Wireless Networks II
Minimodul 5:
Performance aspects and performance modelling

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Overview of course

1. Wireless Systems: GSM, GPRS and 3G
2. Reliability aspects in Service and Context management architectures
3. WiMax and Radio over Fiber Networks - 4G
4. QoS
5. Performance aspects and performance modelling
## Content

- **Motivation & Background**
  - Performance Analysis in Wireless Settings
  - Review of Basic Concepts: Random Variables, Exponential Distributions, Stochastic Processes
  - Birth-Death Processes, Discrete and Continuous time Markov chains

- **M/M/k/k type queueing models (revision)**
  - Kendall Notation,
  - M/M/k/k type of queue models, steady-state solution
  - Erlang-B formula

- **Wireless Link Models**
  - IID errors, correlated errors, Gilbert Elliot Model
  - Application: Queuing model with varying channel
  - Multiple queues, multi-class models

- **Simple Traffic Models**
  - Traffic measurements and characteristics
  - ON/OFF Markov models

- **Summary/Exercises**
**Intro: Packet-Based Transport**

- **Advantages of Packet-Based Transport (as opposed to circuit switched)**
  - Flexibility
  - Optimal Use of Link Capacities, Multiplex-Gain for bursty traffic
- **Drawbacks**
  - Buffering/Queueing at routers can be necessary
  - Delay / Jitter / Packet Loss can occur
  - Overhead from Headers (20 Byte IPv4, 20 Byte TCP)

... and it makes performance modeling harder!!

**Main motivation for Performance Modeling:**

- Network Planning
- Evaluation/optimization of protocols/architectures/etc.
Real-time requirements: Parameters

- **User Plane QoS/Network Performance**
  - End-2-End Packet Delay (in particular interactive applications)
  - Delay Jitter
  - Packet Loss
  - Throughput/Goodput

- **Application Level QoS**
  - e.g. Video/Voice Quality (depending on codecs)

- **Signalling Plane**
  - Call Setup Delays
  - Fraction of blocked Calls

- **Reliability Aspects**
  - Failure probabilities of entities
  - Downtime distribution

- **Behavior at Handover**
  - Dropped Calls
  - Delayed / Lost packets
Extended layered communication model

Ultimate goal of service provisioning: user satisfaction

Focus in this course:
Network and application aspects, i.e. L2-5 and application layer

Relevant functionalities:
• PHY Layer
  • Bit/Symbol transmission → Throughput
  • Symbol error probabilities (channel conditions, interference)
• Power Control
• Propagation delays
Relevant Functionalities (cntd.)

- **Link Layer (L2)**
  - Medium Access Delays
  - Collisions/unsuccesful transmissions
  - Fragmentation
  - Forward error correction (FEC) and error detection (CRC)
  - Link-layer Retransmission Mechanisms (ARQ)
  - L2 scheduling, switching, buffering

- **Network Layer (L3)**
  - Path selection (routing)
  - Processing delays (e.g. for routing table lookup)
  - L3 buffering, scheduling, buffer management (RED)
  - [L3 fragmentation]
Relevant Functionalities (cntd.)

- Transport Layer (L4)
  - Multiplexing/de-multiplexing (UDP/TCP)
  - Error detection/checksums
  - In-order delivery, sequence numbers (TCP)
  - Acknowledgements and Retransmissions (TCP)
  - Flow/Congestion Control (TCP)

- Application Layer/Codecs
  - FEC/CRC
  - Application Layer Retransmissions
  - Application Layer sequence numbers

All Layers
- Increased volume due to headers
Challenges in IP networks:
- Multiplexing of packets at nodes (L3)
- Burstiness of IP traffic (L3-7)
- Impact of Dynamic Routing (L3)
- Performance impact of transport layer, in particular TCP (L4)
- Wide range of applications → different traffic & QoS requirements (L5-7)
- Feedback: performance → traffic model, e.g. for TCP traffic, adaptive applications

