# Bi-directional Amplification of Throughput in a Wireless Multi-Hop Network

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*Abstract*— **In wireless networks, the shared broadcast medium enables interactions among nodes and thus introduction of novel communication modes. This paper introduces and analyzes relaying techniques that increase the achievable throughput in multi–hop wireless networks by taking advantage of the bi–directional traffic flow. Such a relaying technique is termed relaying with *Bi–directional Amplification of Throughput (BAT–relaying)*. The BAT–relaying is utilizing the concept of anti–packets, defined for bi–directional traffic flows. The relay node combines the packets (anti–packets) that are destined for different nodes and broadcasts the combined packet. The first variant, termed Decode–and–Forward (DF) BAT relaying, has been proposed before in the literature. It combines the packets by using the XOR operation, which makes such proposal closely related to the network coding approaches. We proposed another type of BAT–relaying based on Amplify–and–Forward (AF), which utilizes the inherent packet combining that emerges from simultaneous utilization of a multiple access channel. We analyze the achievable throughput of the DF and AF BAT–relaying, regarding the impact of the traffic asymmetry and the channel errors. The unconventionality of this relaying, in particular AF BAT–relaying, opens many possibilities for further research.**

## I. Introduction

The shared wireless medium is the chief factor that limits the capacity of wireless multi–hop networks [1]. On the positive side, the wireless broadcast medium enables enhanced interaction among the wireless transceivers and thereby it allows introduction of novel communication modes. Such is, for example, the *cooperative diversity* [2], where two or more wireless terminals cooperate to achieve a reliable reception at the destination. In a separate significant development, the emergence of network coding [3] has shifted the paradigm under which the network communication is designed. While the traditional routing replicates a packet from an incoming to an outgoing link at an intermediate network node, the network coding allows the intermediate nodes to process the packets in a more general way. Originally, the network coding has exhibited its benefits for multicast in wireline packet networks. Nevertheless, the unreliability and the broadcast nature of the wireless setting appear to be a fertile ground for developing network–coding solutions ([4] and references therein).

In this paper we consider wireless relaying with bi–directional unicast flows. We introduce relaying techniques that offer increase in the achievable throughput over the conventional relaying. Each of those relaying techniques is termed *relaying with bi–directional amplification of the throughput (BAT–relaying)*. For the first BAT–relaying technique, which has previously appeared in [5], [6], we use the term *Decode–And–Forward (DF) BAT–relaying*. The conventional relaying is depicted on Fig. 1(a) and the DF–BAT–relaying is depicted on Fig. 1(b). The packets, each of length $N$ bits, $\mathbf{b}_{AC}, \mathbf{b}_{CA} \in \{0,1\}^N$ are destined from $A$ to $C$ and $C$ to $A$, respectively. It takes 4 slots to relay $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$ by conventional relaying. In DF–BAT–relaying, after decoding $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$, the relay $B$ applies a canonical network coding operation and broadcasts the packet $\mathbf{b}_B = \mathbf{b}_{AC} \oplus \mathbf{b}_{CA}$, where $\oplus$ denotes the bitwise XOR operation. Since the node $A$ already has $\mathbf{b}_{AC}$, it extracts the required packet $\mathbf{b}_{CA}$ through $\mathbf{b}_{CA} = \mathbf{b}_B \oplus \mathbf{b}_{AC}$. Analogously, $C$ extracts $\mathbf{b}_{AC} = \mathbf{b}_B \oplus \mathbf{b}_{CA}$. The relaying method on Fig. 1(b) requires only 3 slots to transfer the packets $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$, which means that the bi–directional throughput is "amplified" with respect to the conventional relaying.

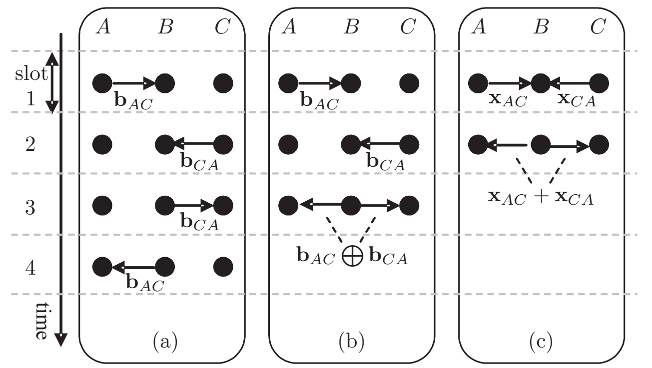


Fig. 1. (a) Conventional bi–directional relaying (b) Decode–and–Forward (DF) BAT–relaying (c) Amplify–and–Forward (AF) BAT–relaying

In this paper we propose a new method for efficient BAT–relaying, depicted on Fig. 1(c). This method is termed *Amplify–and–Forward (AF) BAT relaying* and it goes beyond the network coding, since it utilizes the inherent packet combining provided by the multiple access channel. Let $\mathbf{x}_{AC}$ and $\mathbf{x}_{CA}$ be the complex baseband representations of the packets $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$, respectively. If in the first slot both $A$ and $C$ are transmitting, then $B$ is receiving a signal $\mathbf{y}_B$ that is a result of interference between $\mathbf{x}_{AC}$ and $\mathbf{x}_{CA}$. If node $A$ knows $\mathbf{y}_B$, then $A$, already knowing $\mathbf{x}_{AC}$, can transform $\mathbf{y}_B$ into a

