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Correlation measures for MIMO systems

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Correlation Measures for MIMO Systems

Abstract

Multiple input- multiple output (MIMO) systems have the potential to achieve very high capacities, under the condition that the channel gains for different transmitter/ receiver pairs are decorrelated. In this paper, we compare the channel gains in terms of their complex, envelope and power correlation, based on measurements of a MIMO system under strong and weak line-of-sight conditions in an indoor environment. We study how similar the different correlation measures are and how they correspond to the physical propagation environment. Moreover we show how they affect the channel capacity.

I: INTRODUCTION

In recent years, a lot of attention has been drawn to systems with multiple element transmitter and receiver arrays, because they can achieve very high spectral efficiencies [1], [2].

Assume a system with M transmitters and N receivers. Each transmitter sends an independent data stream with power E_x. Let $\underline{x}$,$\underline{y}$ be the transmitted and the received signal vectors respectively. In the case of a flat fading channel, the channel gain from transmitter j to receiver i is a scalar quantity, denoted T_{ij}. The transmitted and received vectors are related by the equation $\underline{y}=\mathbf{T}\underline{x}+\underline{n}$, where $\underline{n}$ is the receiver noise vector. The channel transfer matrix **T** incorporates the channel transfer gains from each transmitter to each receiver. The noise at the receivers is assumed to be Gaussian, of equal power σ^2 and its components are independent of each other. The Shannon capacity for this stationary channel is

$$C_{\text{stationary}} = \log_2(\det(\mathbf{I} + \frac{E_x}{\sigma^2}\mathbf{T}\mathbf{T}^H)), \quad (1)$$

where $\mathbf{T}^H$ is the Hermitian (complex conjugate transpose) of the matrix **T**.

The signals used in our formulation are discrete-time complex base-band, so the vectors $\underline{x}$,$\underline{y}$,n and the entries of the channel transfer matrix **T** are complex. We assume perfect down-conversion, filtering and sampling.

It has been theoretically demonstrated that the channel capacity decreases as the correlation of the entries of the channel transfer matrix increases [3].

Let us assume two complex random variables u and v. We can define their *complex correlation coefficient* as

$$\rho_{\text{complex}}(u,v) = \frac{E[uv^*] - E[u]E[v^*]}{\sqrt{E\left[|u|^2 - (|E[u]|)^2\right]E\left[|v|^2 - (|E[v]|)^2\right]}} \quad (2)$$

We also define the *envelope correlation coefficient* as

$$\rho_{\text{env}}(c,d) = \frac{E[cd] - E[c]E[d]}{\sqrt{E\left[c^2 - (E[c])^2\right]E\left[d^2 - (E[d])^2\right]}}, \quad (3)$$

$c = |u|, d = |v|$

Finally we define the *power correlation coefficient* as

$$\rho_{\text{pwr}}(f,g) = \frac{E[fg] - E[c]E[g]}{\sqrt{E\left[f^2 - (E[f])^2\right]E\left[g^2 - (E[g])^2\right]}}, \quad (4)$$

$f = |u|^2, g = |v|^2$

Definitions (3) and (4) are similar to definition (2). Since they are operations on real (and positive) random variables we can omit the conjugation and absolute value operation.

The complex correlation is a complex number whereas the envelope and the power correlations are real numbers. However they are all limited in absolute value by 1.

For Gaussian random variables [4]

$$\rho_{\text{env}} \approx \rho_{\text{pwr}} \quad (5)$$

$$\rho_{\text{pwr}} \approx |\rho_{\text{complex}}|^2 \quad (6)$$

The study of the envelope/ power correlation originates in diversity systems where the quantity of interest is the signal power on the diversity branches. However in the context of MIMO systems where the quantity of interest is the channel capacity and the phase of the variables affects the result, the complex correlation coefficient might be of interest. In the MIMO case, the complex random variables are the entries T_{ij} of the channel transfer matrix **T**. We define *transmitter* and *receiver* correlation as:

$$\rho^{\text{XMTR}} = \rho(T_{ij}, T_{il}),\ \rho^{\text{RCVR}} = \rho(T_{ij}, T_{kj}) \quad (7)$$

Receiver correlation describes the local scattering around the receivers, whereas transmitter correlation describes the correlation of the transmitted signals and does not provide insight into the scattering of the environment close to the transmitter array.