Challenges in Wireless Networks:
- Wireless link models (channel models)
- MAC & LLC modeling
- RRM procedures
- Mobility models
- Cross layer optimization
  → Analysis frequently with 'stochastic' models
Basic concepts: Probabilities

Probabilities

- 'Random experiment' with set of possible results $\Omega$

- Axiomatic definition on event set $\mathcal{P}(\Omega)$
  - $0 \leq \Pr(A) \leq 1$;
  - $\Pr(\emptyset) = 0$;
  - $\Pr(A \cup B) = \Pr(A) + \Pr(B)$ if $A \cap B = \emptyset$ [ $A, B \in \mathcal{P}(\Omega)$ ]

- Conditional probabilities: $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$
Basic concepts: Random variables

Random Variables (RV)
- Definition: $X: \Omega \rightarrow \mathbb{R}$; $\Pr(X=x) = p(x)$
  - Probability density function $f(x)$,
  - Cumulative distribution function $F(x) = \Pr(X \leq x)$,
  - Reliability function (complementary distr. Function) $R(x) = 1 - F(x) = \Pr(X > x)$
- Expected value, moments: $E(X^n) = \int x^n f(x) \, dx$

- Relevant Examples, e.g.:
  - Number of packets that arrive at the access router in the next hour (discrete)
  - Buffer occupancy (#packets) in switch x at time y (discrete)
  - Number of downloads (‘mouse clicks’) in the next web session (discrete)
  - Time until arrival of the next IP packet at a base station (continuous)
Basic concepts II: Geometric distributions

Important example:

- Geometrically distributed RV X (parameter p, 0<p<1)
  - \( \Pr(X=n) = (1-p)^{n-1}p, \ n=1,2,3,... \)
  - Expected value, \( E(X) = \frac{1}{p} \), and variance, \( \text{Var}(X) = \frac{1-p}{p^2} \)
  - Example:
    1. Initialize counter \( c=0 \)
    2. \( c=c+1 \)
    3. throw a 'biased' coin ('heads' with probability p)
    4. If coin shows 'heads' go to step 2
    5. Otherwise stop and output c

This function creates samples from a geometrically distributed RV with parameter p
Basic concepts III: Exponential Distributions

Important Case: Exponentially distributed RV
- Single parameter: rate $\lambda$
- Density function $f(x) = \lambda \exp(-\lambda x)$, $x>0$
- Cdf: $F(x) = 1 - \exp(-\lambda x)$, Reliability function: $R(x) = \exp(-\lambda x)$
- Moments: $E\{X\} = 1/\lambda$; $Var\{X\} = 1/\lambda^2$, $C^2 = Var\{X\} / [E\{X\}]^2 = 1$

Important properties:
- Memory-less: $Pr(X>x+y \mid X>x) = \exp(-\lambda y)$
- Properties of two independent exponential RV: $X$ with rate $\lambda$, $Y$ with rate $\mu$
  - Distribution of $\min(X,Y)$: exponential with rate $(\lambda+\mu)$
  - $Pr(X<Y) = \lambda/(\lambda+\mu)$

Exercise 1: Proof the relation in the last bullet! (10min)
Basic concepts IV: Stochastic Processes

Definition of process \((X_i)\) (discrete) or \((N_t)\) (continuous)

- Simplest type: \(X_i\) independent and identically distributed (iid)
- Relevant Examples:
  - Inter-arrival time process: \(X_i\)
  - Counting Process:
    \(N(t) = \max\{n \mid \sum_{i=1}^{n-1} X_i \leq t\}\), alternatively \(N_i(\Delta) = N(i\Delta) - N([i-1]\Delta)\)
Basic concepts IV: Stochastic Processes

Important Example: Poisson Process

- Assume i.i.d. exponential packet inter-arrival times (rate $\lambda$): $X_i := T_i - T_{i-1}$

- Counting Process: Number of packets $N_t$ until time $t$
  - $Pr(N_t = n) = (\lambda t)^n \exp(-\lambda t) / n!$

