Modeling and Control of a Single-Phase Marine Cooling System

Michael Hansen\textsuperscript{a,}\textsuperscript{*}, Jakob Stoustrup\textsuperscript{b}, Jan Dimon Bendtsen\textsuperscript{b}

\textsuperscript{a}A.P. Møller - MaERSK A/S, Esplanaden 50, DK-1098, Copenhagen, Denmark, also affiliated with Aalborg University, Fredrik Bajers Vej 7, DK-9220, Aalborg, Denmark
\textsuperscript{b}Aalborg University, Fredrik Bajers Vej 7, DK-9220, Aalborg, Denmark

Abstract

This paper presents two model-based control design approaches for a single-phase marine cooling system. Models are derived from first principles and aim at describing significant system dynamics including nonlinearities and transport delays, while keeping the model complexity low. The two approaches investigated are: A baseline design for performance comparison, and a nonlinear robust control design. Performance and robustness of performance for the two control designs are evaluated through a simulation example. Both designs show good robustness towards parameter variations, while the nonlinear robust design performs better in terms of disturbance rejection.

Keywords: Modelling, Robust control, Feedback linearization, Nonlinear systems

1. Introduction

To this day, maritime transportation remains the most energy efficient means of transportation when considering fuel consumption per ton freighted goods (Rodrigue et al., 2006). However, there still exist a significant potential for energy optimization when it comes to container vessels, especially when considering ship subsystems and their interconnections. With a growing attention to reduction of CO\textsubscript{2}, SO\textsubscript{x} and NO\textsubscript{x} emissions from maritime transportation (Faber et al., 2009), combined with fluctuating oil prices in recent years (Beverelli et al., 2010), there are now strong incentives to improve the energy efficiency of ocean-going vessels.

This paper presents a number of results on modeling and control of a cooling system that can be found aboard several classes of container vessels in operation today. The overall aim is to lower the energy consumption of this cooling system, while ensuring both sufficient cooling and stability in the presence of disturbances such as main engine load conditions and seawater temperature.

Improving control of this particular system has two significant impacts on the overall energy efficiency of the vessel: The first and most direct impact is through reduction of pump power consumption by lowering flow rates in the system such that only necessary cooling is generated. From the affinity laws it is known that there is a cubic relationship between pump flow rate and pump power consumption, which in other terms means that a flow rate reduction of e.g. 10\% results in a reduction of pump power consumption by about 27\%.

The second impact concerns machinery connected to the cooling system, i.e., the consumers of the system. Since the energy efficiency of each consumer often depends on the operating conditions provided by the cooling system, there is a potential for lowering the energy consumption by ensuring that these
operating conditions are optimal. An example of this could be cooling of the scavenge air for the main engine (typically a two-stroke diesel engine) of the ship, where the temperature of the scavenge air influences the specific fuel oil consumption (SFOC) of the main engine. From an energy efficiency point of view, the scavenge air temperature should be kept as low as possible as this decreases the SFOC (MAN Diesel and Turbo, 2010). However, the temperature cannot be arbitrary low as this increases condensation and the risk of water carry-over to the cylinders which can cause severe wear on the cylinder liner (Aeberli, 2005). By controlling the temperature of the scavenge air to a predefined optimal set point it is possible to lower the SFOC of the main engine, while avoiding too much condensation with resulting water carry-over to the cylinders.

Related work can be found in (Mrakovic et al., 2004) which considers the derivation of a numerical model for the same type of marine cooling system as considered here, but with focus on simulation and transient analysis when designing the cooling system layout. Derivation of models for the marine cooling system aimed specifically at control design in terms of structure and complexity was considered previously by the authors in (Hansen et al., 2011b). Based on these models, design of simple control laws for baseline comparison was dealt with in (Hansen et al., 2011a). In (Hansen et al., 2012) a heuristic but structured approach to a nonlinear robust control design for the thermodynamic part of the cooling system aimed specifically at control design in terms of structure and complexity was considered previously by the authors in (Hansen et al., 2011b). Based on these models, design of simple control laws for baseline comparison was dealt with in (Hansen et al., 2011a). In (Hansen et al., 2012) a heuristic but structured approach to a nonlinear robust control design for the thermodynamic part of the cooling system is presented and applied to the cooling system to compensate for nonlinearities and transport delays. This paper summarizes, expands and compares results from these papers to give a full overview of the status as of today.

The structure of the paper is as follows: Section 2 briefly outlines the structure and mode of operation for the cooling system considered in this work. This is followed by first principle models for the hydraulics and thermodynamics of the cooling system. In Section 3 control laws for the cooling system hydraulics are initially designed. This is followed by a baseline control design for the thermodynamic part of the cooling system, using classical control theory. The final part of Section 3 presents a heuristic but structured approach to a nonlinear robust control design for the thermodynamic part of the cooling system. Performance of the nonlinear robust control is compared to the baseline control in Section 4 through a simulation example and concluding remarks are presented in Section 5.

The following notation is used: \( \mathbb{R} \) denotes the set of real numbers while \( \mathbb{R}_+ \) denotes the set of non-negative real numbers. \( \mathbb{R}^{n \times m} \) is the set of real \( n \times m \) matrices, and \( C^k(M,N) \) is the set of continuous functions mapping from vector space \( M \) to vector space \( N \) with continuous derivatives up to order \( k \). \( I_n \) denotes the \( n \times n \) identity matrix, while \( 0_{n,m} \) denotes the \( n \times m \) zero matrix.

2. Modeling

The cooling system consists of three circuits; a seawater (SW) circuit, a low temperature (LT) circuit and a high temperature (HT) circuit. This setup is illustrated in Figure 1, where \( q_{LT} \) and \( q_{SW} \) are volumetric flows in the LT and SW circuits, while \( q_{HT} \) is the volumetric flow to the HT circuit.