The purpose of this paper is to compare these correlation measures based on measurements that were taken in the Lucent Technologies Crawford Hill building. Moreover we test the validity of (4) and (5) in two different environments: under strong/ weak line-of-sight conditions.

Section II describes the measurement setup and the way the correlation was calculated. Section III summarizes the capacity measurements. Sections IV and V present the transmitter and receiver correlation results from the local statistics studies with respect to antenna polarization and element separation. Section VI compares the different correlation measures and includes the conclusions of this work.

II. MEASUREMENT SETUP AND PROCESSING

A. *Measuring equipment*

The measurements were taken with two antenna arrays: the transmitter array comprised 12 elements arranged on the sides of a square, and the receiver array comprised 15 elements, arranged in a grid. Adjacent antennas were separated by 8cm, and the elements were arranged with alternate polarizations. Fig. 1 shows the two arrays as seen from the front. V (H) stand for vertically (horizontally) polarized elements. The lowest elements of the arrays were placed at a height of 2m.

TRANSMITTER SIDE

V1	**H6**	**V6**	**H5**
H1			**V5**
V2			**H4**
H2	**V3**	**H3**	**V4**

RECEIVER SIDE

H5	**V6**	**H6**	**V1**
V5	**H8**		**H1**
H4	**V7**	**H7**	**V2**
V4	**H3**	**V3**	**H2**

Fig. 1: Antenna array layout

The antennas were folded cavity backed slot antenna elements mounted on 2’x2’ panels. Their gain pattern is nearly hemispherical. The operating frequency was 1.95 GHz, the system bandwidth was 30 kHz, the filters were raised cosine filters with a bandwidth expansion factor of α = 0.23, the symbol rate was 24.3 ks/sec and the constellation size used was 4 QPSK.

A cable connecting the two arrays provided synchronization between the transmitter and the receiver.

B. *Measurement environment*

The measurements were taken in the Lucent Crawford Hill Building. A rough layout of the building is shown in Fig. 2. In reality the building extends in both the 0^o and the 180^o direction, and the secondary corridor extends in the 90^o direction

The hallway measurements were taken in the building’s second floor main corridor. This straight 390-ft corridor is 6 feet wide and lined with offices (typically 10ft x10ft) on one side and laboratories (typically 12ftx24ft) on the other. The secondary corridor intersects the first one in a T-shape, but no measurements were taken in that environment. Inside walls are built of wallboard, outside walls are mainly glass. Ceilings and floors are reinforced concrete.

The transmitter was placed 82.5 ft from the end of the hallway and 2ft from its 90^o wall, pointing in the 180^o direction. This point is the origin of our axis system and the distances were measured with respect to that in feet. The transmitter was kept at this location for all the hallway and lab measurements.

C. *Local statistics measurements*

Local statistics measurements were performed for three locations in the hallway at distances 21ft, 117 ft, 240 ft from the transmitter and in the corresponding labs. Fig. 3 shows the arrangement of the grids.