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noised copy of $\mathbf{x}_{CA}$ and try to decode it (analogously for node $C$). In the absence of noise, if the multiple access channel at $B$ is an additive channel, then $\mathbf{y}_B = \mathbf{x}_{AC} + \mathbf{x}_{CA}$. Then $B$ needs to amplify and broadcast $\mathbf{y}_B$, such that $A$ can extract $\mathbf{x}_{CA} = \mathbf{y}_B - \mathbf{x}_{AC}$ (similar for $C$). Thus, the AF–BAT relaying takes only 2 slots to transfer $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$, thus doubling the throughput compared to Fig. 1(a).

The BAT–relaying methods take advantage of the packets $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$ that are relayed by the same node into opposite directions. We call $\mathbf{b}_{AC}$ an *anti–packet* for $\mathbf{b}_{CA}$ and vice–versa. There are two benefits from the utilization of the anti–packets. First, the *overall system throughput is increased*, at it takes less time to convey the same amount of data. Second, *the relay node saves energy* having less transmissions for the same amount of relayed data. The latter benefit has motivated us to use the term anti–packet through a relation to physics, where the collision of a particle and an anti–particle produces energy. In this paper we analyze the achievable throughput of the BAT–relaying schemes, with an emphasis on the impact of the traffic asymmetry and presence of channel errors.

## II. Basic Notions for BAT–relaying

We will use $\mathbf{b} = (b[1], b[2], \ldots b[N])$ where $b[n] \in \{0, 1\}$ to denote the string of $N$ bits that represents a data packet. Through the application of certain modulation and coding technique, these $N$ bits are mapped into $M$ symbols $\mathbf{x} = (x[1], x[2], \ldots, x[M])$, where $x[m]$ is a complex number and we assume that the expected values are $E\{x[m]\} = 0$ and $E\{|x[m]|^2\} = 1$. The duration of a time slot in which one packet can be accommodated is denoted by $T_s$. The nominal data rate for a packet of $N$ bits is $R_s = \frac{N}{T_s}$ [bps].

For a quick glimpse in the gain of the BAT–relaying techniques, we assume noiseless channels. The conventional relaying takes a time of $4T_S$, while DF BAT–relaying takes $3T_S$ to transfer $2N$ bits in total. Thus, the maximal amount of throughput amplification for DF BAT–relaying is $G_{DF} = \frac{\frac{2N}{3T_s} - \frac{2N}{4T_s}}{\frac{2N}{4T_s}} = \frac{\frac{2}{3}R_s - \frac{R_s}{2}}{\frac{R_s}{2}} = \frac{1}{3} = 33.33\%$. The AF BAT–relaying for two packets with $2N$ bits in total consumes time of $2T_s$, such that the gain is $G_{AF} = \frac{\frac{2N}{2T_s} - \frac{2N}{4T_s}}{\frac{2N}{4T_s}} = 100\%$. Note that these maximal gains are present only when the traffic intensity is symmetric in both directions.

We introduce some notation that will be used further. For AF BAT–relaying, in the first slot, both $A$ and $C$ transmit their packets to $B$, such that the $m-$th received symbol at $B$ is:

$$y_B^{(AF)}[m] = \sqrt{\mathcal{E}} L_{AB} x_A[m] + \sqrt{\mathcal{E}} L_{BC} x_C[m] + z_B[m] \quad (1)$$

$\mathcal{E}$ denotes the average transmitted signal energy from $A$ or $C$ over one symbol period. We assume that both $A-B$ and $B-C$ are static, line–of–sight links, without fast fading. The factor $L_{AB}(L_{BC})$ denotes the path loss on the link $A-B(B-C)$. The value $z_B[m]$ is a complex–valued additive Gaussian white noise at the receiver of $B$ with variance $\sigma_B^2$. In the second slot, the relay $B$ amplifies $y_B^{(AF)}[m]$ for a factor $\beta$ and broadcasts it to both $A$ and $C$. The signal received by $A$ can be written:

$$y_A^{(AF)}[m] = \beta L_{AB} y_B^{(AF)}[m] + z_A[m] =$$
$$\beta\sqrt{\mathcal{E}} L_{AB}^2 x_A[m] + \beta\sqrt{\mathcal{E}} L_{AB} L_{BC} x_C[m] + \beta L_{AB} z_B[m] + z_A[m]$$

The average transmitted signal energy over one symbol period at node $B$ is also $\mathcal{E}$, such that:

$$\beta = \sqrt{\frac{\mathcal{E}}{\mathcal{E} \cdot L_{AB}^2 + \mathcal{E} \cdot L_{BC}^2 + \sigma_B^2}} \quad (2)$$

Having that $A$ knows $x_A[m]$ and $\mathcal{E}$ and assuming that $A$ knows the values $\beta, L_{AB}$, then $A$ receives the symbols of $C$ through an equivalent AWGN channel, represented by:

$$r_A^{(AF)}[m] = \beta\sqrt{\mathcal{E}} L_{AB} L_{BC} x_C[m] + \beta L_{AB} z_B[m] + z_A[m] \quad (3)$$