The SW circuit pumps water from the sea through the cold side of the central coolers for lowering the temperature of the coolant in the LT and HT circuits. The LT circuit contains auxiliary machinery such as diesel generators, air condition units, and turbochargers, all placed in a parallel configuration. The HT circuit contains the main engine (ME) of the ship,
and since the cooling demand for this component is very strict there is little room for energy optimization in this part of the system from a cooling point of view. As a result, the main focus in this work is therefore the LT and SW circuits.

In the current mode of operation, all pumps in the system are controlled in open loop; that is, pumps are running at one of two speeds depending on the temperature of the seawater and the ME load. Coolant temperature is controlled in two places by the three-way valves illustrated in Figure 1. This is done by either shunting coolant past the central coolers, or adjusting the amount of coolant that is recirculated in the HT circuit. The layout of the system is designed such that sufficient cooling can be provided for the machinery even under worst-case conditions, i.e. at high seawater temperature and maximum ME load. However, this is at the expense of excess cooling being generated when the ship is not operating under worst case conditions, which is most of the time.

In the following it is assumed that all flows are turbulent and there are no laminar flow effects. There is also no heat loss to surroundings, i.e. heat exchange only takes place in the consumers or in the central coolers. The coolant does not undergo phase changes and its density as well as specific heat is assumed to be constant in the temperature range of interest. Finally, the coolant in the system is assumed to be incompressible.

A model-based approach to control design is taken in this work, which entails deriving dynamic models for the marine cooling system. The model is divided into two parts, configured in series: A hydraulic part and a thermodynamic part. Time is denoted as \( t \in \mathbb{R}_+ \), while pump head, \( \Delta h_p(t) \in \mathbb{R}_+ \), and hydraulic resistance of control valves, \( \varphi_{cv}(t) \in \mathbb{R}_+ \), are the manipulated inputs for the hydraulic model. Similar, the system flow rates, \( q_v(t) \in \mathbb{R}^{v+1} \), are outputs of the hydraulic model and act as manipulated inputs for the thermodynamic model. The coolant temperature at the outlet of the central coolers and consumers, \( T(t) \in \mathbb{R}^{v+1} \), are thermodynamic model outputs, while disturbances acting on the thermodynamic model, \( W(t) \in \mathbb{R}^{v+1} \), cover heat transfer from the consumers and the SW temperature. This setup is illustrated in Figure 2.

**Figure 2: Model overview and configuration.**

\[
\begin{align*}
\varphi_{cv}(t) & \quad \Delta h_p(t) & \quad \varphi_{cv}(t) & \quad T(t)
\end{align*}
\]

**Hydraulic Model**

It is assumed that the hydraulic layout of the SW and LT circuits conforms to the structure depicted in Figure 3. It should be noted that the 3-way valve and central cooler bypass from Figure 2 is not included in Figure 3. The argument is that a main objective with improving control of the cooling system is to make the 3-way valve redundant such that no coolant flows through the bypass.

\[
\begin{align*}
q_p, k & \quad h_i & \quad h_j
\end{align*}
\]

In this work the general framework from (De Persis & Kallesøe, 2011) and (De Persis & Kallesøe, 2008) is adopted for modeling the hydraulic network. The method is based on network theory and the well known analogy between electrical and hydraulic circuits, where voltage and current corresponds to pressure and flow, respectively. The following models for the hydraulic components in the cooling system are used:

**Valve model:**

\[
egin{align*}
\mu \in C^\infty(\mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R})
\end{align*}
\]

where \( \mu = \mu(K_{v,k}, q_{v,k}) \),

\[
\begin{align*}
\begin{align*}
q_p, k & \quad h_i & \quad h_j
\end{align*}
\end{align*}
\]

\[
\begin{align*}
q_v, k & \quad h_i & \quad h_j
\end{align*}
\]

\[
\begin{align*}
Pump & \quad Valve & \quad Pipe
\end{align*}
\]

In the current mode of operation, all pumps in the system are controlled in open loop; that is, pumps are running at one of two speeds depending on the temperature of the seawater and the ME load. Coolant temperature is controlled in two places by the three-way valves illustrated in Figure 1. This is done by either shunting coolant past the central coolers, or adjusting the amount of coolant that is recirculated in the HT circuit. The layout of the system is designed such that sufficient cooling can be provided for the machinery even under worst-case conditions, i.e. at high seawater temperature and maximum ME load. However, this is at the expense of excess cooling being generated when the ship is not operating under worst case conditions, which is most of the time.

In the following it is assumed that all flows are turbulent and there are no laminar flow effects. There is also no heat loss to surroundings, i.e. heat exchange only takes place in the consumers or in the central coolers. The coolant does not undergo phase changes and its density as well as specific heat is assumed to be constant in the temperature range of interest. Finally, the coolant in the system is assumed to be incompressible.

A model-based approach to control design is taken in this work, which entails deriving dynamic models for the marine cooling system. The model is divided into two parts, configured in series: A hydraulic part and a thermodynamic part. Time is denoted as \( t \in \mathbb{R}_+ \), while pump head, \( \Delta h_p(t) \in \mathbb{R}_+ \), and hydraulic resistance of control valves, \( \varphi_{cv}(t) \in \mathbb{R}_+ \), are the manipulated inputs for the hydraulic model. Similar, the system flow rates, \( q_v(t) \in \mathbb{R}^{v+1} \), are outputs of the hydraulic model and act as manipulated inputs for the thermodynamic model. The coolant temperature at the outlet of the central coolers and consumers, \( T(t) \in \mathbb{R}^{v+1} \), are thermodynamic model outputs, while disturbances acting on the thermodynamic model, \( W(t) \in \mathbb{R}^{v+1} \), cover heat transfer from the consumers and the SW temperature. This setup is illustrated in Figure 2.

\[
\begin{align*}
\varphi_{cv}(t) & \quad \Delta h_p(t) & \quad \varphi_{cv}(t) & \quad T(t)
\end{align*}
\]

**Figure 2: Model overview and configuration.**

**Hydraulic Model**

It is assumed that the hydraulic layout of the SW and LT circuits conforms to the structure depicted in Figure 3. It should be noted that the 3-way valve and central cooler bypass from Figure 2 is not included in Figure 3. The argument is that a main objective with improving control of the cooling system is to make the 3-way valve redundant such that no coolant flows through the bypass.