- Properties:
  - **Merging**: arrivals from two independent Poisson processes with rate $\lambda_1$ and $\lambda_2$ \Rightarrow Poisson process with rate $(\lambda_1 + \lambda_2)$
  - **Thinning**: arrivals from a Poisson process of rate $\lambda$ are discarded independently with probability $p$ \Rightarrow Poisson process with rate $(1-p)\lambda$
  - **Central Limit Theorem**: superposition of $n$ independent processes results in the limit $n \to \infty$ in a Poisson process (under certain conditions on the processes)
Discrete Time Markov Processes

- Definition
  - State-Space:
    finite or countable infinite, \( E=\{1,2,...,N\} \) (\( N=\infty \) also allowed)
  - Transition probabilities: \( p_{jk}=Pr(\text{transition from state } j \text{ to state } k) \)
  - \( X_i = \text{RV indicating the state of the Markov process in step } i \)
  - ‘Markov Property’: State in step \( i \) only depends on state in step \( i-1 \)
    - \( \Pr(X_i=s \mid X_{i-1}=s_{i-1},X_{i-2}=s_{i-2},...,X_0=s_0) = \Pr(X_i=s \mid X_{i-1}=s_{i-1}) \)

- Computation of state probabilities
  - Initial state probabilities (Step 0): \( \pi_0 \)
  - Probability of state-sequence \( s_0,s_1,...,s_i \): \( \Pr(X_0=s_0,X_1=s_1,...,X_i=s_i) = ... \)
  - \( \Pr(X_i=k) = \sum_j [\Pr(X_{i-1}=j)*p_{jk}] \)
  - \( \pi_i = \pi_{i-1}P \)
  - State-holding time: geometric with parameter \( p_{ii} \)
Discrete Time Markov Processes (cntd.)

- Properties
  - Homogeneity: $P$ independent of step $i$
  - Irreducibility: each state is reachable from any other state (in potentially multiple steps)
  - Transient states, positive recurrent states
  - Periodicity

- Steady-state probabilities
  - $\pi = \lim_{i \to \infty} \pi_i$
  - Limit exists and is independent of $\pi_0$ if Markov chain irreducible and aperiodic
  - Aperiodic & positive recurrent = ergodic
    $\Rightarrow \pi$ is probability distribution
Continuous Time Markov Processes

• Defined by
  • State-Space: finite or countable infinite, \( E=\{0,1,2,\ldots,K\} \) (\( K=\infty \) also allowed)
  • Transition rates: \( \mu_{jk} \)
    • Holding time in state \( j \): exponential with rate \( \sum_{k \neq j} \mu_{jk} \)
    • Transition probability from state \( j \) to \( k \): \( \mu_{jk} / \sum_{l \neq j} \mu_{jl} =: \mu_{jk} / \mu_j \)

• \( X_t = \text{RV indicating the current state at time } t; \quad \pi_i(t) := \Pr(X_t=i) \)
• ’Markov Property’: transitions do not depend on history but only on current state \( t_0 < t_1 < \ldots < t_n, \quad \forall i_0, i_1, \ldots, i_n, i, j \in E \)

\[
P\{X(t_{n+1}) = j \mid X(t_n) = i_n, \ldots, X(t_0) = i_0\} = 
\]

• Computation
  • Chapman Kolmogorov Equations: \( d\pi_i(t)/dt = -\mu_i \pi_i(t) + \sum_{j \neq i} \mu_{ji} \pi_j(t) \)
  • Flow-balance equations, steady-state: \( \mu_i \pi_i(t) = \sum_{j \neq i} \mu_{ji} \pi_j(t) \)
  • Here: restriction to irreducible, homogeneous processes

Exercise 2: Derive the C-K equations! (20min)
Example: general Birth-Death Processes