In analogous manner we can obtain:

$$r_C^{(AF)}[m] = \beta\sqrt{\mathcal{E}} L_{AB} L_{BC} x_A[m] + \beta L_{BC} z_B[m] + z_C[m] \quad (4)$$

We assume that the noise level at $A, B$, and $C$ is identical $\sigma_A^2 = \sigma_B^2 = \sigma_C^2 = \sigma^2$. At this point, it is instructive to write the expressions for the $m-$th received symbols at $A$ and $C$ when a DF BAT–relaying is applied. Assuming that $x_B[m]$ represents the $m-$th symbol transmitted by $B$, then:

$$\begin{aligned} r_A^{(DF)}[m] &= \sqrt{\mathcal{E}} L_{AB} x_B[m] + z_A[m] \\ r_C^{(DF)}[m] &= \sqrt{\mathcal{E}} L_{BC} x_B[m] + z_C[m] \end{aligned} \quad (5)$$

## III. Decode–and–Forward (DF) BAT–relaying

### A. Impact of Traffic Asymmetry

We first define the traffic asymmetry factor $\alpha$ as follows.

*Definition 1:* Let there be $K_A$ packets $\mathbf{b}_{AC}$ and $K_C$ packets $\mathbf{b}_{CA}$ within a sufficiently long observation period $T_o$. If $K_A \leq K_C$, then the asymmetry factor $\alpha$, where $0 \leq \alpha \leq 0.5$, is:

$$\alpha = \frac{K_A}{K_A + K_C} \quad (6)$$

To quantify the effect of traffic asymmetry, let us again consider an errorless channel. Let $B$ collect $K$ packets in $K$ slots, such that there are $\alpha K$ packets $\mathbf{b}_{AC}$ and $(1-\alpha)K$ packets $\mathbf{b}_{CA}$. The total number of slots for $B$ to relay these $K$ packets to $A$ and $C$ is $(1-\alpha)K$: First $\alpha K$ transmissions are done by XOR–ing the $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$ packets, and other $(1-2\alpha)K$ slots are used to send the rest of the $\mathbf{b}_{CA}$ packets. During the time of $K+(1-\alpha)K$ slots, $KN$ bits are transmitted in total, which makes the throughput:

$$R_{DF}(\alpha) = \frac{KN}{K(2-\alpha)T_s} = \frac{R_s}{2-\alpha} \quad (7)$$

where $R_s = \frac{N}{T_s}$ and the gain of DF BAT–relaying:

$$G_{DF}(\alpha) = \frac{\frac{R_s}{2-\alpha} - \frac{R_s}{2}}{\frac{R_s}{2}} = \frac{\alpha}{2-\alpha} \quad (8)$$

which, clearly, is maximized for $\alpha = 0.5$.

### B. Impact of Channel Errors

Let $p_{e1,DF}$ denote the packet error rate (PER) for the link $A-B$. Note that this PER does not depend on whether $B$ is transmitting a packet exclusively to $A$ or an XOR–ed packet to $A$ and $C$. The PER on link $B-C$ is denoted by $p_{e2,DF}$. For shortness, we use $p_{e1,DF} = p_{e1}$ and $p_{e2,DF} = p_{e2}$.

To find the maximal achievable throughput of DF BAT–relaying, we assume that $A$ and $C$ are backlogged and each

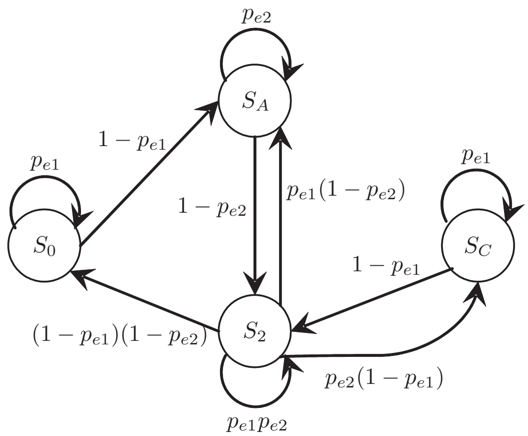


Fig. 2. Markov chain used in the analysis of the DF BAT protocol.

of them has a packet to send whenever polled by $B$. The protocol for DF BAT–relaying can be described as follows:

1) The buffer of $B$ is *empty* if $B$ has delivered all received $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$ packets. With an empty buffer, $B$ polls $A$ to retrieve a new $\mathbf{b}_{AC}$ packet. $A$ retransmits $\mathbf{b}_{AC}$ consecutively until it is correctly received at $B$.
2) If $B$ has $\mathbf{b}_{AC}$, but not $\mathbf{b}_{CA}$ packet in the buffer, then $B$ polls $C$ to retrieve a new $\mathbf{b}_{CA}$ packet. $C$ retransmits $\mathbf{b}_{CA}$ consecutively until it is correctly received at $B$.
3) If $B$ has $\mathbf{b}_{CA}$, but not $\mathbf{b}_{AC}$ packet in the buffer, then $B$ polls $A$ to retrieve a new $\mathbf{b}_{AC}$ packet. $A$ retransmits $\mathbf{b}_{AC}$ consecutively until it is correctly received at $B$.
4) If $B$ has both $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$ packet in the buffer, then $B$ creates the packet $\mathbf{b}_B = \mathbf{b}_{AC} \oplus \mathbf{b}_{CA}$ and transmits it. Due to the channel errors, the following can occur:
   a) With probability $(1-p_{e1})(1-p_{e2})$, the packet $\mathbf{b}_B$ is received correctly by both $A$ and $C$. Then the buffer of $B$ is empty.
   b) With probability $(1-p_{e1})p_{e2}$, the packet $\mathbf{b}_B$ is received correctly by $A$ and erroneously by $C$. Then the buffer of $B$ contains $\mathbf{b}_{AC}$ packet.
   c) With probability $p_{e1}(1-p_{e2})$, the packet $\mathbf{b}_B$ is received correctly by $C$ and erroneously by $A$. Then the buffer of $B$ contains $\mathbf{b}_{CA}$ packet.
   d) With probability $p_{e1}p_{e2}$, the packet $\mathbf{b}_B$ is received erroneously by both $A$ and $C$. Then $B$ retransmits the packet $\mathbf{b}_B$ in the next slot.

The analysis of the described protocol can be made by using the Markov chain from Figure 2. Each state of this chain represents the state of the buffer at node $B$. It is straightforward to prove that, by using the above protocol for DF BAT–relaying, the buffer of $B$ can have 4 possible states: $S_0$ (empty), $S_A$ ($\mathbf{b}_{AC}$ present), $S_C$ ($\mathbf{b}_{CA}$ present), and $S_2$ (both $\mathbf{b}_{AC}$ and $\mathbf{b}_{CA}$ are in the buffer). Let $P(S_0), P(S_A), P(S_C)$, and $P(S_2)$ represent the probability that in a certain slot the system is in state $S_0, S_A, S_C$, and $S_2$, respectively. Then:

$$\begin{aligned}
P(S_0) &= P(S_0)p_{e1} + P(S_2)(1-p_{e1})(1-p_{e2}) \\
P(S_A) &= P(S_0)(1-p_{e1}) + P(S_A)p_{e2} + P(S_2)p_{e2}(1-p_{e1}) \\
P(S_C) &= P(S_C)p_{e1} + P(S_2)p_{e1}(1-p_{e2}) \\
1 &= P(S_0) + P(S_A) + P(S_C) + P(S_2)
\end{aligned} \tag{9}$$

The throughput is proportional to $P(S_2)$, since we are concerned with the end–to–end throughput and the contribution to the throughput can only be made when $B$ transmits the combined packet $\mathbf{b}_B$. If in a given slot the system is in state $S_2$ and both $A$ and $C$ receive $\mathbf{b}_B$ correctly, then the throughput achieved in that slot is $2\frac{N}{T_s} = 2R_s$. If either $A$ or $C$ receives $\mathbf{b}_B$ correctly, the throughput in that slot is $R_s$. Therefore, the throughput achieved is calculated as:

$$R_{DF} = R_s P(S_2)(p_{e2}(1-p_{e1}) + p_{e1}(1-p_{e2}) + 2(1-p_{e1})(1-p_{e2}))$$

$$R_{DF} = R_s P(S_2)(2 - p_{e1} - p_{e2}) \tag{10}$$

where $P(S_2)$ is found by solving the system of equations (9):

$$P(S_2) = \frac{(1-p_{e1})(1-p_{e2})}{3 - 3p_{e1} - 3p_{e2} + p_{e1}p_{e2} + p_{e1}^2 + p_{e2}^2} \tag{11}$$

For errorless channel $p_{e1} = p_{e2} = 0$, we obtain $P(S_2) = \frac{1}{3}$ and $R_{DF} = \frac{2R_s}{3}$, which coincides with the analysis in the previous section. When the links $A-B$ and $B-C$ have the same PER $p_{e1} = p_{e2} = p_{e,DF}$, it can be easily seen that:

$$R_{DF} = \frac{2}{3}R_s(1-p_{e,DF}) \tag{12}$$

This is an interesting result: the throughput of each link $A-B$ and $B-C$ is given by $R_s(1-p_e)$ and, for the symmetric links, the overall throughput of the system under conventional relaying is $\frac{R_s}{2}(1-p_e)$. We see that the combining of the packets/anti–packets does not change that relationship.