\[
\begin{align*}
q_p, k & \quad h_i & \quad h_j
\end{align*}
\]

In this work the general framework from (De Persis & Kallesøe, 2011) and (De Persis & Kallesøe, 2008) is adopted for modeling the hydraulic network. The method is based on network theory and the well known analogy between electrical and hydraulic circuits, where voltage and current corresponds to pressure and flow, respectively. The following models for the hydraulic components in the cooling system are used:

**Valve model:**

\[
egin{align*}
\mu \in C^\infty(\mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R})
\end{align*}
\]

where \( \mu = \mu(K_{v,k}, q_{v,k}) \),

\[
\begin{align*}
\begin{align*}
q_p, k & \quad h_i & \quad h_j
\end{align*}
\end{align*}
\]

\[
\begin{align*}
q_v, k & \quad h_i & \quad h_j
\end{align*}
\]

\[
\begin{align*}
Pump & \quad Valve & \quad Pipe
\end{align*}
\]
distinguish them from non-controllable valves.

**Pipe model:**

$$J_k \frac{dq_{p,k}}{dt} = (h_i - h_j) - \lambda(K_{p,k}, q_{p,k}),$$  \hspace{1cm} (2)

where $J_k$ is the inertance for the $k$'th pipe section while $K_{p,k}$ is the hydraulic resistance for the $k$'th pipe section. Also, $(h_i - h_j)$ is the pressure drop along the pipe, $q_{p,k}$ is the flow through the $k$'th pipe and $\lambda \in C^\infty(\mathbb{R}^+ \times \mathbb{R}, \mathbb{R})$ is a strictly monotonically increasing function of both its arguments.

**Pump model:**

$$h_i - h_j = -\Delta h_{p,k},$$  \hspace{1cm} (3)

where $\Delta h_{p,k}$ is the delivered pressure by the $k$'th pump. In this scope, a quasi-static pump response is assumed such that the pump moves from one steady state operating point to another. This means that pump dynamics is neglected, which is justified by the fact that the rate of change for this application is much less than one tenth of the rotational frequency of the shaft (Brennen, 1994). Also, it is assumed for simplicity that pumps are able to deliver the required pressure regardless of the flow through the pump.

By applying Kirchoff’s loop law to each fundamental loop in the hydraulic network it is possible to set up equations for the relationship between pressure and flow for the loop. To this end, the fundamental loop matrix $B \in \mathbb{R}^{p \times m}$ is introduced:

$$B_{ij} = \begin{cases} 
1 & \text{if component } j \text{ is in loop } i \text{ and directions agree} \\
-1 & \text{if component } j \text{ is in loop } i \text{ and directions do not agree} \\
0 & \text{if component } j \text{ is not in loop } i
\end{cases}$$  \hspace{1cm} (4)

where $i = 1, 2, \ldots, m$ and $m$ is the total number of components in the hydraulic network, while $j = 1, 2, \ldots, p$ and $p$ is the number of loops in the associated graph equivalent to the number of consumers. Agreement of direction in this context relates to the reference direction defined for the component and loop, respectively. In this work, the choice of reference directions is defined such that signs in $B$ are positive. The aim is a model of the form:

$$\dot{q} = f(q, \varphi_{cv}, \Delta h_p),$$  \hspace{1cm} (5)

where $q = [q_1, q_2, \ldots, q_p]^T$ is a vector of free flows, i.e. the flows through each loop in the network; see Figure 3. By combining the individual component models it is possible to characterize the differential pressure for each component with:

$$\Delta h_i = J_i \dot{q}_{c,i} + \lambda_i(K_{p,i}, q_{c,i}) + \mu(K_{v,i}, q_{c,i})$$
$$+ \mu_{cv,i}(\varphi_{cv,i}, q_{c,i}) - \Delta h_{p,i},$$  \hspace{1cm} (6)

It applies that: $J_i = \lambda_i = \mu_i = \mu_{cv,i} = 0$ if the $i$'th components is a pump, $\mu_i = \mu_{cv,i} = \Delta h_{p,i} = 0$ if the
\[ i^\text{th} \text{ component is a pipe}, \ J_i = \lambda_i = \Delta h_{p,i} = \mu_{c,i} = 0 \text{ if the } i^\text{th} \text{ component is a valve, and } J_i = \lambda_i = \Delta h_{p,i} = \mu_i = 0 \text{ if the } i^\text{th} \text{ component is a control valve. } \Delta h \text{ and } q_i \text{ are defined as vectors where the elements are the pressure drops of and flows through each component in the network, respectively. Then, from Kirchhoff's loop law and the definitions of } q_i, q_c, \Delta h \text{ and } B, \text{ it is implied that: } 0 = B^T \Delta h \text{ and } q_i = B^T q. \]

To simplify notation, the following definitions are introduced:

\[
\Delta h_p = [\Delta h_{p,1}, \ldots, \Delta h_{p,m}]^T, \quad J = \text{diag}(J_1, \ldots, J_k), \quad \lambda(K_p, q_c) = [\lambda_1(K_{p,1}, q_{c,1}), \ldots, \lambda_m(K_{p,m}, q_{c,m})]^T, \quad \mu(K_v, q_c) = [\mu_1(K_{v,1}, q_{c,1}), \ldots, \mu_m(K_{v,m}, q_{c,m})]^T, \quad \mu_c(\varphi_{cv}, q_c) = [\mu_{c,1}(\varphi_{cv,1}, q_{c,1}), \ldots, \mu_{c,m}(\varphi_{cv,m}, q_{c,m})]^T.
\]