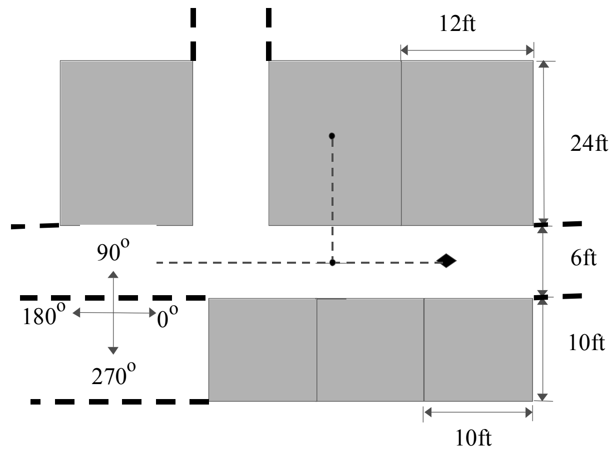

Fig. 2: Building layout

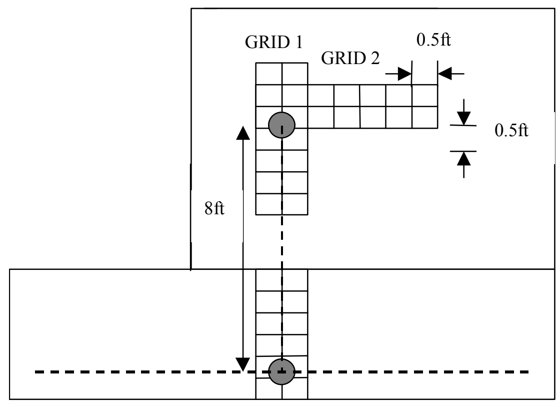

Fig. 3: Grid layout

The grid cell size was for all measurements 0.5 ft.

In the hallway, the grid was formed by three lines across and seven lines along the hallway. We will concentrate on the measurements taken at each point with 0^o oriented receivers (strong line-of-sight).

In the case of the labs, two grids were studied:

The first grid spanned the lab in the 90^o-270^o direction: it was formed by three lines in the 90^o-270^o direction (10 ft to 6.5 ft from the point were the hallway measurement was taken), and eight lines in the 0^o-180^o direction that each spanned ±0.5 ft. The measurements on this grid were for the $0^o$and the 180^o orientation of the receiver array.

The second grid spanned the lab in the 0^o-180^o direction: it was formed by three lines in the 0^o-180^o direction, and eight lines in the 90^o-270^o direction. The receivers were oriented in the 90^o or the 270^o direction.

D. *Measurement procedure*

The prototype used for the measurement campaign processes data in bursts. Each burst consists of 100 symbols. Out of these, the first 20 are training symbols and are used for the measurement of the channel transfer matrix, using orthogonal training sequences as suggested in [5]. The signal power was set to 9.2 dBm, in order to guarantee a signal to noise ratio of at least 15dB, to guarantee the accuracy of our capacity calculation [7]. For each measurement location, about 100 bursts (100 instantiations of the channel transfer matrix **T**) were recorded in order to average over the small-scale temporal variation (opening and closing doors, people walking etc).

Previous measurements in the same building have shown that the maximum delay spread τ_{MAX} is less than 0.8 μs [6]. The symbol time T_s was 41μs. So $T_s > \tau_{MAX}$, and the narrowband assumption holds.

E. Calculation of correlation

We study the correlation when both the transmitting and the receiving ends are of the same polarization (either horizontal or vertical), i.e. when all the antennas i, j, k, l in def. (7) are of the same polarization.

For the transmitter and receiver correlations defined in (7), we average over the element indices i and j respectively. The parameter of interest is the vertical or horizontal separation of the elements (j, l) or (i, k). Due to the fixed array layout we are limited to vertical and horizontal separations of 1λ.

The parameters are:

- propagation environment: hallway or labs
- receiver vs. transmitter correlation
- horizontal vs. vertical polarization
- horizontal vs. vertical element separation

III. Capacity Results

Fig. 4 and 5 show the capacity for two symmetrical single polarization subsystems in the hallway and in the labs. The horizontal polarization subsystem contained the elements H1 through H6, and the vertical polarization subsystem contained the elements V1 through V6, on both the transmitting and the receiving sides (see fig. 1).

The capacities have been calculated for a reference signal to noise ratio of 20dB. This power normalization accounts for the fact that power loss is observed in the real measurements and isolates the effect of channel change. As a measure of comparison we have plotted the median capacity of a 6x6 channel where all the entries of the channel transfer matrix are independent identically distributed random variables. Neither subsystem achieves the capacity of a Gaussian channel.

In the hallway the line-of-sight component is significant, so the channel is not rich in multi-path and the achievable capacity is low. The systems achieve higher capacities in the labs for all antenna orientations (richer scattering). However the channel capacity falls off with distance, which indicates that the channel gains become more correlated for both the hallway and the labs (common dominant propagation path: down the hallway and into the labs). The same effect is observed for both polarizations.

IV. Transmitter Correlation

A. Hallway environment

Fig. 6 shows the transmitter correlation in the hallway for vertically/ horizontally polarized, vertically/ horizontally separated elements.