- Steady-State Probabilities:
  \[ \pi_i := \Pr(Q=i) = \pi_0 \prod_{k=0}^{i-1} \frac{\lambda_k}{\mu_k} / \prod_{k=1}^i \mu_k \]
  \[ \pi_0 := (1 + \sum_{i=1} (\prod_{k=0}^{i-1} \lambda_k / \prod_{k=1}^i \mu_k))^{-1} \]

- Models in this class, e.g.
  - M/M/1/B
  - M/M/C, M/M/C/C
  - Load-dependent services

Exercise 3: multiple server queue, see last slide (15min)
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  • Kendall Notation,
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  • Erlang-B formula

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• Summary/Exercises

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The circuit switched scenario

- K channels
- Users allocate one channel per call for certain call duration
- If all channels are allocated additional starting calls are blocked
- How many channels are necessary to achieve a call certain maximal blocking probability?

Common Model Assumptions:
- Calls are arriving according to a Poission Process (justified for large user population, limit theorems for stochastic processes) with rate $\lambda$
- Call durations are exponentially distributed with mean $T$ (okay for voice calls)
Queueing Models: Kendall Notation

X/Y/C/[B] Queues (example: M/M/1, GI/M/2/10, M/M/10/10, ...)

- **X**: Specifies Arrival Process
  - **M**=Markovian → Poisson
  - **GI**=General Independent → iid

- **Y**: Specifies Service Process (M,G(I),...)

- **C**: Number of Servers

- **B**: size of finite waiting room (buffer) [also counting the packet in service]
  - If not specified: B=∞

- **Often also specified**: service discipline
  - **FIFO**: First-In-First-Out (default)
  - Processor Sharing: PS
  - Last-in-first-out LIFO (preemptive or non-preemptive)
  - Earliest Deadline First (EDF), etc.

Scope here: Point-process models as opposed to fluid-flow queues
**M/M/1 queue**

- Poisson arrival of packets (first 'M' → Markovian) with rate $\lambda$.
- Exponentially distributed service times of rate $\mu$ (second 'M')
- Single Server (1)
- FIFO service discipline
- $Q_t = \text{Number of packets in system is continuous-time Markov Process}$

'Derived' Parameter:
- Utilization, $\rho = \frac{\lambda}{\mu}$ : if $\rho \geq 1$, instable case (no steady-state q.l.d)

Performance Parameters
- Queue-length distribution: $\pi(t)$, steady-state limit: $\pi = \lim \pi(t)$ (if $\rho < 1$)
- Queue-length that an arriving customer sees
- Waiting/System time distribution
M/M/1 queue: Performance

Birth-Death Process

- Probability of idle server: \( \pi_0 = (1-\rho) \)
- Probability of \( i \) packets in queue: \( \pi_i := \Pr(Q=i) = (1-\rho) \cdot \rho^i \), where \( \rho = \frac{\lambda}{\mu} < 1 \)
- Average Queue-length: \( E\{Q\} = \frac{\rho}{1-\rho} \)
- Average Delay (System Time): \( E\{S\} = E\{Q\}/\lambda = 1/(\mu-\lambda) \)
Packet-based link model: M/M/1/K queue

• Assumptions
  • Poisson arrival of packets with rate $\lambda$
  • Exponentially distributed service times of rate $\mu$
  • Single Server
  • Finite waiting room (buffer) for $K$ packets

• Suitable e.g. for modeling ’bottleneck’ link in packet-based wireless networks
  • [Full network models: see Markov Models II lecture]

• Additional performance metric
  • Buffer-Overflow Probability for level $K = \Pr(\text{arriving customers sees buffer occupancy } B \text{ or higher})$
M/M/1/K queue: Performance

From Birth-Death Process Theory:

• Probability of i packets in queue
  \[ \pi_i := \Pr(Q=i) = \frac{(1-\rho)}{(1- \rho^{K+1})} \cdot \rho^i, \text{ where } \rho = \frac{\lambda}{\mu} \neq 1, \ i=0,...,K \]

