Fuzzy Sliding Mode Controller Design for Spacecraft Attitude Tracking in Terms of Quaternion

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Abstract: In this paper a fuzzy sliding mode control (FSMC) for spacecraft attitude tracking in terms of quaternion is designed. For this purpose, at first, the spacecraft dynamic and kinematic equations based on quaternion are described. Then, the design procedure of related variable structure controller is presented. To prove the system stability in sliding mode and also to guarantee the convergence of quaternion vectors to desired state, a lemma is presented which is proved by using Lyapunov direct method. In continuation, in order to improve the controller performance, tuning of the sliding mode gain is accomplished by using fuzzy logic approach. Finally, the proposed controller design is simulated on an assumed case study with uncertain parameters and external disturbances. The simulation results are provided to show the effectiveness of the method and its robustness to parameter uncertainties and disturbances.

Key Words: Sliding Mode Control, Fuzzy Logic, Lyapunov Stability, Spacecraft Attitude Tracking, Quaternion.

1 INTRODUCTION

Spacecraft attitude control is one of the most important subjects in spacecraft control issues for which various linear and nonlinear control methods have been explained in different texts and articles [1-4]. In order to derive the spacecraft model for attitude control, Euler angles and quaternions can be used which leads to a nonlinear dynamics [1]. Because of the presence of structural uncertainty and disturbances in spacecraft attitude system, sliding mode control (SMC) [5, 6] is a suitable control method which considered as a variable structure control approach and is robust against the model uncertainty and disturbances [7]. In order to fast hitting at reaching mode, using of a high gain control is advisable but this lead to chattering increment. Fuzzy logic [8-10] can be used to adapt the sliding gain to avoid chattering. Therefore, the overall controller can utilize both advantages of sliding control and fuzzy logic. Generally, this combination of sliding mode control and fuzzy logic is called fuzzy sliding mode control [11-13]. In [14], an adaptive fuzzy sliding mode control is applied to the attitude stabilization of flexible satellite. In [15], position and attitude control of spacecraft by sliding mode control is explained too, but it doesn’t consider the model uncertainty and disturbance. In other study, [16], by using Rodrigues parameters, a sliding mode controller design for attitude tracking in the presence of structural uncertainty and disturbances is performed. A decoupled sliding mode control combined with observer by using quaternion parameters in spacecraft modeling has been presented in [17], but model uncertainty has not been considered in this design.

In this paper, to avoid some probable singularities and complexities which may arise in the case of using Euler angles, the quaternions are used and a fuzzy sliding mode controller for spacecraft attitude tracking in terms of quaternion is designed. This controller is robust against the parameters uncertainties and presence of external disturbances. Furthermore, the states are assumed to be accessible.

For this purpose, in section 2, the system model is introduced and the spacecraft dynamics is derived. Then, sliding mode controller design for the achieved model is presented in section 3 and the stability analysis is also provided in a lemma which is proved by using the Lyapunov direct method. In section 4, the fuzzy sliding mode controller is described. In order to show the performance of the proposed control law, it is simulated for a numerical example of an assumed multiaxial attitude tracking in section 5 and the simulation results and comparison of FSMC approach with SMC method are provided.

2 SPACECRAFT MODEL DESCRIPTION

As a pre-step of sliding mode controller design for spacecraft attitude, the governing dynamical model of the system is required that is given in this section. By considering the spacecraft as a rigid body, its dynamics and kinematics equations in terms of quaternions can be described as follow:

\[
\dot{q} = \frac{1}{2} ( q_4 I_3 + q^\times ) \omega \\
\dot{q}_4 = -\frac{1}{2} \omega^T q \\
J \dot{\omega} = -\Omega I \omega + u , \quad \Omega = \omega^\times 
\]