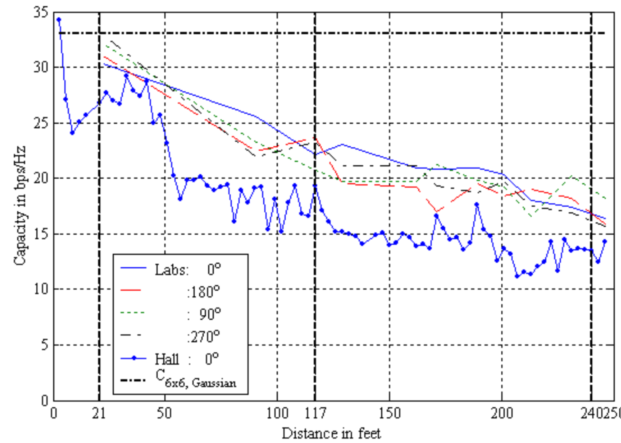

Fig. 4: Horizontal polarization subsystem

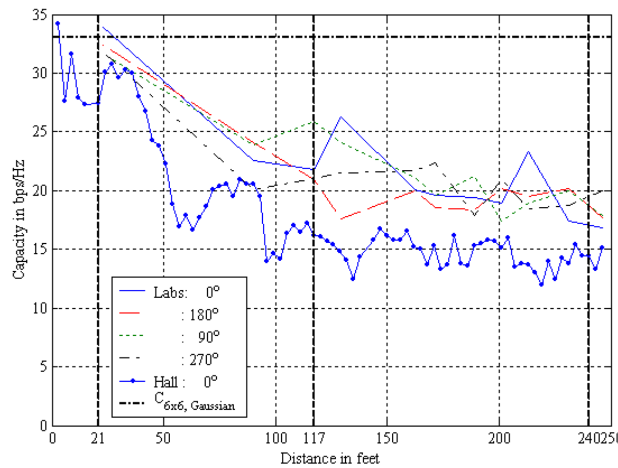

Fig. 5: Vertical polarization subsystem

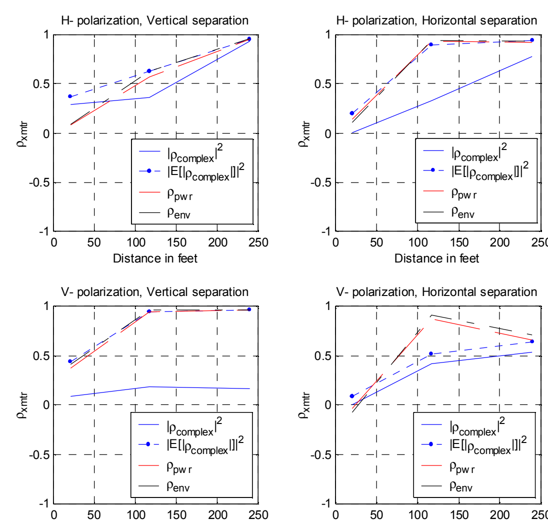

Fig. 6: XMTR correlation in the hallway

As expected, the transmitter correlation increases as the distance between the transmitter and the receiver arrays increases. At larger distances in the hallway, the dominant signal component is the direct line of sight component, the angular spread is limited and the correlation for a given separation increases [8]. Indeed the most drastic changes occur going from 21ft to 117ft. This indicates that by 117ft the line-of-sight component is the dominant mode of propagation.

B. Laboratory environment

The added parameter for the labs is the receiver orientation. Fig. 7-10 compare the different correlation measures in the laboratory environment for the various antenna orientations.