• Probability of packet loss:
  \[ P_{\text{loss}} = \pi_K = \frac{(1-\rho)}{(1- \rho^{K+1})} \cdot \rho^K \]

• Average Delay:
  \[ \hat{D} = \frac{1}{[\lambda \ (1-p_K)]} \cdot \rho/(1- \rho^{K+1}) \cdot [(1- \rho^K)/ (1-\rho) - K \rho^K ] \]
M/M/K/K model, Erlang-B formula

Finite Birth-Death Process:

- Probability of i calls active
  \[ \pi_i := \Pr(n=i) = \pi_0 \left( \frac{\lambda T}{i!} \right)^i, \quad i=1,\ldots,K \]
  where \( \pi_0 = \frac{1}{\sum (\lambda T)^i / i!} \) (sum taken over \( i=0 \) to \( K \))

- Probability of blocked call:
  \[ P_{(\text{Blocking})} = \pi_K = \pi_0 \frac{(\lambda T)^K}{K!} \]
  [also known as Erlang-B formula]
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- **Summary/Exercises**
Independent errors

Simplest error model: every bit (L2 frame, packet) independently experiences the same error probability $p_b$ ($p_f$, $p_p$)
- Number of erroneous bits: Bernoulli distributed
- Probability of correct frame transmission for frame-size $s$
  - Without FEC
  - With FEC (assuming that $n$ bit errors can be corrected)
- Assuming simple ARQ (repeat until successful)
  - Number of needed attempts geometrically distributed with parameter $p_f$
  - Corresponds to geometrically distributed frame-delay in slotted systems
- Properties of error sequence ($E_i$ 0/1 sequence, $E_i=1$ corresponds to transmission error)
  - Steady-state error probability
  - Error burst length: geometric
Correlated Errors

For correlated channel errors: Markov modulated error model:
• While current state $i$: packets corrupted with probability $\varepsilon_i$
• After each packet: state transition with probability $p_{ik}$

Easiest Case:
• 2-state model: ‘Gilbert-Elliot‘
• Long-term packet error rate

$$\gamma = \pi_0 \ast \varepsilon_0 + \pi_1 \ast \varepsilon_1$$

$$\pi_1 = 1 - \pi_0 = 1 - \frac{P_{10}}{P_{01} + P_{10}} = \frac{P_{01}}{P_{01} + P_{10}}$$

• Correlation Properties $\Rightarrow$ Exercise 4
• Error burst-length $\Rightarrow$ Exercise 4

Such models are special cases of so-called Hidden Markov Models
Hidden Markov Models (HMMs): Definition

- Main property
  - In each state \( s \in E \), an 'observation symbol' from some alphabet \( V \) is generated probabilistically
  - The underlying state cannot be observed, only the sequence \( O = [O_1, O_2, ..., O_T] \) of generated symbols

- HMM = \(<E, V, \pi_1, P, B>\)
  - \( E \): state-space (discrete, finite/infinite), \( E = \{1, 2, ..., N\} \)
  - \( V \): set of possible observation symbols (discrete for now), \( V = \{1, 2, ..., M\} \)
  - \( \pi_1 \): initial state probabilities at step 1
  - \( P \): \( N \times N \) matrix of state transition probabilities \( p_{ij} = \Pr(X_{k+1} = j \mid X_k = i) \)
  - \( B \): \( N \times M \) matrix of symbol generation probabilities: \( b_{ij} = \Pr(O_k = j \mid X_k = i) \)

- Note: Discrete time Markov model is special case of HMM, namely each column of \( B \) contains at most one non-zero element
Hidden Markov Models (HMMs): example

The observation sequence above can be produced by the following state sequences.

• 5 3 2 5 3 2
• 5 3 1 2 1 2
• 4 3 2 5 3 2
• 4 3 1 2 1 2
• 3 1 2 5 3 2

But which, and what is the probability of observing one of the sequences?
Hidden Markov Models (HMMs): Computations

• Given the parameters of the model, compute the probability of a particular output sequence, and the probabilities of the hidden state values given that output sequence. This problem is solved by the forward-backward algorithm.

