_n$ is the reference signal from the MPC to the actuator, and $y_n = u_n + w_n$ is the output from the actuator, i.e., the input to the plant, with the tracking error of the actuator given as $w_n$.

Let $x_a = [x_1, \ldots, x_{N_a}]^T$, $u = [u_1, \ldots, u_{N_a}]^T$, $w = [w_1, \ldots, w_{N_a}]^T$, $A_a = \text{diag}(a_1, \ldots, a_{N_a})$, and $B_a = \text{diag}(b_1, \ldots, b_{N_a})$, then

$$x_a^+ = A_a x_a + B_a u$$

$$u + w = x_a.$$  \hspace{1cm} (6)

where $B_a = I - A_a$.

Let the old reference tracking error, $w_o(t) = w(t-1)$, and the old input, $u_o(t) = u(t-1)$, be the states, then the actuator reference tracking error, $w$, can be modelled as

$$w = [w_o]^+ = [A_a I \ 0 \ 0 \ |w_o] + [-I \ I \ 0 \ 0] u$$

$$w = A_a w_o + u_o - u$$

i.e., if the actual actuator configuration is faster than the modelled actuator configuration, the norm of the actuation error $\|w\|$ is smaller than $\sqrt{2} \|([A_a w_o]^T, (\Delta u)^T)\|.$

III. AUXILIARY CONTROLLER

In order to show that a model predictive controller stabilizes the process to be controlled, the existence of an auxiliary controller, that stabilizes the process for all $x$ in a subset of $X$, must be shown.

Reformulate the models given above as

$$\tilde{x}^+ = \tilde{A} \tilde{x} + \tilde{B}_a u + \tilde{B}_w w$$

$$z = \tilde{C} \tilde{x} + \tilde{D} u,$$

where

$$\tilde{x} = \begin{bmatrix} x \\ w_o \end{bmatrix}, \quad z = \begin{bmatrix} k_x \\ A_a w_o \\ \Delta u \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{B}_a = \begin{bmatrix} B \\ 0 \\ I \end{bmatrix}, \quad \tilde{B}_w = \begin{bmatrix} B \\ I \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} kI & 0 & 0 \\ 0 & A_a & 0 \\ 0 & 0 & -I \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \quad k > 0.$$  \hspace{1cm} (12)

This reformulation is seen in Figure 3.

Then, since, for all actuator configurations where $|a_{pn}| \leq |a_n|$,

$$\|w\| \leq \sqrt{2} \left\| \begin{bmatrix} A_a w_o \\ \Delta u \end{bmatrix} \right\| \leq \sqrt{2} \|z\|,$$  \hspace{1cm} (13)
if there exists a control law \( u = K\dot{x} \), final cost, \( V_f(\dot{x}) \), final set, \( X_f \), such that

\[
X_f \subseteq X
\]

\[
K\dot{x} \in U, \forall \dot{x} \in X_f
\]

\[
(\dot{A} + \dot{B}_u K)\dot{x} + B_w w \in X_f, \forall (\dot{x}, w) \in X_f \times W
\]

\[
\Delta V_f(\dot{x}) < \gamma^2 \| w \|^2 - \| z \|^2, \tag{17}
\]

then we can construct a model predictive controller that stabilizes the process. \(^{[14]}\)

Let the auxiliary controller \( K \) be given by the following linked matrix inequalities with \( P > 0 \) and \( \gamma < \frac{1}{\sqrt{2}} \)

\[
0 > \tilde{B}_w^T P \tilde{B}_w - \gamma^2 I \tag{18}
\]

\[
0 > (\dot{A} + \dot{B}_u K)^T P(\dot{A} + \dot{B}_u K) - P + (\dot{C} + \dot{D} K)^T (\dot{C} + \dot{D} K)
\]

\[-(\dot{A} + \dot{B}_u K)^T P \tilde{B}_w
\]

\[\tilde{B}_w^T P \tilde{B}_w - \gamma^2 I)^{-1} \tilde{B}_w^T P(\dot{A} + \dot{B}_u K), \tag{19}\]

the final cost by

\[
V_f(\dot{x}) = \dot{x}^T P \dot{x}, \tag{20}\]

and the final set by

\[
X_f = \{ \dot{x} \in \mathbb{R}^{n_x} | V_f(\dot{x}) \leq \rho \}, \rho > 0. \tag{21}\]

Since the origin is an interior point in \( X \) and \( U \) property (14) and (15) are guaranteed by choosing \( \rho \) sufficiently small. Property (16) follows from (17), the size of \( \gamma \), and the choice of \( X_f \). Property (17) is proved in the following.