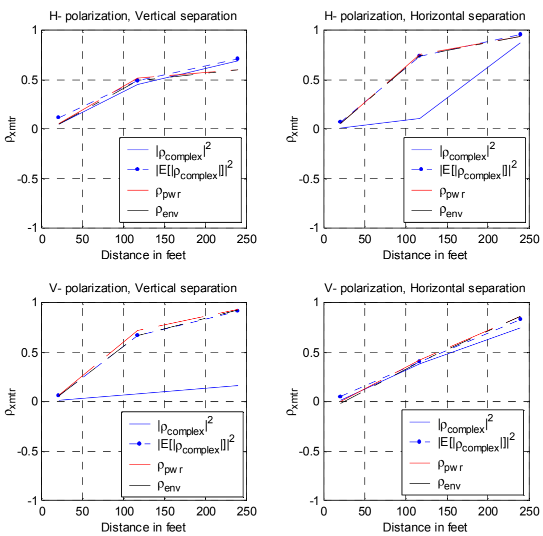

Fig. 7: XMTR correlation in the labs- 0° orientation

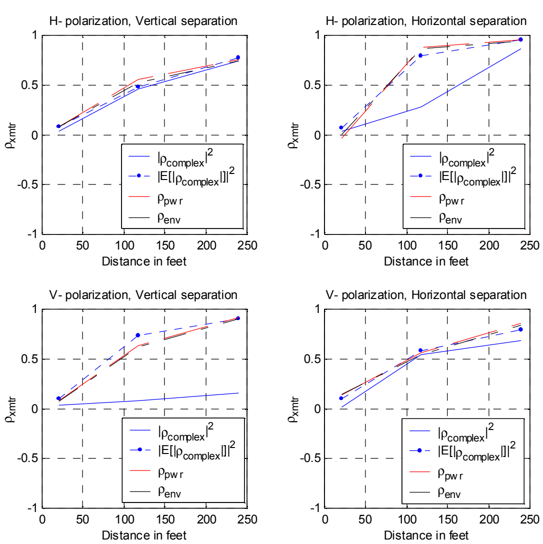

Fig. 8: XMTR correlation in the labs- 180° orientation

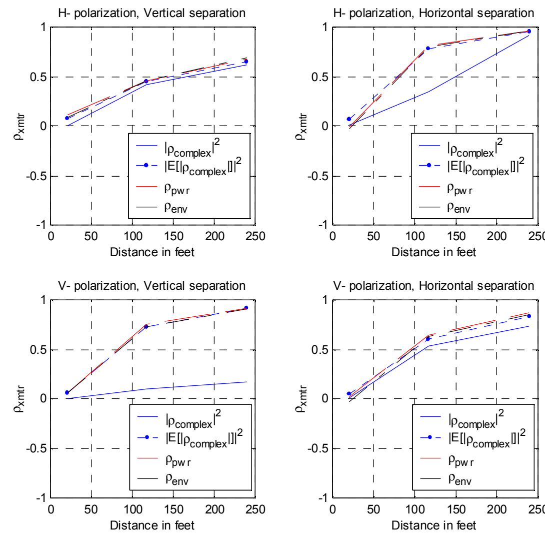

Fig. 9: XMTR correlation in the labs- 90° orientation

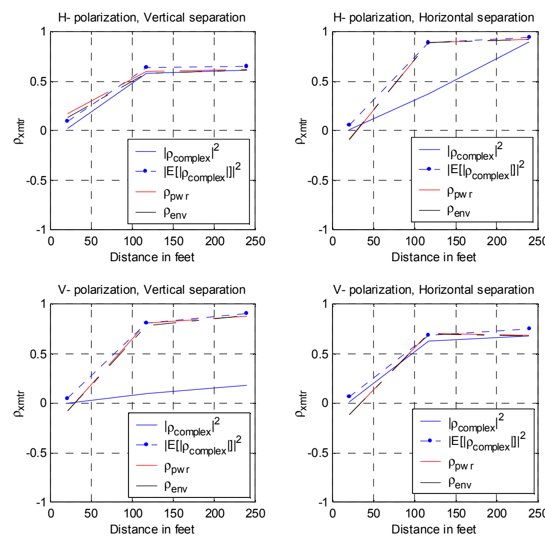

Fig. 10: XMTR correlation in the labs- 270° orientatio

Propagation along the hallway and into the labs is the dominant propagation path into the labs, hence the high transmitter correlation in the labs. This is a demonstration of the effect described in [9], [10]. Typically the transmitter correlation is higher in the hallway than in the labs.

V. RECEIVER CORRELATION

A. Hallway environment

We show the same plots for the receiver correlation in fig. 11.