From this the resulting model can be written as:

\[
BJ^T q = -B\lambda(K_p, B^T q) - B\mu(K_v, B^T q) \quad \text{(8)}
\]

where \( B\lambda(K_p, B^T q) \) is the pressure drop due to pipe elements and \( B\mu(K_v, B^T q) \) is the pressure drop due to valve elements. The changes in potential and kinetic energy in the control volume can be described as the sum of work transfer and internal energy change of the fluid elements in the system. Also, \( \Omega \) is the control volume, \( v = v_{in} + v_{out} \) is the average velocity of the flow at in- or outlet, and \( A \) is the control volume cross section area at the in- or outlet. As there is only a single flow in and out of the volume, it is apparent that:

\[
\dot{m} = \rho_{in} v_{in} = \rho_{out} v_{out}, \quad (14)
\]

where \( \dot{m} \) is the mass flow through the volume. The rate of work transfer can be described as the sum of pressure forces acting on the inlet and outlet of the control volume:

\[
\sum W_s = \dot{W}_{pf} = \rho_{in} v_{in} - \rho_{out} v_{out}, \quad (15)
\]

The changes in potential and kinetic energy in the control volume is neglected, implying that \( e = u \) where \( u \) is the internal energy per mass unit. Furthermore, heat transfer to the system is denoted by \( \sum Q_s = \dot{Q} \). From the definition of specific enthalpy, \( H = u + \frac{e}{\rho} \), the expression becomes:

\[
\frac{d}{dt} \int_{\Omega} e \rho \, dV = \dot{m} (H_{in} - H_{out}) + \dot{Q}, \quad (16)
\]

As there is no phase change of the coolant, it is possible to approximate the specific enthalpy by \( \Delta H = \rho \frac{dH}{dt} \).
\[ c_p \Delta T_c, \text{ where } c_p \text{ is the specific heat for the coolant and } T_c \text{ is the temperature of the coolant (Massoud, 2005).} \]

Preferably, the model should express the change of energy in the volume as a function of the outlet temperature. To achieve this it is approximated that:

\[
\frac{d}{dt} \int_{\Omega} e(t) \rho \, dV \approx \rho c_p V_{CV} \frac{d T_{out}(t)}{dt},
\]

where \( V_{CV} \) is the volumetric size of the control volume. Thereby, the resulting consumer model becomes:

\[
\hat{T}_{out}(t) = \frac{1}{\rho c_p V_{CV}} \left[ \dot{m}(t)c_p(T_{in}(t) - T_{out}(t)) + \dot{Q}(t) \right].
\]

(17)

Delays \( D_1, \ldots, D_p \) arise from transport of coolant from the central coolers to the respective consumers in the cooling system. This means that the delays depend on the layout of the system as well as the flow rates to the individual consumers. Based on the structure in Figure 4 it is assumed that the delays can be described by:

\[
D_i = \sum_{j=1}^{i} \left( a_{m,j} \sum_{k=j}^{n} q_k^{-1} \right) + a_{c,i} q_i^{-1},
\]

(18)

where \( q_i \) is the volumetric flow to the \( i \)th consumer, while \( a_{m,i} \) and \( a_{c,i} \) correspond to inner volumes of the main pipe section \( i \), and consumer line pipe section \( i \), respectively. It is assumed that this model is applicable to all consumers in the cooling system and by use of the notation from Figure 4 it is given that for \( i = 1, \ldots, p \):

\[
\hat{T}_i(t) = \frac{1}{\rho c_p V_i} \left[ q_i(t) c_p (T_{LT,in}(t) - D_i) - T_i(t) \right] + \dot{Q}_i(t).
\]

(19)

For the central coolers the same model is applied as for the consumers, except that the heat transfer between the LT and SW circuit is written as a function of temperatures and mass flow rates. Assuming steady state heat transfer from the LT circuit to the SW circuit the model is given by:

\[
\dot{T}_{LT,in}(t) = \frac{1}{V_{CC}} \left[ q_{LT}(t)(T_{LT,out}(t) - T_{LT,in}(t)) + q_{SW}(t) \frac{\rho_{sw} c_{p,sw}}{\rho c_p} T_{SW,in}(t) - q_{SW}(t) \frac{\rho_{sw} c_{p,sw}}{\rho c_p} T_{SW,out}(t) \right].
\]

(20)

3. Control Structure and Design

With the model structure presented in Section 2 a cascaded control configuration as illustrated in Figure 5 is pursued. This partitioning is justified by the assumption that the thermodynamics in the system does not influence the hydraulics and there is a separation of time scales since the time constants of the heat dynamics are significantly slower than those of the hydraulics. State-feedback is considered for the flow controller in the inner loop of Figure 5. Two designs are considered for the temperature control in the outer loop: A baseline design based on PI control and a nonlinear robust design based on feedback linearization and \( H_\infty \) control. This structure is chosen to help compensate for the nonlinear dynamics in the hydraulic model, and while state-feedback is considered in this work for the inner loop controller, other linear design techniques could have been employed, such as the one presented in (Hansen et al., 2011a).
3.1. Flow Control

Design of control for the inner loop in Figure 5, is based on the model derived in Section 2. For notational simplicity and without loss of generality it is assumed that the number of consumers in the cooling system is \( p = 2 \). From (8) it is possible to write the independent flows through consumer 1 and 2 as:

\[
\begin{align*}
\dot{q}_1 &= -K_{11}\dot{q}_1 - K_{12}\dot{q}_2\varphi_{cv,1} + K_{13}\dot{q}_2\varphi_{cv,2} + K_{14}\Delta h_{p,LT} - K_{15}(q_1 + q_2)^2 + K_{16}\dot{q}_1^2 \\
\dot{q}_2 &= -K_{21}\dot{q}_1 - K_{22}\dot{q}_2\varphi_{cv,2} + K_{23}\dot{q}_2\varphi_{cv,1} + K_{24}\Delta h_{p,LT} - K_{25}(q_1 + q_2)^2 + K_{26}\dot{q}_1^2
\end{align*}
\]

where \( K_{ij} \) are positive system specific parameters, while \( \varphi_{cv,1} \) and \( \varphi_{cv,2} \) denotes the hydraulic resistances of control valve 1 and 2, respectively. It is assumed that the LT pump delivers a constant pressure, \( \Delta h_{p,LT} \). By linearizing (22) and (23) using a first order Taylor approximation at the operating point \((\dot{q}_1, \dot{q}_2, \dot{q}_{SW}, \varphi_1, \varphi_2)\), it is possible to obtain the state equations:

\[
\begin{bmatrix}
\dot{\hat{q}}_1 \\
\dot{\hat{q}}_2 \\
\dot{\hat{q}}_{SW}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 & 0 \\
0 & 0 & a_{33} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{q}_1 \\
\hat{q}_2 \\
\hat{q}_{SW}
\end{bmatrix}
+ \begin{bmatrix}
b_{11} & b_{12} & 0 & 0 & 0 \\
b_{21} & b_{22} & 0 & 0 & 0 \\
0 & 0 & b_{33} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\varphi_{cv,1} \\
\varphi_{cv,2} \\
\Delta h_{p,SW}
\end{bmatrix},
\]

or:

\[
\dot{\hat{q}}_s = M\hat{q}_s + N\hat{\psi},
\]

where:

\[
\begin{align*}
a_{11} &= 2(-K_{11}\dot{q}_1 - K_{12}\dot{q}_2\varphi_{cv,1} - K_{15}\dot{q}_1 - K_{15}\dot{q}_2) \\
a_{12} &= 2(K_{13}\dot{q}_2\varphi_{cv,2} - K_{15}\dot{q}_2 - K_{15}\dot{q}_1 + K_{16}\dot{q}_2) \\
a_{21} &= 2(K_{23}\dot{q}_1\varphi_{cv,1} - K_{25}\dot{q}_1 - K_{25}\dot{q}_2 + K_{26}\dot{q}_1) \\
a_{22} &= 2(-K_{21}\dot{q}_2 - K_{22}\dot{q}_2\varphi_{cv,2} - K_{25}\dot{q}_2 - K_{26}\dot{q}_1) \\
a_{33} &= (-2K_{SW}\dot{q}_{SW})J_{SW}^{-1}
\end{align*}
\]

\[
\begin{align*}
b_{11} &= K_{12}\dot{q}_1^2 \\
b_{12} &= K_{13}\dot{q}_2^2 \\
b_{21} &= K_{23}\dot{q}_1^2 \\
b_{22} &= K_{22}\dot{q}_2^2 \\
b_{33} &= J_{SW}^{-1}
\end{align*}
\]

A control law of the form: \( \hat{\psi} = -F\hat{q}_s \) is pursued, where \( F \) is chosen such that \( M - NF \) is Hurwitz. This could for instance be done by solving an LQR problem using the cost function:

\[
J = \int_0^\infty (\hat{q}_s^T Q\hat{q}_s + \hat{\psi}_s^T R \hat{\psi}_s) dt ,
\]

where the matrices \( Q \) and \( R \) for example may be determined using Bryson’s rule (Bryson & Ho, 1969).

Finding the feedback matrix \( F \) is then a matter of solving the algebraic Ricatti equation, which can be done using standard software. In the following, the controlled hydraulic subsystem is considered as a simple finite gain due to the difference in timescale compared to the thermodynamics of the system.

3.2. Baseline Temperature Control Design

Two temperature control designs are considered in the following: A baseline design based on classical
control theory, and a nonlinear robust control design to compensate for nonlinearities and delays. The baseline design is dealt with in this section, while the nonlinear robust design is described in Section 3.3. As the name implies, the baseline control serves the purpose of providing a base of comparison for more advanced control strategies. This also means that delay compensation is not considered in the baseline control design and standard PI controllers are employed.

First, the linearized small perturbation model is determined at the operating point (\(\bar{T}_i, \bar{q}_i, \bar{T}_{LT,in}\)), and subsequently the transfer function from \(\hat{q}_i(s)\) to \(\hat{T}_i(s)\) is found:

\[
G_i(s) = \frac{\hat{T}_i(s)}{\hat{q}_i(s)} = \frac{1}{\bar{q}_i} (\hat{T}_{LT,in} - \bar{T}_i) \cdot (s \bar{q}_i + 1) .
\]

Similar, for the central coolers:

\[
G_{LT,i}(s) = \frac{\hat{T}_{LT,in}(s)}{\bar{q}_{SW}(s)} = \frac{\bar{C}_p \bar{SW} \bar{p}_{SW}}{\bar{C}_{p} \bar{q}_{LT}} (\bar{T}_{SW,in} - \bar{T}_{SW,out}) (s \bar{V}_{CC} + 1) .
\]

Using phase margin, \(PM\), and crossover frequency, \(\omega_0\), as design parameters and choosing PI controllers of the form:

\[
D(s) = k_p \left( 1 + \frac{1}{s T_{int}} \right) ,
\]

the parameters \(k_p\) and \(T_{int}\) can be determined from:

\[
\begin{align*}
|D(s)G(s)|_{s=j\omega_0} &= 1 , \\
\tan^{-1} \left( \frac{\text{Im}(D(s)G(s))}{\text{Re}(D(s)G(s))} \right) \bigg|_{s=j\omega_0} &= PM - 180^\circ .
\end{align*}
\]

3.3. Nonlinear Robust Temperature Control Design

A robust nonlinear design approach is now considered to deal with both the nonlinearities of the system and the transport delays described in Section 2. Initially, states, control inputs and disturbances are defined as:

\[
x = \begin{bmatrix} T_1(t) \\ \vdots \\ T_p(t) \\ T_{in}(t) \end{bmatrix} , \quad u = \begin{bmatrix} q_1(t) \\ \vdots \\ q_p(t) \end{bmatrix} , \quad w_1 = \begin{bmatrix} \dot{Q}_1(t) \\ \vdots \\ \dot{Q}_p(t) \end{bmatrix} .
\]

With the definitions in (32) the state equations can be represented as:

\[
x(t) = \sum_{i=1}^{n_u} f_i(x(t), x(t-D_i(u)))u_i(t) + B_u w_1(t) ,
\]

where \(x \in \mathbb{R}^n_x, u \in \mathbb{R}^n_u, w_1 \in \mathbb{R}^n_d, B_u \in \mathbb{R}^{n_x \times n_d}\) and \(f_i(\cdot)\) are smooth vector fields defined on a relevant subset of \(\mathbb{R}^n_d\).