• Given the parameters of the model, find the most likely sequence of hidden states that could have generated a given output sequence. This problem is solved by the Viterbi algorithm.

• Given an output sequence or a set of such sequences, find the most likely set of state transition and output probabilities. In other words, discover the parameters of the HMM given a dataset of sequences. This problem is solved by the Baum-Welch algorithm or the Baldi-Chauvin algorithm.
Hidden Markov Models (HMMs): Computations

Problem: Compute probability of observing a certain sequence \( o = [o_1, ..., o_T] \) in a given HMM.

- **First approach (‘brute-force’):**
  - Generate all possible state-sequences of length \( T \): \( q = [q_1, ..., q_T] \)
  - Sum up all \( \Pr(o | q) \) weigthed by \( \Pr(q) \) (total probabilities)
  - Advantage is that it is a simple approach
  - Disadvantage is the time complexity is \( O(T N^T) \)

- **More efficient approach: forward-backward procedure**
  - Given the observed sequence, moves forward through the sequence and calculates probabilities for next sequence element.
  - Once reached through the given observed sequence, the algorithm traverse backwards similar, performing similar steps and does smoothing as well
  - Advantage is the time complexity is good \( O(N^2 T) \)
  - Disadvantage is the complexity
Hidden Markov Models (HMMs): Computations

Problem: Compute probability of observing a certain sequence \( o = [o_1, ..., o_T] \) in a given HMM.

- **First (inefficient) approach (‘brute-force’):**
  - Generate all possible state-sequences of length \( T \): \( q = [q_1, ..., q_T] \)
  - Sum up all \( Pr(o| q) \) weighted by \( Pr(q) \) (total probabilities)
  - Problem: Number of paths grows exponentially as \( N^T \)

- **More efficient (quadratic in \( N \)) approach: forward procedure**
  - Iterative method computing probabilities for pre-fixes of the observation sequence:
    \( \alpha_t := [Pr(O_1=o_1,...,O_t=o_t, X_t=1), ..., Pr(O_1=o_1,...,O_t=o_t, X_t=N)] \)
  - At step \( t=1 \):
    \( \alpha_1(i) = Pr(O_1=o_1, X_1=1) = \pi_1(i) b_{1,o_1} \) [Matlab Notation: \( \alpha_1 = \pi_1 .* B(:, o_1) \)]
  - \( t \rightarrow t+1 \) (\( t=1,2,...,T-1 \)):
    \[
    \alpha_{t+1}(i) = \left( \sum_{j \in E} \alpha_t(j) p_{ji} \right) Pr(O_{t+1}=o_{t+1} | X_{t+1}=i) \\
    \alpha_{t+1} = (\alpha_t P) .* B(:, o_{t+1})'
    \]
  - Finally: \( Pr(O=o) = \sum_{j \in E} \alpha_T(i) \)
  - Computation can be illustrated in Trellis structure

- **Similarly (and identifiers needed later): Backwards procedure**
  - \( \beta_t := [Pr(O_{t+1}=o_{t+1},...,O_T=o_T | X_T=1), ..., Pr(O_{t+1}=o_{t+1},...,O_T=o_T | X_T=N)] \)
  - \( \beta_T = 1 \) (vector with all elements = 1); \( \beta_t = (P * B(:, o_{t+1})) .* \beta_{t+1} \)
Queueing models with varying service quality

- Extension of M/M/1 queue model for channel-dependent service time, e.g. Gilbert-Elliot model with ARQ
  - Poor channel condition → service rate $\mu_0$
  - Good channel condition → service rate $\mu_1 > \mu_0$

- Two different cases
  - Channel changes only after successful transition
  - Channel may change at any moment in time

- Two dimensional Markov chain: $E=\{n=0,1,2,\ldots\} \times \{\text{‘G’, ‘P’}\}$, Transition Rates?