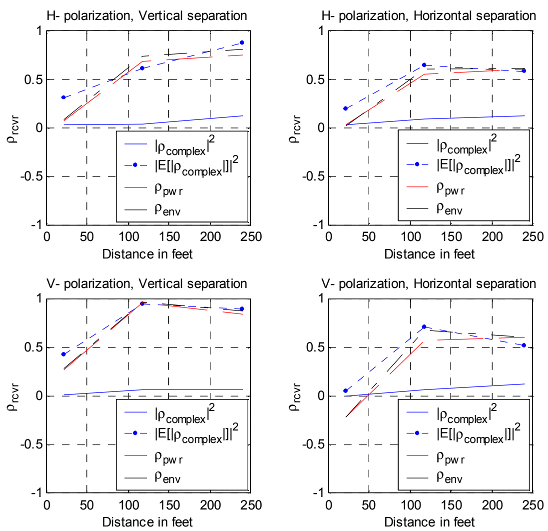

Fig. 11: RCVR correlation in the hallway

B. Laboratory environment

The results for the laboratory environment and the various antenna orientations are shown in fig. 12-14. Receiver correlation typically increases with distance for both polarizations and for both separations. Also, it is higher in the hallway than in the labs, but it is much lower than transmitter correlation.

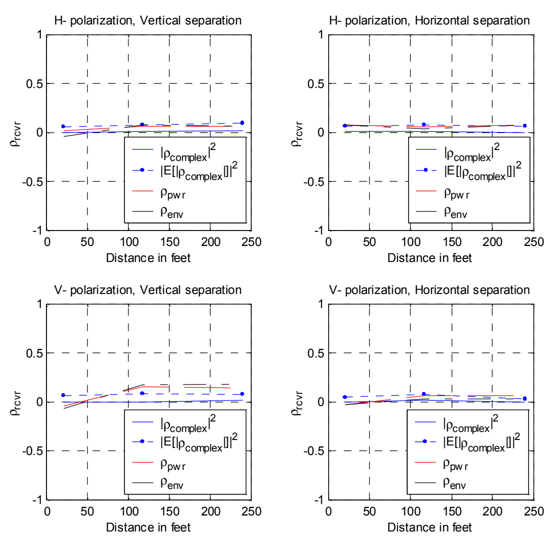

Fig. 12: RCVR correlation in the labs- 0° orientation

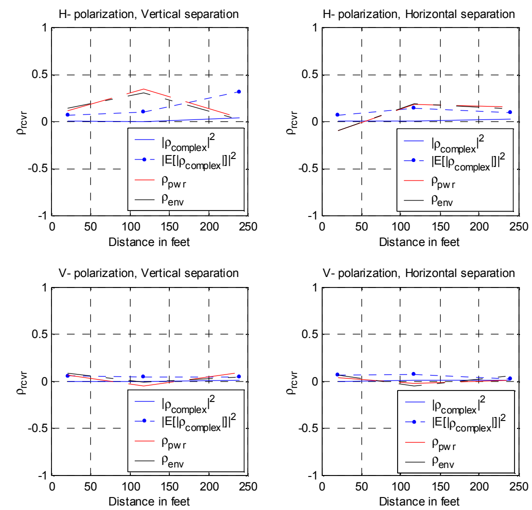

Fig. 13: RCVR correlation in the labs- 180° orientation

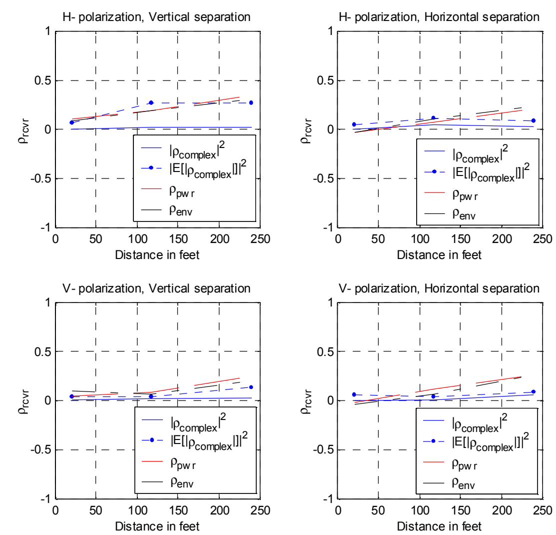

Fig. 14: RCVR correlation in the labs- 90° orientation

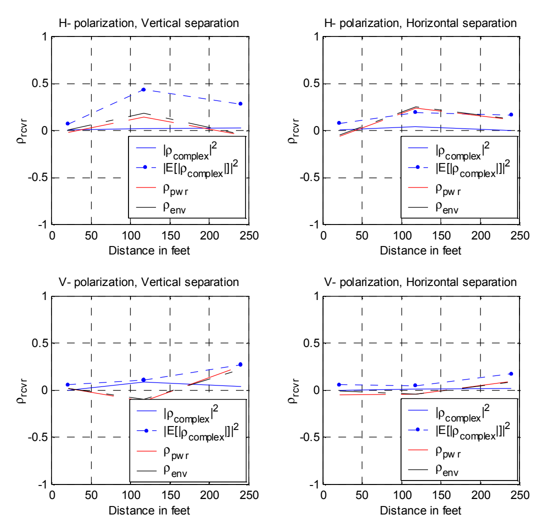

Fig. 15: RCVR correlation in the labs- 270° orientation

VI. Comparison and Conclusions

As expected the receiver correlation which reflects the local scattering around the receiver array remains low and is independent of the distance from the transmitter.

The transmitter correlation increases as the separation from the transmitter increases. In the hallway this is attributed to the existence of a dominant line-of-sight component. In the labs, it indicates that the main propagation path is down the hallway and into the labs, where the signal gets scattered locally. This effect limits the channel capacity as seen in figures 4 and 5.

The comparison of the various correlations measures draws some interesting conclusions. First of all, the power and envelope correlation coefficients relate to real random variables and can be negative numbers as for example in fig. 6. However this cannot be reflected in the square amplitude of the complex correlation coefficient. Also, the power and envelope correlation coefficients grow faster with transmitter- receiver distance than the complex correlation coefficient.

The obvious observation from the previous plots is that for both the hallway and the laboratory environment, the power and the envelope correlations give very similar results, so statement (5) is true.