As a reference, we derive the throughput of the conventional relaying with PERs $p_{e1}$ and $p_{e2}$. Here, $B$ polls $A$ and $A$ (re)transmits the $\mathbf{b}_{AC}$ packet until $B$ receives it. Then $B$ (re)transmits the $\mathbf{b}_{AC}$ packet to $C$ until $C$ receives it. The average number of slots used to transmit a packet from $A$ to $C$ is $\frac{T_s}{1-p_{e1}} + \frac{T_s}{1-p_{e2}}$. Due to symmetry, the same is true for a $\mathbf{b}_{CA}$ packet, which makes the throughput:

$$R_{conv} = \frac{R_s(1-p_{e1})(1-p_{e2})}{2 - p_{e1} - p_{e2}} \tag{13}$$

and for symmetric links $p_{e1} = p_{e2} = p_e$:

$$R_{conv} = \frac{R_s(1-p_e)}{2} \tag{14}$$

## IV. Amplify–and–Forward (AF) BAT–relaying

### A. *Impact of Traffic Asymmetry*

Analogously to the DF case, we quantify the asymmetry effect in the AF case by neglecting the channel errors. Let us observe $K$ slots in which $B$ receives packets from $A$ and $C$. In the first $K_1 < K$ slots both $A$ and $C$ transmit simultaneously to $B$, while in the last $(K-K_1)$ slots only $C$ transmits to $B$. Using the asymmetry factor as in Section III-A:

$$\alpha = \frac{K_1}{K+K_1} \Rightarrow K_1 = \frac{K\alpha}{1-\alpha} \tag{15}$$

The total number of slots used to transmit $K_1$ packets $\mathbf{b}_{AC}$ and $K$ packets $\mathbf{b}_{CA}$ is $2K$, such that the throughput is:

$$R_{AF}(\alpha) = \frac{(K+K_1)N}{2KT_s} = \frac{R_s(1+\frac{\alpha}{1-\alpha})}{2} = \frac{R_s}{2(1-\alpha)} \quad (16)$$

and the gain over the conventional relaying is:

$$G_{AF}(\alpha) = \frac{\frac{R_s}{2(1-\alpha)} - \frac{R_s}{2}}{\frac{R_s}{2}} = \frac{\alpha}{1-\alpha} \quad (17)$$

and is maximized for the maximal possible value of $\alpha = 0.5$.

### *B. Impact of Channel Errors*

The first thing to note about the AF BAT–relaying is that the bit errors that occur when $A$ receives the (amplified–and–forwarded) transmission from $B$ are not independent from the bit errors that occur at $C$ which receives the same transmission from $B$. This is because, for the $m-$th transmitted symbol from $B$, there is an identical noise component $z_B[m]$ which present in both received symbols $y'_A[m]$ and $y'_C[m]$, though with different amplification. Consequently, packet errors at $A$ are dependent with the packet errors at $C$. Let us define the following events $\Omega_A$ (a packet is received correctly at $A$) and $\Omega_C$ (a packet is received correctly at $C$). The complementary events are $\bar{\Omega}_A$, and $\bar{\Omega}_C$, respectively.

We use the following scheme for AF BAT–relaying. We again assume that $A$ and $C$ are backlogged with packets and each of them has a new packet whenever requested by $B$. Here is a simple variant of the AF BAT–relaying:

1) In the odd slots, both $A$ and $C$ transmit to $B$ packets simultaneously.
2) In an even slot, $B$ amplifies and retransmits the signal that it has received in the previous odd slot. By assuming short acknowledgement packets, we assume that at the end of the even slot, $A$ is informed via $B$ whether its packet has been received by $C$ and $C$ is informed whether its packet has been received by $A$. If $A$ and/or $C$ gets negative acknowledgement, it retransmits the same packet to $B$ in the next odd slot.

To calculate the throughput of this scheme, we first note that exactly in half of the slots there is an opportunity to deliver packets to the destinations. Using $P(\Omega)$ to denote the probability of the event $\Omega$, we can express the throughput as:

$$R_{AF} = \frac{R_s}{2}[2P(\Omega_A,\Omega_C) + P(\Omega_A,\bar{\Omega}_C) + P(\bar{\Omega}_A,\Omega_C)] =$$
$$\frac{R_s}{2}[P(\Omega_A,\Omega_C) + P(\Omega_A,\bar{\Omega}_C) + P(\Omega_A,\Omega_C) + P(\bar{\Omega}_A,\Omega_C)] =$$
$$\frac{R_s}{2}[P(\Omega_A) + P(\Omega_C)] = \frac{R_s}{2}(2 - p_{e1,AF} - p_{e2,AF}) \quad (18)$$

where the error probabilities are given as $p_{e1,AF} = 1 - P(\Omega_A)$ and $p_{e2,AF} = 1 - P(\Omega_C)$. Note that the dependency between the errors at $A$ and $C$ has disappeared in (18). For symmetric links $A-B$ and $B-C$, $p_{e1,AF} = p_{e2,AF} = p_{e,AF}$ such that:

$$R_{AF} = R_s(1 - p_{e,AF}) \quad (19)$$

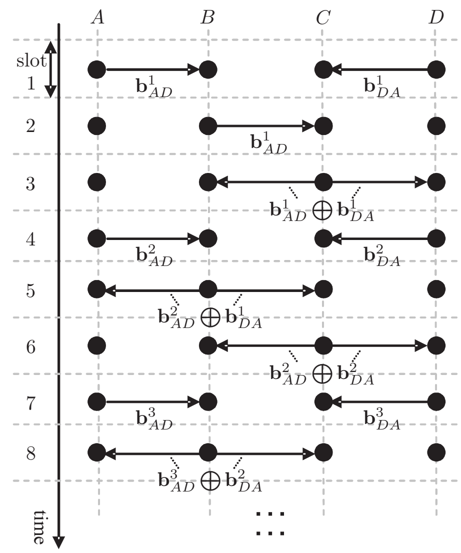


Fig. 3. Illustration of DF BAT–relaying over multiple hops.