- Solutions: Truncate and compute $\pi$ numerically or use Quasi-Birth-Death Process solution (→Markov Models II, 9th semester)
Application to scheduling

Scenario:
- GPRS type of system (TDMA), slotted (discrete time)
- Two users in radio cell:
  - User 1 with good radio condition, service probability $p_1$
  - User 2 worse, service probability $p_2 < p_1$
- Each user has a downstream packet flow, assumed to be Bernoulli arrivals with parameter $a_i$
- Downstream packets are stored in two different queues

The scheduler needs to make a decision which queue to serve (non preemptive!)
- Round-robin
- Random
- Strict priority
- Compute buffer occupancy, avg. delay, etc.!
  - Discrete time Markov chain
  - State-space: $E = \{ (n_1, n_2) \} = G_0 \times P_0$
  - Extended state space (odd/even slot?) for RR
  - Transition probabilities ?

→ Exercise 5: Analysis of scheduling strategies
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Traffic models: General hierarchical models

Frequently used: Several levels with increasing granularity
- E.g. 3 levels: sessions, connections, packets
- Or: 5-level model:

![Diagram showing traffic models with 5 levels: Access Session Level (min – h), App. Session Level (min – h), Dialogue Level (sec – min), Connection Level (ms – Min.), Burst Level (μs – s), Packet Level (μs – s)]
Example: HTTP traffic model

‘Main’ objects contain zero or more embedded objects that the browser retrieves

→ Correlated requests for embedded objects within retrieval of main object

- Statistics:
  - Session arrivals: Renewal process (Poisson)
  - Idle time: heavy-tail

- # embedded objects: geometric (measurements e.g. mean 5)
- Object size: heavy-tailed
Daily Profiles/Non-stationarity

• Utilization of networks varies during the course of a day
  • mainly due to human behavior

• Depending on network types Fr-Sun may show different utilization profiles

• For network design, interest in ‘busy hour’ traffic

• Analysis of WLAN data (AAU) and GPRS data
  • indication of ‘stationarity periods’ of 4 hours resp. 1 hour
  • wrt. Poisson process assumptions for the user session arrivals

Figure source: M. Crovella
Burstiness

• **On packet-level (corresponding to shorter time-scales)**
  • Poisson assumption with constant rate for packet arrivals may be applicable only for highly aggregated traffic (core networks)
  • In access networks: typically varying arrival rates, e.g.
    • High data rates in ftp download but less activity between downloads
    • http: activities after mouse-clicks
    • Video streaming: high data rates in frame transmissions
    • Interactive Voice: talk and silent periods

• **Model types:**
  • Inhomogeneous Poisson process: time-varying rate $\lambda(t)$
  • ON/OFF models
  • Hierarchical models
Inhomogeneous Poisson Process

- Inhomogeneous Poisson process
  - time-varying rate $\lambda(t)$

- Approaches for simulation
  - Piece-wise constant approximation
  - Probabilistic discarding
    - Simulate Poisson process with rate $\lambda_m > \lambda(t)$ for all $t$
    - For event at time $t$, discard with probability $(\lambda_m - \lambda(t)) / \lambda_m$
More realistic models: ON/OFF Models

Parameters:

- N sources, each average rate $\kappa$
- During ON periods: peak-rate $\lambda_p$
- Mean duration of ON and OFF times

$\kappa = \lambda_p \frac{\text{ON}}{\text{ON} + \text{OFF}}$

'bursty' traffic, when $\lambda_p >> \kappa$
Performance Model: On-OFF/M/1 (single source)

- Two dimensional Markov chain: $E=\{n=0,1,2,\ldots\} \times \{\text{‘ON’, ’OFF’}\}$
- Transition Rates $\lambda$
- Solutions: Truncate and compute $\pi$ numerically or use Quasi-Birth-Death Process solution ($\Rightarrow$ Markov Models II, 9th semester)
Performance Impact: Multiplexed Case