Let \( \hat{A} = \dot{A} + \dot{B}_u K \) and \( \hat{C} = \dot{C} + \dot{D} K \), then (18) and (19) can be written as:

\[
0 > \hat{B}_w^T \hat{A} \hat{B}_w - \gamma^2 I \tag{22}
\]

\[
0 > \hat{A}^T P \hat{A} - P + \hat{C}^T \hat{C}
\]

\[-\hat{A}^T \hat{B}_w (\hat{B}_w^T \hat{B}_w - \gamma^2 I)^{-1} \hat{B}_w^T P \hat{A}. \tag{23}\]

The final cost of \( \dot{x}^+ \) is then given as:

\[
V_f(\dot{x}^+) = (w^T \hat{B}_w^T + \dot{z}^T \hat{A}^T) P(\hat{B}_w w + \hat{A} \dot{x})
\]

\[
= w^T \hat{B}_w \dot{P} \hat{B}_w w + \dot{z}^T \hat{A}^T P \hat{A} \dot{x}
\]

\[+ 2w^T \hat{B}_w \dot{P} \hat{A} \dot{x} - \dot{z}^T \dot{z} + \dot{x}^T \dot{C}^T \dot{C} \dot{x}
\]

\[+ \gamma^2 w^T \dot{w} - \gamma^2 \dot{w}^T \dot{w}
\]

\[
= w^T (\hat{B}_w \dot{P} \hat{B}_w - \gamma^2 I) w
\]

\[+ \dot{z}^T (\hat{A}^T \hat{P} + \dot{C}^T \dot{C}) \dot{x}
\]

\[+ 2w^T \hat{B}_w \dot{P} \hat{A} \dot{x} - \| z \|^2 + \gamma^2 \| w \|^2 \tag{24}\]

Letting \( S = \hat{B}_w^T \hat{P} \hat{B}_w - \gamma^2 I \), we know from (18) that \( S < 0 \), and we can show that

\[
\Delta V_f(\dot{x}) = V_f(\dot{x}^+) - V_f(\dot{x})
\]

\[
= \gamma^2 \| w \|^2 - \| z \|^2 + w^T (\hat{B}_w^T \hat{P} \hat{B}_w - \gamma^2 I) w
\]

\[+ \dot{z}^T (\hat{A}^T P + \dot{C}^T \dot{C} - P) \dot{x}
\]

\[+ 2w^T \hat{B}_w \dot{P} \hat{A} \dot{x}
\]

\[
< \gamma^2 \| w \|^2 - \| z \|^2 + w^T (\hat{B}_w^T \hat{P} \hat{B}_w - \gamma^2 I) w
\]

\[+ \dot{z}^T (\hat{A}^T P \hat{B}_w - \gamma^2 I) \dot{B}_w^T \hat{B}_w \dot{P} \hat{A} \dot{x}
\]

\[+ 2w^T \hat{B}_w \dot{P} \hat{A} \dot{x}
\]

\[
\leq \gamma^2 \| w \|^2 - \| z \|^2, \tag{25}\]

where the first inequality follows from (23) and the second from

\[
[ S^{-1} I I S ] \leq 0, \tag{26}\]

which is easily seen from the Schur complement

\[
S \leq 0 \tag{27}\]

\[
S^{-1} - IS^{-1} I \leq 0 \tag{28}\]

\[
I(I - SS^{-1}) = 0, \tag{29}\]

and the proof is complete.