Where \((q, q_4) \in \mathbb{R}^3 \times \mathbb{R}\) is quaternion and \(q = [q_1 \ q_2 \ q_3]^T\) is its vector part, \(\omega\) is the spacecraft angular velocity vector, and \(u\) represents the control torque. Also, \(I_3\) is the \(3 \times 3\) identity matrix, and \(J\) is known as the inertia matrix which is inherently symmetric and
positive definite by choosing the body fixed frame appropriately. The matrix \( J \) has constant components, thus its time derivative is zero and its uncertainty is assumed bounded. It should be noted that \( a^\times \) operator denotes the skew symmetric matrix operation on vector \( a = [a_1 \ a_2 \ a_3]^T \) that is given by following relation:
\[
a^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}
\]
The matrix \( M \) is also defined as follows which will be used in continuation:
\[
M = q_4 I_3 + q^\times
\] (4)
To avoid the singularity in \( M^{-1} \) that occurs at \( q_4 = 0 \) the workspace is restricted as follows [2]:
\[
\| q \| \leq \beta < 1 ; \ q_4 \geq \sqrt{1 - \beta^2}
\] (5)
Where, \( \| \cdot \| \) denotes the norm of the vector part of the quaternion. This limitation is equivalent to restricting the range of equivalent Euler angle rotation between \( \pm \cos^{-1}(\sqrt{1 - \beta^2}) \).
Now the decoupling tracking control objective can be defined as following relation:
\[
\lim_{t \to \infty} \| q_d(t) - q(t) \| = 0
\] (6)
where, \( q_d(t) \) is the desired quaternion tracking profile, and the quaternion error components \( q_e = q_d(t) - q(t) \) approach zero independently. It should be noted that it is sufficient to track the reduced quaternion [17].

3 SLIDING MODE CONTROLLER DESIGN
In this section, sliding mode controller design for multiaxial attitude tracking of the spacecraft that is modeled by (1), (2) and (3) is presented. In the design procedure, the required feedback signals \( \omega \) and \( q \) are assumed to be measurable via available sensors. Moreover, to discuss the robustness of the designed controller, it is allowed the dynamic equation (3) to possess bounded input disturbance \( d \) and parameter uncertainty \( \Delta J \). Thus, the dynamic equation is rewritten as follows:
\[
J \dot{\omega} = -\Omega J \omega + u + d
\] (7)
Where, \( J = J_0 + \Delta J \) and \( J_0 \) represents the nominal part of \( J \). It should be noted that the inertia matrix \( J \) is assumed to be time invariant and its variation \( \Delta J \) is for example due to payload changes of the spacecraft. The objective of the tracking control is to guide the spacecraft such that the quaternion vector \( q(t) \) is controlled to follow the given reference vector \( q_d(t) \). In this paper this is accomplished by a sliding mode controller. The design procedure of sliding mode control generally has two steps. In the first step, sliding vector is chosen such that in the sliding mode the control objectives are achieved. In the second step, the control law is designed such that the reaching conditions to sliding surface are satisfied and keep the system in this mode. For this purpose, the sliding surface equation is defined as follows:
\[
s = \dot{q}_e + Kq_e = 0
\] (8)
Where \( q_e = q_d(t) - q(t) \) and \( K \) is a diagonal positive definite matrix. The form of the sliding surface dictates that constant angle maneuvers are restricted to rotations about the eigenaxes.
The control strategy that guaranties the system’s motion to the surface \( s = 0 \) in a limited reaching time and keep the system’s motion on \( s = 0 \) thereafter, is given in the following lemma.

Lemma 1. The spacecraft dynamics described by the equations (1) – (5) is given. The control input of the relation (9) in which \( u_{eq} \) is as the relation (10), causes the convergence of the state vector to the sliding surface and guaranties the system stability.
\[
u = u_{eq} + 2L \text{sgn}(s)
\] (9)
\[
u_{eq} = -J_0 M^{-1} \dot{q}_d \omega + \Omega J_0 \omega + 2J_0 M^{-1} \dot{q}_d
\] (10)
where \( \text{sgn} \) denotes sign function.
Proof. The dynamical equations of the system can be rewritten in \( s \)-space as follows:
\[
s = \left[ \dot{q}_e - \frac{1}{2} \dot{q}_e \omega \omega + \frac{1}{2} MJ^{-1} \Omega J \omega - \frac{1}{2} MJ^{-1} u + Kq_e \right]
\] (11)
In order to provide asymptotic stability of \( s = 0 \), the following candidate of Lyapunov function is selected:
\[
V = \frac{1}{2} s^T J M^{-1} s > 0
\] (12)
where \( JM^{-1} \) is a symmetric positive definite matrix within the defined workspace. The derivative of \( V \) is calculated as follows:
\[
\dot{V} = s^T \left[ -\frac{1}{2} JM^{-1} \dot{q}_d \omega + \frac{1}{2} \Omega J \omega + JM^{-1} \dot{q}_d + JM^{-1} Kq_e - \frac{1}{2} u + JM^{-1} s + JM^{-1} s \right]
\] (13)
In the above equation, \( J = J_0 + \Delta J \) and \( \dot{J} = 0 \).
Now, by considering the control torque as (9) and \( u_{eq} \) of relation (10), the derivative of the Lyapunov function can be calculated as follows:
\[
\dot{V} = s^T \left[ \delta - L \text{sgn}(s) \right] = \sum_{i=1}^{3} s_i \left( \delta_i - L_i \text{sgn}(s_i) \right)
\]
\[
= \sum_{i=1}^{3} L_i \left[ s_i \left[ \frac{1}{L_i} \delta_i - \text{sgn}(s_i) \right] \right]
\] (14)
Where
\[
\delta = \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 \end{bmatrix}^T \\
= -\Delta J M^{-1} \dot{q}_d \omega + \Omega \Delta J \omega + 2 \Delta J M^{-1} \ddot{q}_d \\
+ 2 \Delta J M^{-1} K_\delta e + 2 \Delta J M^{-1} s + d 
\]

