Minimization of Distribution Grid Losses by Consumption Coordination

Smart & Cool project

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This talk

- coordination of active power consumption of a community
- local objectives concerning price and discomfort (consumers)
- global objective of loss minimization while obeying grid capacity (DSO)
Problem outline

consider a community of flexible consumers:

comunity consists of \( n \) households in \( m \) radials

Figure: Illustration of community
Consumer modeling

- we consider a set of sample times $\mathcal{T} = \{1, \ldots, L\}$
- consumers have:
  - inflexible consumption $\hat{u}_i(t) \in \mathbb{R}$, $t \in \mathcal{T}$
  - flexible consumption $\tilde{u}_i(t) \in \mathbb{R}^2$
  - local production $u_{s,i}(t) \in \mathbb{R}_+$
- inflexible consumption represents lights, range etc.
- flexible consumption is from EHP and EV
- local production could be from solar panel of wind turbine
- total consumption

$$u_i(t) = \hat{u}_i(t) + 1^T \tilde{u}_i(t) - u_{s,i}(t)$$
Consumer modeling

consumer discomfort

- flexible consumption $\tilde{u}_i(t)$ effects indoor temperature $T_i = (T_i(1), \ldots, T_i(L))$, and EV charge level $V_i(t), t \in \mathcal{T}$
- consumer discomfort is measured as deviation of temperature and EV charge level, from desired values

$$d_i(u_i) = (T_{sp,i} - T_i)^T (T_{sp,i} - T_i) + (V_i(L) - V_{sp,i})^T (V_i(L) - V_{sp,i})$$

- consumer objective is to minimize local discomfort and incurred cost, subject to local constraints
Price modeling

cost of energy:

- each household has a consumption profile
  \( u_i = (u_i(1), \ldots, u_i(L)) \in \mathbb{R}^m, i \in \{1, \ldots, n\} \)

- let \( w(t) \in \mathbb{R}_+, t \in \mathcal{T} \) be the cost of energy at each time instance, and define
  \( W = \text{diag}(w(1), \ldots, w(L)) \in \mathbb{R}_{+}^{L \times L} \)

- accumulated cost of energy for each household is
  \( c_{e,i}(u_i) = \mathbf{1}^T W u_i, \ i = 1, \ldots, n \)
Price modeling

cost of losses:

- denote the energy dissipated in the tie-line by $u_{\text{tie}}(t) \in \mathbb{R}$
- we approximate these losses by $u_{\text{tie}}(t) = \beta \left( \sum_{j=1}^{n} u_{j}(t) \right)^{2}$
- total losses becomes

$$c_l(u_1, \ldots, u_n) = \sum_{t=1}^{L} w(t) u_{\text{tie}}(t)$$

$$= \beta (u_1 + \cdots + u_n)^T W (u_1 + \cdots + u_n)$$
Grid modeling

- grid is organized as radials $r_j \subset \{1, \ldots, n\}, j \in \{1, \ldots, m\}$
- radials and tie-line has capacity limitations:

$$\left| \sum_{i \in r_j} u_i(t) \right| \leq \nu_j, \quad \left| \sum_{i=1}^{n} u_i(t) \right| \leq \nu_{m+1}$$

for all $t, j$
The combined problem for all consumers and the DSO:

\[
\text{minimize } \sum_{i=1}^{n} \left( c_{e,i}(u_i) + \lambda_i d_i(u_i) \right) + c_l(u_1, \ldots, u_n)
\]

subject to

\[
\begin{align*}
    x_i(t) &\in \mathcal{X}_i, & \tilde{u}_i(t) &\in \mathcal{U}_i, & \forall i \\
    f_j(u_1(t), \ldots u_n(t)) &\leq \bar{v}_j, & t &\in \mathcal{T}, & j &\in \{1, \ldots, m + 1\},
\end{align*}
\]
Problem strategy

- centralized problem is convex and may be solved by known algorithms
- however, consumers may have agreements with different companies who do not wish to cooperate
- we therefore wish to solve the problem decentralized
Problem strategy

- each consumer has local objectives and constraints
- consumers are tied together through grid losses and grid constraints
- problem may be solved iteratively in a distributed fashion
Distributed optimization

Our approach to distributed optimization:

<table>
<thead>
<tr>
<th>Initialize $\bar{z}^{(0)}_j, u^{(0)}_i, \bar{y}^{(0)}_i, j = 1, \ldots, m, \quad i = 1, \ldots, n$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>for</strong> $k=0,1, \ldots$ <strong>do</strong></td>
</tr>
<tr>
<td><strong>Local optimization at each household:</strong></td>
</tr>
<tr>
<td>$u^{(k+1)}<em>i = \arg\min</em>{u_i} \left( h_i(u_i) + \frac{\rho}{2} |u_i - \bar{z}^{(k)}_j + \bar{y}^{(k)}_j|_2^2 \right), ; i = 1, \ldots, n$</td>
</tr>
<tr>
<td><strong>Central coordination:</strong></td>
</tr>
<tr>
<td>$\bar{z}^{(k+1)} = \arg\min_{\bar{z}} \left( \tilde{g}(\bar{z}) + \frac{\rho}{2} \sum_{j=1}^m n_j |\bar{z}_j^{(k)} - u^{(k+1)}_j - \bar{y}^{(k)}_j|_2^2 \right)$</td>
</tr>
<tr>
<td>$\bar{y}^{(k+1)}_j = u^{(k+1)}_j + \bar{y}^{(k)}_j - \bar{z}^{(k+1)}_j, \quad j = 1, \ldots, m$</td>
</tr>
<tr>
<td><strong>end</strong></td>
</tr>
</tbody>
</table>

method is known as:
Alternating Direction Method of Multipliers (ADMM)
Distributed optimization

- limited communication between households

**Figure:** Communication requirement for the ADMM approach
Numerical results

Example 1: Benefit of including concern toward losses

- Recall the centralized problem

\[
\begin{align*}
\text{minimize} \quad & \sum_{i=1}^{n} (c_{e,i}(u_i) + \lambda_i d_i(u_i)) \\
& + c_l(u_1, \ldots, u_n) \\
\text{subject to} \quad & x_i(t) \in X_i, \quad \tilde{u}_i(t) \in U_i, \quad \forall i \\
& f_j(u_1(t), \ldots u_n(t)) \leq \bar{v}_j, \\
& t \in T, \quad j \in \{1, \ldots, m + 1\},
\end{align*}
\]

- We compare results when including losses, to the case where losses are disregarded.
Numerical results

- here $n = 30$, $m = 3$,
- 20 consumers have solar panels installed
Numerical results

Example 2: Convergence of distributed optimization

- $n = 342$, $m = 19$ (approx. same size as the town of Aasted)
- we let 144 consumers be installed with solar panels
Concluding remarks

- we have included cost of distribution losses as consumption tariffs
- concerns towards losses carries a reduction of grid loading in general
- distributed methods exists for converging on global optimum
Thank you