## V. BAT–RELAYING OVER MULTIPLE HOPS

This section describes the potential effect of BAT–relaying for multi–hop bidirectional flows. In the following examples, we consider a completely synchronized slotted channel where packet transmissions in each slot can be perfectly scheduled for each node. All the packets have the same length. There are no transmission errors. All the nodes operate in half–duplex mode i. e. a node cannot transmit and receive simultaneously.

Fig. 3 depicts a simple realization of DF BAT–relaying over a 3–hop network with 4 nodes. The node *A* has a packet flow sent to *D*, and vice versa. In the slot 1, *A* and *D* transmit packets to *B* and *C*, respectively. Here, $\mathbf{b}^1_{AD}$ ($\mathbf{b}^1_{DA}$) means the first packet in the packet flow from *A* to *D* (from *D* to *A*). In the slot 2 there are still no anti–packets; *B* relays $\mathbf{b}^1_{AD}$ to *C*, while *A* and *D* are quiet due to the half–duplex condition. In slot 3 *C* broadcasts $\mathbf{b}^1_{AD} \oplus \mathbf{b}^1_{DA}$ to *B* and *D*. Since *D* (*B*) has $\mathbf{b}^1_{DA}$ ($\mathbf{b}^1_{AD}$) from slot 1, it extracts the packet $\mathbf{b}^1_{AD}$ ($\mathbf{b}^1_{DA}$) through XOR operation between the combined packet and $\mathbf{b}^1_{DA}$ ($\mathbf{b}^1_{AD}$). In the slot 4, *B* has the packet $\mathbf{b}^1_{DA}$, therefore, *A* transmits $\mathbf{b}^2_{AD}$ which is an anti–packet for $\mathbf{b}^1_{DA}$. In the same slot, *D* can transmit a new packet to *C*. In slot 5, $B$ broadcasts to $A$ and $C$, which retrieve packets $\mathbf{b}^1_{DA}$ and $\mathbf{b}^2_{AD}$, respectively. This state is identical to the one after the slot 2: *C* has anti–packets to be transmitted to *B* and *D*. Thus, the identical procedure from slot 3 to 5 can be repeated at slots 6 to 8, 9 to 11, etc. In such a "stable" state, 2 new packets are put into the multi–hop network from the packet–originating nodes every 3 time slots (e.g. *A* and *D* transmit the 2nd packet in the flow at slot 4, the 3rd packet at slot 7, etc). The throughput of both flows is $\frac{2R_s}{3}$. Without BAT–relaying, at most 2 packets can be transmitted over 4 time slots between *A* and *D*. This yields maximal possible gain identical to the 3–node case.

Fig. 4 shows a possible realization of AF BAT–relaying over a 4–hop network. Node *A* has a packet flow for node *E*, and vice versa. As in the DF case, the start is at slot 1 in which *A*

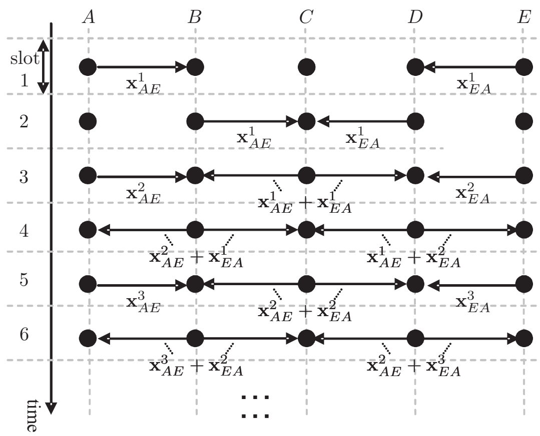


Fig. 4. Illustration AF BAT–relaying over multiple hops.