Since the external disturbances \(d\) and uncertain parameter \(\Delta J\) are both bounded, the upper bound of \(|\delta_i|\) can be found and denoted as \(\delta_i^{max}\). Therefore, if we choose equality (16), the relation (14) becomes as (17):

\[
L_i = \delta_i^{max}, \quad i = 1, 2, 3 
\]

\[
\dot{V} = -\sum_{i=1}^{3} \delta_i^{max} |s_i| \left(1 - \frac{\delta_i}{\delta_i^{max}} \text{sgn}(s_i)\right) < 0 \quad \text{for} \quad s \neq 0 
\]

This implies that \(V\) is a Lyapunov function and the stability of the sliding mode, \(s = 0\) is guaranteed.

However, the control law \(u\) in (9) always suffers from the chattering problem due to the sign function \(\text{sgn}(s)\). To avoid this, one way is to replace the sign function by the following saturation function:

\[
\text{sat}(s, \varphi) = \begin{cases} 
1 & s > \varphi \\
\varphi & |s| < \varphi \\
-1 & s < -\varphi 
\end{cases} 
\]

The system is now no longer forced to stay in the sliding mode but is constrained within the sliding layer \(|s_i| \leq \varphi\).

The cost of such substitution is a reduction in the accuracy of the desired performance. One other idea to overcome the chattering problem is the fuzzy tuning of the sliding controller which is described in the next section.

### 4 FUZZY SLIDING MODE CONTROLLER DESIGN

Now, the sliding gain \(L\) is adjusted by a fuzzy method such that the gain is high when the states are far from the sliding surface and it is reducing while the states are approaching to the sliding surface. Thus, the reaching phase performance is improved by starting with such a high gain. On the other hand, the chattering is avoided by using of this fuzzy gain tuning.

For this purpose, a double-input single-output fuzzy system is designed with Mamdani’s fuzzy implication, center of area defuzzification method and fuzzy rule base as presented in table 1 [18].

<table>
<thead>
<tr>
<th>(\dot{s})</th>
<th>NB</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>B</td>
<td>B</td>
<td>M</td>
<td>S</td>
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</tbody>
</table>

The abbreviations used in above table are NB: Negative Big; NS: Negative Small; Z: Zero; PS: Positive Small; PB: Positive Big and M: Medium. For example, one rule is: if \(s\) is PS and \(\dot{s}\) is Z then \(L_{fuzz}\) is M. Parameters \(s\) and \(\dot{s}\) can be considered as distance and approaching velocity between trajectory and surface, respectively. These parameters are assumed as inputs of the fuzzy system with membership functions illustrated in figure 1. The output of the fuzzy system is \(L_{fuzz}\) and related membership function is shown in figure 2. So the gain \(L\) is as follows:

\[
L = N \cdot L_{fuzz} 
\]
5 NUMERICAL EXAMPLE AND SIMULATION RESULTS

In this section, in order to evaluate the effectiveness of the proposed control system in previous section, the following multiaxial attitude tracking case study is assumed. Then by considering parameter uncertainties and bounded external disturbances, the proposed methodology for designing free-chattering fuzzy sliding mode controller is performed.