- Here: case of two ON/OFF sources, mean packet delay
- For small utilization $<0.1$ (large $\nu$): less than factor of 2 compared to M/M/1 queue
- For larger utilization close to 1: Approx factor of 10 for the given parameters
- [Ignore curves labeled with $T>1$ for now]
Defining Burstiness for ON/OFF sources

- One possibility: Burstiness = Peak Rate of Source $\lambda_p$ / Average Rate of Source $\kappa$

- Transformed to range 0 (not bursty) to 1 (very bursty): $b := 1 - \kappa / \lambda_p$

- Investigating burstiness:
  - Vary $\lambda_p \geq \kappa$
  - Keep #packets per ON period $\rightarrow$ scale ON duration accordingly
  - Extreme Cases:
    - $b=0$ ($\lambda_p = \kappa$): Poisson arrivals, no burstiness
    - $b=1$ ($\lambda_p = \infty$): Bulk arrivals
  - Average Queue length: see dotted curve on the right (here low utilization $\rho = 5\%$)
Bulk Arrival Models

• Queue-length at arrival instances increases not only by 1, but by a Random Variable \( B \), the bulk-size

• Parameter set of model
  • Bulk arrival process, e.g. exponential with rate \( \lambda \)
  • Bulk-Size distribution: \( p_i \) (e.g. geometric)
  • Service rate (single packet)

• Steady-state solution for mean system time
  [Chaudhry & Templeton 83]:
  \[
  E\{S\} = \frac{[E\{B\} + E\{B^2\}]}{2 E\{B\} \mu (1-\rho)}
  \]
Non-exponential models

- Long-range dependent and self-similar properties in network traffic
- Indications for the need to include heavy-tail distributions (see e.g. Web-file sizes below, right graph)
- However, Markovian models (with certain structure) can mimic such distributions

Source: Barford et al., *World Wide Web, 1999*
Content

- **Motivation & Background**
  - Performance Analysis in Wireless Settings
  - Review of Basic Concepts: Random Variables, Exponential Distributions, Stochastic Processes
  - Birth-Death Processes, Discrete and Continuous time Markov chains

- **M/M/k/k type queueing models (revision)**
  - Kendall Notation,
  - M/M/k/k type of queue models, steady-state solution
  - Erlang-B formula

- **Wireless Link Models**
  - IID errors, correlated errors, Gilbert Elliot Model
  - Application: Queuing model with varying channel
  - Multiple queues, multi-class models

- **Simple Traffic Models**
  - Traffic measurements and characteristics
  - ON/OFF Markov models

- **Summary/Exercises**
1. **Exponential Distribution**, see Slide 11
2. **Continuous Time Markov Chains**, Chapman Kolmogorov Equation, see Slide 16
3. **Multi-server queue**: One router in a network (to be modelled as M/M/1 queue) is apparently creating large delays. The admin decides to add a second routing processor to it. In principle, the admin now has the choice to upgrade to an M/M/2 system, but he could also configure separate queues for the new routing module (packets will be randomly directed to one of the two queues); alternatively, he could replace the routing processor with another one that is twice as fast. Which solution is performance-wise better? Plot the ratio of the mean queue-lengths for varying arrival rate lambda.
4. **Correlated Errors**: Given is a 2-state Markov modulated bit error model with parameters \( P = [0.4, 0.6; 0.2, 0.8], \epsilon_1 = 0.5, \epsilon_2 = 0. \)
   - Compute the long-term average bit-error rate.
   - Write a Matlab program to simulate the error sequence \( E_i \). Compute the coefficient of correlation \( r(k) \) and a histogram of error burst lengths. Compare the latter with a geometric distribution. Furthermore, compare with an independent error model.
5. **Inhomogeneous Poisson Process**: Write a MATLAB program that creates samples from a time-varying Poisson process with \( \lambda(t) = 1 + \sin(2\pi t/100) \). Estimate the mean and the variance of this sample series (inter-event times) using \( n = 1e4 \) samples. Compute an estimate of the autocorrelation.