and *E* transmit their packets to their neighboring nodes. Here, $\mathbf{x}^1_{AE}$ ($\mathbf{x}^1_{EA}$) means the set of symbols mapped from bits of the first packet in the packet flow from *A* to *E* (from *E* to *A*). In slot 2, *B* and *D* amplify and forward the signals received in slot 1. *C* receives the combined signal that is the symbol–by–symbol addition of two signals transmitted from *B* and *D*. In the next slot, *C* amplifies and forwards the combined signal, but also *A* and *E* transmit their second packets in their flows to *B* and *D*, respectively. Now $B$ receives the symbol–by–symbol combination of mapped symbols from 3 packets, that is, $\mathbf{x}^1_{AE}$, $\mathbf{x}^1_{EA}$, and $\mathbf{x}^2_{AE}$. However, $B$ knows $\mathbf{x}^1_{AE}$ since it had sent it at slot 2. Thus, by subtracting the known set of symbols from the combined signal, $B$ can extract the combination of $\mathbf{x}^1_{EA}$ and $\mathbf{x}^2_{AE}$. Then, $B$ amplifies and relays the combined signal to *A* and *C*. At slot 4, *A* receives the combined signal of $\mathbf{x}^1_{EA}$ and $\mathbf{x}^2_{AE}$, but it can extract only $\mathbf{x}^1_{EA}$ since it has the content of $\mathbf{x}^2_{AE}$. These operations are completely symmetric, and *E* can extract $\mathbf{x}^1_{AE}$ at slot 4 with the completely same process. Furthermore, at this slot, $C$ receives the combined signal of symbols from 4 different packets, that is, $\mathbf{x}^1_{AE}$, $\mathbf{x}^1_{EA}$, $\mathbf{x}^2_{AE}$, and $\mathbf{x}^2_{EA}$. Note that $C$ knows $\mathbf{x}^1_{AE}+\mathbf{x}^1_{EA}$ from slot 3 and thus $C$ can extract the combined signal $\mathbf{x}^2_{AE}+\mathbf{x}^2_{EA}$. The status after slot 4 is identical to the one after slot 2 except for the sequence number of the packets: *C* has the combined signal from *B* and *D*, which consists of symbols mapped from packets originated in *A* and *E*. As in DF case, the same procedure from slot 3 to 4 can be repeated at slots from 5 to 6, etc. In the "stable" state, every 2 slots there are 2 new packets injected in the network from both flows, making the maximal throughput gain identical to the 3–node case.

The two examples above illustrate idealized operation of the BAT–relaying in multi–hop networks. For a realistic application, we certainly need practical protocols that operate in a distributed manner, control the packet identification/addressing/transmission, handle packet errors, etc. In fact, the BAT–relaying can be considered as a new link–layer protocol, which lies under Network layer and is in charge of packet delivery among neighboring nodes. These practical protocol–related issues are left as a task for future work.

## VI. Numerical results

To compare the different relaying schemes, we introduce the following notations for the per–symbol SNR. For DF BAT–relaying, by using (5) the SNRs are given by:

$$\gamma_A^{(DF)}=\gamma_1=\frac{\mathcal{E}L_{AB}^2}{\sigma^2}\qquad \gamma_C^{(DF)}=\gamma_2=\frac{\mathcal{E}L_{BC}^2}{\sigma^2} \tag{20}$$

Note that the same SNR values in (20) are also valid for the conventional relaying. With such notation, the value of the amplification factor (2) can be expressed as:

$$\beta=\sqrt{\frac{\mathcal{E}}{\sigma^2(\gamma_1+\gamma_2+1)}} \tag{21}$$

Starting from (3), the SNR at node $A$ when AF BAT–relaying is applied can be expressed as:

$$\gamma_A^{(AF)}=\frac{\beta^2\mathcal{E}L_{AB}^2L_{BC}^2}{(\beta^2L_{AB}^2+1)\sigma^2}=\frac{\gamma_1\gamma_2}{2\gamma_1+\gamma_2+1} \tag{22}$$

Analogously, the per–symbol SNR at node $C$ is:

$$\gamma_C^{(AF)}=\frac{\gamma_1\gamma_2}{2\gamma_2+\gamma_1+1} \tag{23}$$

For simplicity, let us assume that the packets are transmitted by using uncoded BPSK. Let $P_b(\gamma)$ denote the bit error probability for BPSK at SNR equal to $\gamma$. For packets consisting of $N$ bits, the packet error rates that are required to calculate the throughput can be expressed as follows:

$$\begin{aligned}&p_{e1,DF}=1-(1-P_b(\gamma_1))^N\quad p_{e2,DF}=1-(1-P_b(\gamma_2))^N\\&p_{e1,AF}=1-(1-P_b(\gamma_A^{(AF)}))^N\\&p_{e2,AF}=1-(1-P_b(\gamma_C^{(AF)}))^N\end{aligned} \tag{24}$$

Figure 5 depicts the normalized throughput $\frac{R_{DF}}{R_s}$, $\frac{R_{AF}}{R_s}$ and $\frac{R_{conv}}{R_s}$ for the three different relaying schemes as functions of the SNR for the link $A-B$, $\gamma_A^{(DF)}=\gamma_1$. The packet size is $N=100$ bits and the links $A-B$ and $B-C$ are symmetric, such that $\gamma_2=\gamma_1$ and $R_{DF},R_{AF}$ and $R_{conv}$ are calculated by using (12), (19), and (14), respectively. While it is always $R_{DF}>R_{conv}$, at low SNR the noise amplification degrades the performance of AF BAT–relaying as compared to DF, but AF yields the highest throughput at high SNR.