5.1 Case Study

It is assumed that the under study spacecraft has the following nominal diagonal inertia matrix as relation (20) and parameter uncertainties are as relation (21):

\[
J_0 = \begin{bmatrix}
78.212 & 0 & 0 \\
0 & 86.067 & 0 \\
0 & 0 & 114.562
\end{bmatrix} \tag{20}
\]

\[
\Delta J = \begin{bmatrix}
\Delta J_{11} & 0 & 0 \\
0 & \Delta J_{22} & 0 \\
0 & 0 & \Delta J_{33}
\end{bmatrix} \tag{21}
\]

\[
|\Delta J_{11}| < 8.7212 (10\% of \ J_{011})
\]

\[
|\Delta J_{22}| < 8.6067 (10\% of \ J_{022})
\]

\[
|\Delta J_{33}| < 11.4562 (10\% of \ J_{033})
\]

It should be noted that the general form of inertia matrix is not diagonal but for case of an orbiting satellite, without the loss of generality, the inertia matrix can be transformed to diagonal form by a transformation matrix [1]. The diagonal terms of the diagonal inertia matrix are known as the principal moments of inertia, and the corresponding new axes are called principal axes.

In the operational mode of spacecraft, usually some kinds of disturbances are occurred in the attitude control system due to different sources such as earth’s magnetic field, reaction forces produced by expulsion of gas or ion particles and solar radiation pressure on spacecraft surfaces. To evaluate the performance of the proposed design against the disturbance, in our simulation, the following model of disturbance which has relatively large magnitude is considered:

\[
d = \begin{bmatrix}
0.5 \sin t \\
0.5 \sin t \\
0.5 \sin t
\end{bmatrix} \tag{22}
\]

Additionally, in order to control the attitude of the above spacecraft under such disturbance, it is assumed that the related actuator can provide sufficient control effort.

5.2 Simulation Results

In order to simulate the proposed controller on the above mentioned case study, the initial conditions are assumed as \(\omega_0 = 0\), \(q_0 = [0 \ 0.5 \ 0.5]^T\) and \(q_w = 0.7071\) with restricted workspace by defining \(\beta^2 = 0.75\).

The desired quaternion profile for multiaxial tracking is given as follows:

\[
q_d = \begin{bmatrix}
0.5 \cos(\pi t/12.5) \\
0.5 \sin(\pi t/12.5) \\
-0.5 \sin(\pi t/12.5)
\end{bmatrix} \tag{23}
\]

For comparing the performance of the proposed FSMC method with respect to the conventional SMC one, the simulation results of both approaches are provided in this subsection.

For SMC case, the required parameters are selected as \(K = 0.8I_3\) and \(\phi = 0.1\), then from (15) and (16), parameter \(L\) is determined by upper bounds of \(\delta\) as:

\[
L = \Delta J_{\max} [M^{-1}\dot{q}_d\dot{q}_d] + [2\Delta J_{\max} + 2\Delta J_{\max} K \dot{q}_d] + 2\Delta J_{\max} [M^{-1}K\dot{q}_d] + \delta_{\max} + \delta_1 \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T \tag{24}
\]

The tracking error vector in the case of using SMC has been shown in figure 3. In this point of view, the performance is acceptable. The control torque vector of this case is also shown in figure 4. Figure 5 demonstrates the sliding vector "s". As it can be seen from the above mentioned figures, the reaching time is about 28 seconds. Moreover, the value of \(s\) has some variations around zero.
For the case of using FSMC design, the design parameters "K" and "ϕ" are selected as previous case but "L" is selected from (19) where \( N = 30 \). The tracking error vector of this case has been shown in figure 6 which is well acceptable. The control torque vector is also illustrated in figure 7. The time behaviors of attitude and reference vectors are shown in figure 8. Finally, figure 9 shows the sliding vector "s". As it can be seen through these figures, the reaching time is decreased to 8 second in the case of using FSMC design which is more desirable with respect to the conventional SMC design. It should also be noted that, in this approach using higher control gain is allowable while fuzzy gain tuning avoids chattering problem. In fact, the cost of attenuating the chattering is the increment of the control effort but it is still in an available practical range.

**CONCLUSION**

In this paper, a fuzzy sliding mode control has been exerted on spacecraft attitude tracking issue. For this purpose, the dynamical description of the spacecraft in terms of quaternion has been considered. Then, sliding mode theory has been used to design a robust attitude controller. The robust performance and stability guarantee of the controlled system are proved by Lyapunov direct method. In the second stage, with the purpose of improving the reaching phase, fuzzy logic has been applied to tuning the SMC gain. The resulted FSMC is a free-chattering sliding mode controller. Finally, the simulation results show acceptable performance and robustness of the SMC method in the presence of disturbance and parameter uncertainty and also indicate the improvement of sliding mode controller by using fuzzy approach in the controller gain tuning.

**REFERENCES**