It is assumed that the parameter varying delays are bounded, i.e., all \(D_i(u), i = 1, \ldots, n_u\), belong to the set:

\[
D := \{D \in C(\mathbb{R}, \mathbb{R}); 0 \leq D(u) \leq \overline{D} < \infty \quad \forall u \in \mathbb{R}^n_u \}.
\]

Ensuring delays are bounded requires that flow rates must be strictly positive for the system to be at an equilibrium, and it is thus reasonable to constrain the flows, and thereby the inputs such that:

\[
0 < \underline{u} \leq u_i \leq \overline{u} \quad \forall t \in \mathbb{R}_+ .
\]

Similar, disturbances are assumed to be bounded but unknown, i.e. they belong to the set:

\[
W := \{Q \in C(\mathbb{R}, \mathbb{R}); 0 < \underline{Q} \leq Q(t) \leq \overline{Q} < \infty \quad \forall t \in \mathbb{R}_+ \}.
\]

Initial conditions for the system in (33) are governed by:

\[
x(0) = x_0 , \quad x(\theta) = \phi(\theta) , \quad \theta \in [-\overline{D}, \theta] ,
\]

8
and the state history for \( t \in \mathbb{R}_+ \) is defined as:

\[
x_t(\theta) = x(t + \theta), \quad \theta \in [-T, 0].
\]  

(38)

It is assumed that \( x_t(\theta) \) is available to the controller. 

To ease notation in the following it is defined that:

\[
\Lambda = \frac{\rho_p c_p}{\mathcal{C}_p} (T_{SW,in}(t) - T_{SW,out}(t)) ,
\]

\[
\Phi = T_{LT,out}(t) - T_{LT,in}(t) ,
\]

\[
\Psi_i = (T_{LT,in}(t - D_i) - T_i(t)) .
\]  

(39)

From the definitions in (32) and (39) it is possible to write the nonlinear thermodynamic model as:

\[
\dot{x}(t) = B\gamma(\cdot)u(t) + B_ww_1(t) ,
\]  

(40)

where

\[
B = \begin{bmatrix}
\frac{1}{\mathcal{C}_p} & 0 & \ldots & 0 \\
0 & \ddots & \vdots & \vdots \\
\vdots & \ddots & \frac{1}{\mathcal{C}_p} & 0 \\
0 & \ldots & 0 & \frac{1}{\mathcal{C}_p}
\end{bmatrix} ,
\]

\[
\gamma(\cdot) = \begin{bmatrix}
\psi_1(\cdot) & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \ldots & \psi_p(\cdot) & 0 \\
\phi(\cdot) & \ldots & \phi(\cdot) & \Lambda(\cdot)
\end{bmatrix} ,
\]

\[
B_w = \begin{bmatrix}
\frac{1}{\rho_p \mathcal{V}_1} & 0 & \ldots & 0 \\
0 & \ddots & \vdots & \vdots \\
\vdots & \ddots & \frac{1}{\rho_p \mathcal{V}_p} & 0 \\
0 & \ldots & \ldots & 0
\end{bmatrix} .
\]  

(41)

(42)

(43)

Under the assumption that the flow rates are known, the delays can be estimated using (19). Also, if \( \gamma(\cdot) \) is nonsingular in the domain of interest, one can obtain an equivalent delay free and linear system through feedback linearization by use of the control law:

\[
u(t) = \gamma^{-1}(\cdot)v(t) ,
\]  

(44)

where \( v(t) \) is a linear control input (Khalil, 1996). For \( \gamma \) to be nonsingular in the domain of interest it is sufficient to ensure that the product of the diagonal elements is nonzero, i.e.:

\[
\Lambda \prod_{i=1}^{p} \Psi_i \neq 0 , \quad \forall t \in \mathbb{R}_+ .
\]  

(45)

In the specific case of the marine cooling system it applies that \( \Lambda \) is strictly negative as the domain of interest is when the system is in operation. This means there is a positive heat transfer from the LT circuit to the SW circuit such that \( T_{SW,in}(t) < T_{SW,out}(t) , \forall t \), and the state history for \( x \) is assumed that:

\[
\gamma(\cdot) \text{ is the estimate of system nonlinearities and } \mathbf{n} \in \mathbb{R}^{n_x} \text{ is measurement noise. This may be represented as:}
\]

\[
\dot{x} = B(v + \mathbf{n}) + B_ww_1 ,
\]

\[
y = x + \mathbf{n} ,
\]  

(46)

(47)

(48)

where \( \Delta \) represents the mismatch due to uncertainties and \( w_2 \in \mathbb{R}^{n_x} \) is a disturbances term included to account for these uncertainties in the design. It is assumed that:

\[
\frac{1}{\rho_0} ||\Delta||_{\infty} = \frac{1}{\rho_0} \sup_{\omega} \sigma_{\max}(\Delta(j\omega)) \leq 1 ,
\]  

(48)

where \( \rho_0 \in \mathbb{R}_+ \) can be considered as the maximum percentage error between the nominal input, \( v \), and the actual input. Furthermore, \( B_w \) is scaled to nor-
nalize \( w_1 \) such that \( ||\dot{w}_1||_\infty \leq 1 \). Hence:

\[
\dot{B}_w = \begin{bmatrix}
\overline{v} & 0 & \ldots & 0 \\
0 & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots \\
0 & \ldots & 0 & \overline{v}
\end{bmatrix},
\]

(49)

where \( \overline{v}_1, \overline{v}_2, \ldots, \overline{v}_p \) are upper bounds for the disturbances in \( w_1 \). The problem can then be formulated as in Figure 6. To reject constant disturbances, the linearized model is augmented to include integral error states such that the feedback linearized system is represented as:

\[
\begin{bmatrix}
\dot{x} \\
\dot{e}
\end{bmatrix} = \begin{bmatrix} 0_{n_x} & 0_{n_x} \\ I_{n_x} & 0_{n_x} \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0_{n_x,n_u} \end{bmatrix} v + \begin{bmatrix} \dot{w}_1 \\ 0_{n_x,n_d} \end{bmatrix} \dot{w}_1 + \begin{bmatrix} B \\ 0_{n_x,n_u} \end{bmatrix} w_2 + \begin{bmatrix} 0_{n_x} \\ I_{n_x} \end{bmatrix} n
\]

\[
\begin{bmatrix} y \\ y_e \end{bmatrix} = \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} I_{n_x} \\ 0_{n_x} \end{bmatrix} n,
\]