Remark: The maximum \( \rho \) that guarantees properties (14) and (15) can be found as \( \min \{ \rho_1, \ldots, \rho_n \} \), where the individual \( \rho_i \)'s are given as:

\[
\begin{align}
\min_{\dot{x}, \rho_i} & \quad \rho_i \\
\text{s.t.} & \quad C_{x,u}(i) \dot{x} \geq k_{x,u}(i) \\
& \quad \dot{x}^T P \dot{x} \leq \rho_i,
\end{align}
\]

where \( C_{x,u}(i) \) is the \( i^{th} \) row in the matrix

\[
[ \hat{C}_x \\ C_{u} K ], \tag{30}\]

and \( \hat{C}_x = [ C_x \ 0 ] \), so that only \( x \) is chosen from the vector \( \dot{x} \), and \( k_{x,u}(i) \) is the \( i^{th} \) element in the vector

\[
[ k_x \\ k_u ]. \tag{31}\]

The above approach finds the \( \rho_i \) for each half plane, with index \( i \), in \( X \) and \( U \), such that the line describing the half
plane is a tangent to the ellipse $V_f(\tilde{x}) = \rho_i$. This approach is shown in Figure 4.

Remark: The linked matrix inequalities (18) and (19) can be rewritten as LMIs to make synthesis of the auxiliary controller easier. Using Schur’s complement (22) and (23) can be written as:

$$
\begin{bmatrix}
\hat{A}^T P \hat{A} - P + \hat{C}^T \hat{C} & \hat{A}^T P \hat{B}_w \\
(*)^T & \hat{B}_w^T P \hat{B}_w - \gamma^2 I
\end{bmatrix} < 0. 
$$

Inserting $\hat{A} = \hat{A} + \hat{B}_u K$ and $\hat{C} = \hat{C} + \hat{D} K$, and using Schur’s Complement again gives

$$
\begin{bmatrix}
P^{-1} & 0 & \hat{A} + \hat{B}_u K & \hat{B}_w \\
(*)^T & I & \hat{C} + \hat{D} K & 0 \\
(*)^T & (*)^T & P & 0 \\
(*)^T & (*)^T & (*)^T & \gamma^2 I
\end{bmatrix} > 0. 
$$

Pre and post multiplication with $\text{diag}\{I, I, P^{-1}, I\}$, and a change of variables such that $\gamma^2 = \mu$, $P^{-1} = G$, and $KP^{-1} = L$ gives the LMI

$$
\begin{bmatrix}
G & 0 & \hat{A} G + \hat{B}_u L & \hat{B}_w \\
(*)^T & I & \hat{C} G + \hat{D} L & 0 \\
(*)^T & (*)^T & G & 0 \\
(*)^T & (*)^T & (*)^T & \mu I
\end{bmatrix} > 0, 
$$

in the variables $G = G^T$, $L$, and $\mu$.

A. Reference Tracking

By adding a reference such that the plant controlled by the auxiliary controller has a structure as seen in Figure 5, then (13), (14), (15), (16), and (17) still holds.

![Fig. 5. The structure after the reference is added.](image)

IV. MODEL PREDICTIVE CONTROLLER

The high level model predictive control computes the control signal $u(t)$ at each time step $t$, in accordance with the receding horizon principle.

$$
\min_{u(t), \ldots, u(t+N-1)} V_f(\tilde{x}(t + N))
$$

subject to

$$
\begin{align*}
\sum_{i=t}^{t+N-1} & \|z(i)\|^2 - \gamma^2 \|w(i)\|^2 \\
\tilde{x}^+ & = \tilde{A} \tilde{x} + \tilde{B}_u u + \tilde{B}_w w \\
z & = \tilde{C} \tilde{x} + \tilde{D} u \\
\tilde{x}(t) & = \tilde{x}_i \\
w & = f_w(\tilde{x}, u) \\
x(i) & \in X, u(i) \in U, \\
\forall i \in \{t, \ldots, t + N - 1\}, \\
x(t + N) & \in X_f,
\end{align*}
$$

where

$$
f_w(\tilde{x}, u) = A_{pa} w_o + u_o - u
$$

is a local simulator of the actuator tracking error given an actuator configuration with parameters $A_{pa}$.