The impact of the asymmetry of the links $A-B$ and $B-C$ in terms of SNR can be examined by using Figure 6. This figure depicts the normalized throughput values $\frac{R_{DF}}{R_s}$ and $\frac{R_{AF}}{R_s}$ as a function of $\gamma_1$ and for three different values of $\gamma_2$. The value of $\frac{R_{conv}}{R_s}$ is not plotted, since for each $\gamma_1,\gamma_2$ it is less than $\frac{R_{DF}}{R_s}$ and does not bring additional information compared to Figure 5. From Figure 6 we can conclude that the DF BAT relaying is more sensitive to the link asymmetry. To see this, consider the two curves (DF and AF) when $\gamma_1=\gamma_2$ and observe, for example, the value of the normalized throughput equal to $0.4$. If $\gamma_2$ increases, then the required $\gamma_1$ to reach $R_{DF}=0.4$ is around 0.5 dB less than the required $\gamma_1$ when $\gamma_1=\gamma_2$; while this value for AF is more than 1 dB. In short, for given normalized throughput, the increase of $\gamma_2$ over $\gamma_1$ leads to better throughput improvement in case of AF

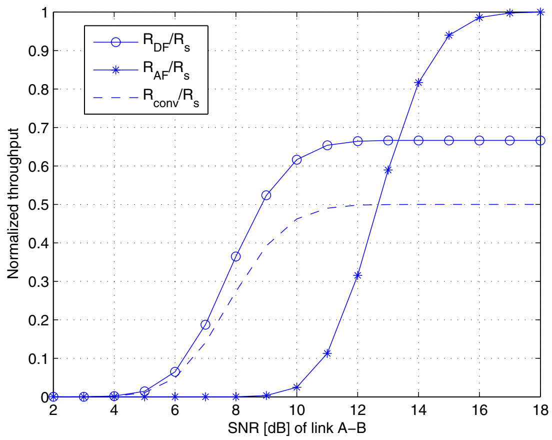


Fig. 5. Normalized throughputs $R_{DF}, R_{AF}$ and $R_{conv}$ versus the SNR for the link $A - B$ $\gamma_A^{(DF)} = \gamma_1$. The SNR for the link $B - C$ is chosen to be equal $\gamma_2 = \gamma_1$. Packet size is $N = 100$ bits.

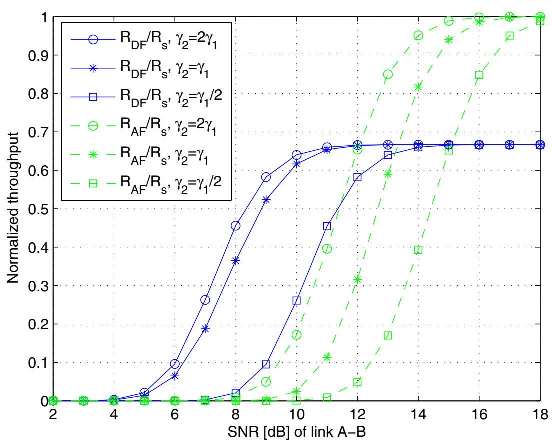


Fig. 6. Normalized throughputs $R_{DF}$ and $R_{AF}$ versus the SNR for the link $A - B$ $\gamma_A^{(DF)} = \gamma_1$ for different SNR values on the link $B - C$: $\gamma_2 = 2\gamma_1$, $\gamma_2 = \gamma_1$, $\gamma_2 = \frac{\gamma_1}{2}$. Packet size is $N = 100$ bits.

compared to DF. Conversely, if $\gamma_2$ drops below $\gamma_1$, then the degradation in the throughput is more severe for DF than AF BAT relaying. This conclusion can be reached analytically by inspecting the derivatives $\frac{\partial R_{DF}}{\partial \gamma_2}$ and $\frac{\partial R_{AF}}{\partial \gamma_2}$, but such analysis is outside the scope of this paper.

## VII. Conclusion

This paper introduces and analyzes relaying techniques that increase the achievable throughput in multi–hop wireless networks by taking advantage of the bi–directional traffic flow. Such a relaying technique is termed relaying with Bi–directional Amplification of Throughput (BAT–relaying). The BAT–relaying is utilizing the concept of anti–packets, defined for bi–directional traffic flows. The relay node combines the packets (anti–packets) that are destined for different nodes and broadcasts the combined packet. This operation has a two–fold effect: it decreases the time to transmit the data (therefore increases the achievable throughput) and decreases the energy that the relay node spends to forward the data (therefore, introduces energy–efficient operation). The first variant, Decode–and–Forward (DF) BAT relaying has been proposed before in the literature. It combines the packets by using the XOR operation, which makes such proposal closely related to the network coding approaches. In this paper, we have proposed another type of BAT–relaying based on Amplify–and–Forward (AF). AF BAT–relaying utilizes the inherent packet combining that emerges from simultaneous utilization of a multiple access channel. In an errorless channel, AF BAT–relaying is always superior to DF, but we have shown that in noisy channels the noise amplification can severely degrade the performance of AF compared to DF.

The work presented in this paper opens many issues for a future work. In this paper we have been concerned with the maximal achievable throughput and thereby assumed that the nodes that originate the traffic are backlogged with packets. The case of random packet arrivals introduces interesting tradeoff between the energy consumption and the delay in the system: The relay node can decide to wait for an anti–packet and thus save energy or it can relay a packet without waiting to combine it with anti–packet and thus reduce the overall delay. Next, it is interesting to investigate the AF BAT–relaying over multiple access channels that are not additive. Finally, the BAT–relaying can change the way in which the routing is designed — one can think of a routing algorithm where the packets are directed to routes in a way to achieve symmetric bi–directional flow along the routes and thus foster the BAT–relaying.

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