(50)

where \( e \in \mathbb{R}^{n_x} \) are the integral error states. To deal with the system uncertainties and disturbances, the linear control input \( v \) is designed using robust control theory. To formulate a standard \( H_\infty \) problem (Doyle et al., 1989) the exogenous inputs, \( \dot{w}_1, w_2 \) and \( n \), are combined into a single vector. Also, an error vector, \( z \), penalizing states and control inputs is introduced. This yield:

\[
w = \begin{bmatrix} \dot{w}_1 \\ w_2 \\ n \end{bmatrix}, \quad z = \begin{bmatrix} x \\ \rho_0 v \end{bmatrix}.
\]

(52)

The partitioning from (Doyle et al., 1989) is applied such that:

\[
G(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix},
\]

(53)

where system matrices have been partitioned according to \( z, y, w \) and \( v \), respectively.

As a consequence of all poles being placed on the imaginary axis the problem is non-standard and it cannot be solved with standard \( H_\infty \) theory directly. To overcome this problem a bilinear transformation is applied to shift the poles of the feedback linearized system from the origin and transform the system model into a close approximation which allows for standard \( H_\infty \) control design. Once the controller is designed for the approximate model, the inverse bilinear transformation is applied to get the final controller for the original plant model (Chiang & Safonov, 1991). In this paper, the \( j\omega \)-axis pole shifting transformation is applied, which is described in details in (Chiang & Safonov, 1992). This is given by:

\[
s = \frac{s}{\rho_0} + \frac{p_1}{p_2} + 1,
\]

(54)

where \( p_1, p_2 < 0 \) are the endpoints of the diameter of a circle being mapped by (54) from the left \( s \)-plane into the \( j\omega \)-axis of the \( \tilde{s} \)-plane. The parameters \( p_2 \) and especially \( p_1 \) in (54) plays essential roles when placing dominant closed loop poles in the \( s \)-plane and thus becomes important design parameters for the \( H_\infty \) control design along with the weighting factor, \( \rho_0 \).

![Figure 6: Linear equivalent of feedback linearized marine cooling system with uncertainties and disturbances.](image)

![Figure 7: Structure of flow controller in configuration with the nonlinear robust temperature controller.](image)
4. Simulation Example

To evaluate performance and robustness of performance for the control designs proposed in this paper a simulation scenario is considered, where the number of consumers is two, i.e. $p = 2$.

The simulation model is implemented in MATLAB Simulink using the ODE45 solver, and consists of the nonlinear models from Section 2 i.e., equations (8), (12) and (19)-(21). Saturation of pump pressure has also been included in the simulation model, and so has heat exchanger characteristics based on the logarithmic mean temperature difference (LMTD) approach. By neglecting potential and kinetic energy as well as assuming constant specific heat and that no phase change occur, the heat transfer through the heat exchanger can be expressed by (Massoud, 2005):

$$Q_{HE} = \frac{(T_{LT,out} - T_{SW,in})(\beta - 1)}{q_{LT,p} - \frac{1}{q_{SW,p},SW,SW}}.$$  

(55)

with

$$\beta = e^{U_{HE,HE} \frac{1}{q_{LT,p}} \frac{1}{q_{SW,p},SW,SW}}.$$  

(56)

where $U_{HE}$ is the overall heat transfer coefficient for the heat exchanger and $A_{HE}$ is the heat exchanger surface area. From (55) the steady state heat transfer between the LT and SW circuit is calculated, which is used for modeling the seawater outlet temperature as a function of seawater inlet temperature and flow rates.

Responses for the baseline temperature control design are compared with those for the nonlinear robust control design using the same flow controller as the inner control loop in both cases. The design of flow controller, baseline temperature controller and nonlinear robust temperature controller for this simulation example follows the methodology presented in Section 3. Design parameters for the baseline design are illustrated in Table 1, where $\omega_{c,LT}$ is the crossover frequency for the consumer temperature controllers, and $\omega_{c,LT}$ is the crossover frequency for the LT inlet temperature controller.

The design parameter, $\rho_0$, is estimated numerically from noise bounds on the measurements included in $\gamma$, and uncertainty bounds on the parameters in the delay estimation. Delay estimation errors are included in $\rho_0$ by bounding the derivative of the time delayed variable and evaluating the temperature deviation for the worst case delay estimation error. In this case, it is assumed that delay model parameters are subject to $\pm30\%$ uncertainty and that the noise is bounded by $\pm3\sigma_n$, where $\sigma_n$ is the standard deviation of the signal noise. The standard deviation for noise on temperature measurements is denoted with sub index $T$, and with sub index $q$ for flow measurements. The resulting $\rho_0$ is listed in Table 1 along with the noise standard deviations and other parameters used for controller design. Model parameters are shown in Table 2 and 3 for respectively the hydraulic and thermodynamic model.

<table>
<thead>
<tr>
<th>$PM$</th>
<th>$\omega_{c,LT}$</th>
<th>$\omega_{c,LT}$</th>
<th>$\rho_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60°</td>
<td>0.01</td>
<td>0.05</td>
<td>0.78</td>
</tr>
<tr>
<td>$\sigma_{n,T}$</td>
<td>$\sigma_{n,q}$</td>
<td>$p_1$</td>
<td>$p_2$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.004</td>
<td>-0.003</td>
<td>-100</td>
</tr>
</tbody>
</table>

Table 2: Parameters for linearized hydraulic model.