For the laboratory environment, where we have uniform angle of arrival (the signal gets scattered locally by the furniture and the equipment in the labs), we have a better approximation of the Gaussian channel gain distribution. In these cases, the envelope, power and complex correlation coefficients give similar results. In the hallway environment this assumption does not hold and the correlation measures deviate. The greatest deviations occur for vertically polarized, vertically separated and horizontally polarized, horizontally separated elements.

A possible explanation is shown for the vertically polarized, vertically separated case in figures 16 and 17. The pairs of vertically polarized, vertically separated elements are: (V1, V2), (V5, V4) (and (V6, V7) on the receiver side). Each one would correspond to a cluster of points that relate to the correlations calculated at each element (of the same polarization) at the other side of the communication link. At small distances, the phases are randomized, but the amplitudes are similar. At larger distances, the correlations have similar values in both amplitude and phase within a cluster, but the clusters are offset in phase. Considering them as an ensemble leads to a low absolute value of the correlation coefficient.
This indicates that the specific location of the elements in the cross-section of the hallway has a significant effect on the correlation calculation, and limits our ensemble size.

Since the capacity of the channel does not collapse to that of a single input- single output system, there is some benefit in the phase offset, which is not reflected in the power/ envelope correlation coefficients or in the averaging of the complex correlation coefficient in the absolute value sense.

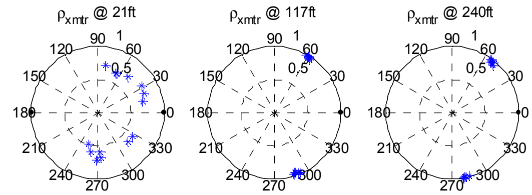

Fig. 16: ρ_{XMTR} (V pol, V separation, Hall)

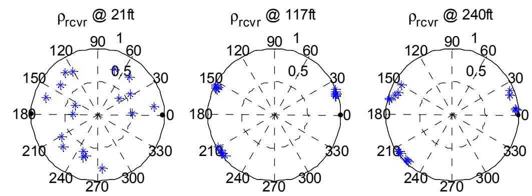

Fig. 17: ρ_{RCVR} (V pol, V separation, Hall)

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