This problem is the same as the finite horizon closed-loop differential game reported in [14], but since the uncertainty, $w$, is given at each time step by the local simulator of the tracking error, there is no need to maximize the performance function with respect to it, and open loop control policies can be used.

Furthermore, because of (13), (14), (15), (16), and (17), if there exists an auxiliary controller $K$ such that $\gamma < \frac{1}{\sqrt{2}}$ then the given MPC stabilizes the uncertain system given by the high level model and the corresponding set of actuators.

V. EXAMPLE

The above was used for designing a stabilizing model predictive controller for a linear, constrained process, where the actuator is changed after the initial commissioning. An overview of the process and control system is seen in Figure 6.

![Fig. 6. Overview of the process and control system.](image)

The process model is given as the following first order system:

$$
x^+ = 1.1 x + u_1 + w_{p1}, \quad x \in X, \quad u_1 \in U,
$$

with

$$
X = \{x|\underline{x} \leq x \leq \bar{x}\}
$$

$$
U = \{u|u_1 \leq u_1 \leq \bar{u}_1\}
$$
The initial actuator configuration is given as:

\[ x_1^+ = 0.3x_1 + 0.7u_1 \]  
\[ w_1 = x_1 - u_1. \]  

Reformulating the process in order to find an auxiliary controller and adding a reference gives the following model:

\[ \tilde{x}^+ = \tilde{A}\tilde{x} + \tilde{B} \begin{bmatrix} u \\ w \end{bmatrix} \]  
\[ z = \tilde{C}\tilde{x} + \tilde{D} \begin{bmatrix} u \\ w \end{bmatrix}, \]  

with

\[ \tilde{A} = \begin{bmatrix} 1.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \]  
\[ \tilde{C} = \begin{bmatrix} 0.2394 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0.3 & 0 \end{bmatrix}, \]  

and

\[ \tilde{D} = \begin{bmatrix} 0 & 0 & -0.2394 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \]  

From finding an auxiliary controller that satisfies \( \gamma < \frac{1}{\sqrt{2}} \) the final cost is found to be

\[ V_f(\tilde{x}) = \tilde{x}^T \begin{bmatrix} 0.4673 & -0.0570 & 0.5664 \\ -0.0570 & 0.1470 & -0.0691 \\ 0.5664 & -0.0691 & 1.0838 \end{bmatrix} \tilde{x}, \]  

and the maximum final set, \( X_f \), was found as

\[ X_f = \{ \tilde{x} | V_f(\tilde{x}) \leq 0.6727951 \}. \]  

With the above, a model predictive controller, as shown in (35), for the plant, with a horizon of \( T = 10 \) was formulated, and the closed loop system was simulated.

The output from the simulation, where at time \( t < 15 \) the reference is given as \( r(t) = \bar{x} \) and for \( t \geq 15 \), \( r(t) = x \) can be seen in Figure 7. Also, at time \( t = 15 \) the actuator is changed from the initial actuator to a faster actuator, with dynamics given as

\[ x^+_{1p} = 0.1x_{1p} + 0.9u_1 \]  
\[ w_{1p} = x_{1p} - u_1, \]  

and a new simulator of the tracking error is formulated.

The actuator output, \( u + w \), is shown with the bounds on the actuator output, \( u + \sqrt{2}||aw_o, \Delta u|| \) and \( u - \sqrt{2}||aw_o, \Delta u|| \), in Figure 8.

VI. CONCLUSION AND FUTURE WORK

This paper presented a high level model predictive controller for plug-and-play process control, that maintains stability even under reconfiguration of the process to be controlled by improving the actuator configuration.

The developed model predictive controller was used in an example where the actuator configuration was changed after commissioning, showing the potential of this approach for Plug-and-Play Process control.

Though the approach in this paper has been taken in order to ensure stability in the face of plant improvements, it can also be used to ensure stability for a plant with degrading actuators, as long as the actuator does not degrade past the initially chosen parameters describing it.

Many extensions can be pursued, such as the combination of this approach with heuristic tuning algorithms for hierarchical systems as in [7], or the combination of this work and the work in [15].
REFERENCES