<table>
<thead>
<tr>
<th>$a_{11}$</th>
<th>$a_{12}$</th>
<th>$a_{21}$</th>
<th>$a_{22}$</th>
<th>$a_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15.94</td>
<td>5.47</td>
<td>5.92</td>
<td>-15.03</td>
<td>-8.80</td>
</tr>
<tr>
<td>$b_{11}$</td>
<td>$b_{12}$</td>
<td>$b_{13}$</td>
<td>$b_{21}$</td>
<td>$b_{22}$</td>
</tr>
<tr>
<td>-0.0029</td>
<td>0.0016</td>
<td>0.0015</td>
<td>-0.0033</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Parameters for thermodynamic model.

MATLABS's hinfsyn is employed for the $H_\infty$ control design in this context. hinfsyn uses the two-Riccati formula from (Doyle & Glover, 1988) (Doyle et al., 1989) and computes a controller with an order equivalent to that of the augmented system, which in this case is 6th order. Plotting the multiplicative-error singular values for the $H_\infty$ controller as shown

<table>
<thead>
<tr>
<th>$Q_{11}$</th>
<th>$a_{1,1}$</th>
<th>$a_{1,2}$</th>
<th>$V_1$</th>
<th>$V_{CC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.8 \times 10^6$</td>
<td>20±30%</td>
<td>30±30%</td>
<td>13.5</td>
<td>20</td>
</tr>
<tr>
<td>$Q_{21}$</td>
<td>$a_{2,1}$</td>
<td>$a_{2,2}$</td>
<td>$V_2$</td>
<td></td>
</tr>
<tr>
<td>$7.2 \times 10^6$</td>
<td>10±30%</td>
<td>40±30%</td>
<td>13.5</td>
<td>20</td>
</tr>
</tbody>
</table>
in Figure 8 reveals the possibility of reducing the controller to 3 states with little change in performance. This is achieved through application of the Balanced Stochastic Truncation technique from (Safonov & Chiang, 1988), and only the response for the reduced order \( H_{\infty} \) controller is presented in the following.

![Figure 8: Multiplicative-error singular values of \( H_{\infty} \) controller.](image)

In the first simulation scenario the system is subjected to step-wise disturbances while at the operating point used in the design of the baseline controller. This is reasonable because the heat load for several consumers directly depends on the main engine load and it is not uncommon for this to change in relative large steps during maneuvering.

To evaluate and compare robustness of performance for the two designs, parameter perturbations of \( \pm 30\% \) for \( a_{c,1} \), \( a_{c,2} \), \( a_{m,1} \) and \( a_{m,2} \), resulting in 16 combinations of extreme values, are also considered. These combinations are all tested, and responses are plotted along the response for the nominal system for both controller design. Disturbances are plotted in Figure 9 and responses are plotted in Figure 10 and 11.

![Figure 9: Disturbances \( Q_1(t) \) and \( Q_2(t) \).](image)

Comparing responses for \( T_1(t) \) and \( T_2(t) \) in Figure 10 and 11 shows that the robust nonlinear design is superior to the baseline design in terms of disturbance rejection while the temperature response for the LT inlet temperature, \( T_{LT,in} \), is very similar for both design.

In the second simulation scenario the system is subjected to step-wise changes in the reference for \( T_{LT,in}(t) \) while at the operating point used in the design of the baseline controller. Also, the same parameter perturbations are evaluated, and responses are plotted along the response for the nominal system in Figure 12 and 13. The step-like response of the flow rates in Figure 12 and 13 when the temperature reference is changed from 36 °C to 30 °C is due to saturation of pump pressure. Comparing responses for the 2nd simulation example in Figure 12 and 13 shows similar robustness of performance and reference tracking performance of \( T_{LT,in}(t) \). However, the robust nonlinear design again shows an improvement over the baseline design in terms of disturbance rejection for \( T_1(t) \) and \( T_2(t) \).

5. Concluding Remarks

This paper presents two model-based approaches to design of controllers for a single-phase marine cooling system. Derivation of models for both control design and simulation is based on first-principles methods and aims at obtaining models of low complexity while still encompassing important system dynamics. This includes nonlinear flow characteristics in the hydraulic model, input affine behavior in the thermodynamic model and transport delays. The first control design is based on classical control theory and constitutes a baseline design mainly for performance comparison. The second design applies principles from feedback linearization to deal with delays and nonlinearities, while a \( H_{\infty} \) control design is used to ensure robustness towards model uncertainties and disturbances. Robustness of performance for the two control designs is illustrated through a simulation example that includes unmodeled dynamics and considers both parameter variations as well as changes in operating conditions. Both control designs show good robustness towards parameter variations, while
the nonlinear robust design performs better in terms of disturbance rejection for consumer temperatures. While energy optimization has not been addressed explicitly in the control designs, the relationship between pump flow rate and pump power consumption as given by the Affinity Laws, means that a reduction in pump power is achieved whenever the SW and LT circuit flow rates are reduced from the nominal system design. In this respect, the baseline and nonlinear robust design do not differ significantly since pump power is mainly determined by operating conditions and set points, which can be optimized separately from the feedback control design. Good disturbance rejection capabilities is a strong complementary benefit to set point optimization, however, since it allows the system to operate consistently close to the optimal conditions.

Future work involves verification of the proposed design through implementation on a full scale cooling system aboard a container vessel in operation.


Figure 10: 1st run: Temperature and flow rate responses for baseline control configuration. Responses for nominal parameters are plotted in color, while responses for parameter perturbations are in shades of gray.

Figure 11: 1st run: Temperature and flow rate responses for nonlinear robust control configuration with reduced (3rd) order $H_\infty$ controller. Responses for nominal parameters are plotted in color, while responses for parameter perturbations are in shades of gray.

Figure 12: 2nd run: Temperature and flow rate responses for baseline control configuration. Responses for nominal parameters are plotted in color, while responses for parameter perturbations are in shades of gray.

Figure 13: 2nd run: Temperature and flow rate responses for nonlinear robust control configuration with reduced (3rd) order $H_\infty$ controller. Responses for nominal parameters are plotted in color, while responses for parameter perturbations are in shades of gray.